International Journal of scientific research and management (IJSRM)
\||Volume||3||Issue||5||Pages|| 2812-2815||2015||
Website: www.ijsrm.in ISSN (e): 2321-3418

Step Function Model For Forecasting Project Cash Flow

Vivek C. Datey

Research Scholar pursuing Doctoral studies (Ph.D) at the Global Business School and Research Centre, Tathwade, Pune - 411033

Of Padmashree D. Y. Patil Vidyapeeth, Pune, India

Abstract:

Cash flow forecasting for projects has assumed a special interest among researchers for the past 30 years or more. Due to their peculiar execution cycle, projects follow a sigmoid or 'S' curve pattern in their progress from start to completion. Many researchers have developed different empirical and mathematical models and also computer programs that forecast the cash flows by using a variety of characteristic 'S' curves for different project activities and incorporating related time lags between expenditure and cash flow. However, it must be noted that the cash flows do not occur on a continuous basis but are discrete and occur at specific instances. Therefore mathematically, cash flows are not continuous functions but are step functions and any projection made on the assumption of them being continuous functions would be inaccurate and therefore unreliable in practice. While modeling cash flows as continuous functions may be a mathematically reasonable approximation, it tends to average out the values occurring at different times by a smooth curve and may hide the likely cash deficits that may occur between the two periods. This difference cannot be ignored since cash flow shortfall, even for an extremely short duration of a few days, can have acute real consequences on a project. The paper critically examines various mathematical approaches by different researchers, and suggests a novel approach for developing a step function model for discrete cash flow forecasting for projects. This lays a foundation for further work that needs to be done to develop the actual equations based on project data and validate the approach for practical use.

Key Words: Project Cash Flow, Project Working Capital

INTRODUCTION

Cash flow forecasting for projects has assumed a special interest among researchers for the past 30 years or more. Due to their peculiar execution cycle, projects follow a sigmoid or 'S' curve pattern in their progress from start to completion as compared to a steady level maintained by continuously running manufacturing or trading activity. As per the standard accounting practice of booking the cost on accrual basis, incurrence of cost on a project that is directly linked to the physical progress of project activities is also proportional to the physical progress and therefore follow the 'S' curve.

It has also been established that the cash flows relating to the activities have different timings than the incurrence of costs. For a contractor working on a project, the cost is incurred on a continuous and regular basis while the payment for the work completed on a periodic basis is received from the client after certain time lag as per the terms of the contract. Similarly, payments for the materials, equipment and sub-contractor also occur with a time lag except for the advances given initially. Therefore it is generally assumed that the cash flows – inflows and outflows – follow a pattern of 'S' curve albeit with a time lag from the 'S' curve for project expenditure.

Many researchers have developed different empirical and mathematical models and also computer programs that forecast the cash flows by using a variety of characteristic 'S' curves for different project activities and incorporating related time lags between expenditure and cash flow. The statistical tests on the overall basis show an acceptable level of reliability of these models for forecasting of project cash flows.

However, it must be noted that the cash flows do not occur on a continuous basis but are discrete and occur at specific instances. Therefore mathematically, cash flows are not continuous functions but are step functions and any projection made on the assumption of them being continuous functions would be inaccurate and therefore unreliable in practice. While modeling cash flows as continuous functions may be a mathematically reasonable approximation, it tends to average out the values occurring at different times by a smooth curve and may hide the likely cash deficits that may occur between the two periods. This difference cannot be ignored since cash flow shortfall, even for an extremely short duration of a few days, can have acute real consequences on a project.

The paper critically examines various mathematical approaches by different researchers, and suggests a novel approach for developing a step function model for discrete cash flow forecasting for projects.

LITERATURE REVIEW

Kenley [1] developed a 'Logit net cash flow' model using actual component inward and outward cash flow data in order to form a residual cash flow model. While developing the model, he had recognized the stepped nature of cash flows and although it was quite suitable for representing real data, he felt that it could not be represented for any form of modeling, forecasting or simulation. He therefore smoothened the steps to remove the jagged nature of the net cash flow. Kenley's model was based on empirical project data that used the actual component inward and outward cash flow data in order to form the residual model. This model had adopted the Logit gross cash flow model by Kenley and Wilson [2], to model the component curves. When combined, the component curves form a net cash flow model reflecting the true net cash position for each project. The final model is still based on smoothened out ‘S’ curves instead of step curves.

Park, Han & Russell [3] attempted to estimate cash flows by categorization of different project activities on the basis of time lags involved between their physical occurrence of different stages and the related cash flows and identifying the characteristic movement of cumulative cash flow against time of the activity. Various cost categories were identified for material, labour, equipment, sub-contracts and overheads and depreciation since each of these categories had a different time lag and a cash flow profile. Typical ‘cash flow curves’ were developed for each of the different categories based on moving weights system and were converted to mathematical equations for cash inflows and outflows. The overall cash flows were worked out by consolidating these equations to the project schedule.

Jarrah, Kulkarni & O’Connor [4] collected actual cash flow data in form of monthly account summary reports for various projects under Texas Department of Transportation. The sample consisted of different category of projects such as construction and replacement of bridges, new non-freeways, road overlay and rehabilitation of existing roads, landscape scenic enhancements, widening of freeways etc. Projects were further classified in different cost ranges. Based on the scatter chart of payments against time for different projects in a given category, a fourth degree polynomial regression analysis was used to obtain the cash flow curves that turned out to be characteristic ‘S’ shaped for most of the projects. Although statistical significance could not be proved due to limited availability of data, a feasible approach for cash flow prediction was established. Since the data was related to payments to the contractor, only the cash inflow curves could be established. Extending the same methodology to data for contractor’s cash outflows, there appears to be a possibility of working out net working capital gap.

A model developed by Görög [5] suggested a set of new measurements and indicators in line with the ‘earned value’ measurements and indicators, for possible integration of both systems. Therefore similar to the earned value measurements such as Budgeted Cost of Work Scheduled (BCWS), Budgeted Cost of Work Performed (BCWP) and the Actual Cost of Work Performed (ACWP) for working out the Cost Performance and Schedule Performance Indices (CPI and SPI); the new set of measurements and indicators was based on the ‘Price Value’ and ‘Invoice Value’ of the contracted work. This could therefore forecast the difference between price to be received by the contractor from client and the cost to be expended by the contractor for the amount of work carried out at any point of time. Hence the differential indicated expected margin based on project status. Since the methodology explained in the model worked out the values on accrual basis and did not recognize the time difference in their occurrence and therefore could not be found useful to forecast and monitor cash flows in financial sense.

There are more models proposed and developed by different researchers to forecast contractor’s project cash flows, such as Garden II and Creese [6], Khosrowshahi [7], and Cheng and Roy [8] However, these models also assumed the ‘S’ curve pattern of cash flows. A closer look of these models also reveals that the models work out only the expenditure / cost flow and do not consider the aspect of timing difference between expenditure and cash flow. Hence these models are not of any significance to our present study.

A software ‘FINCASH for Windows’ developed by CSIRO, Australia, that was specifically designed for project cash flow management was compared by O’Leary and Tucker [9] with present methods for cash flow projections using project management software such as Primavera P3, Suretrak and Microsoft Project. FINCASH made use of a library of cash flow ‘S’ curve profiles that were characteristic of the particular type of project under consideration. These cash flow profiles were developed on the basis of past experience and amenable to modification and refinement as more data becomes available. This eliminates the need for specifying cash flow profiles for individual activities as is the case with other project management software and hence was claimed to be easier to use. However the FINCASH software has the same limitation like other project management software i.e. it gives only the cash outflows based on costs and associated time lags in payments, but does not account for the cash inflows that are very important for the contractor.

A computer program CAFFS (CAsh Flow Forecasting System) was developed by Hwee & Tiong [10] for predicting cash flow profile of a construction project. The program took into account contractual factors in a project as well as working practices and trends that affect the project’s cash flow. Impact of uncertainties and risks such as excess measurement, variation of contract, cost fluctuation etc. could also be reflected on the cash flow profile. Internal Rate of Return (IRR) based on monthly cash flows and maximum working capital requirement were the primary outputs that could be most useful to the contractor and also act as indicators to project’s performance. The input data such as contract value, duration, defect liability period, percentage markup, payment delay period, retention limit and estimated proportions of different cost groups (material, labour, equipment, sub-contracts and overheads) were coupled with typical ‘S’ curve profiles for different cost groups to develop a theoretical ‘S’ curve profile for the project. Output of the program was cumulative monthly cash outflows and cash inflows. The program could therefore be used for predicting the cash flows as the project progressed as well as to understand the impact of risk factors on project cash flows. However, this program is not available commercially and hence has probably remained to the confines of research paper.

NEED TO CONSIDER CASH FLOWS AS STEP FUNCTIONS

The research carried out so far as seen above has always assumed cash flows as continuous functions and have tried to fit them in different types of curves such as second, third, fourth or fifth degree polynomials to see the best fit. It was also observed the cash flow curves were always assumed to follow the sigmoid curves or more commonly known as ‘S’ curves that generally represent typical expenditure curve or a ‘cost-flow’ curve for a project. However, cash flows in reality do not progress continuously, but occur at specific points of time. Therefore all cash flows are to be considered as events as per the terminology of project management. If the cumulative values of cash flows against time are considered, they will jump from one level to another every time the cash flow occurs. Graphically, the curves will look like steps, with change in value from one level to another at the same time rather than the smoothened lines that are assumed in all the mathematical models. This is illustrated in the following diagrams. Figure 1 shows the cash outflow, cash inflow and the net cash flows as step curves while the expenses curve is a smooth curve.

Figure 1: Project expenditure and cash flows

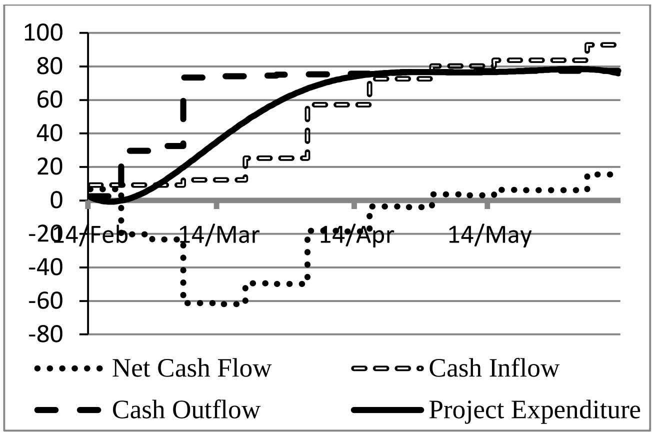
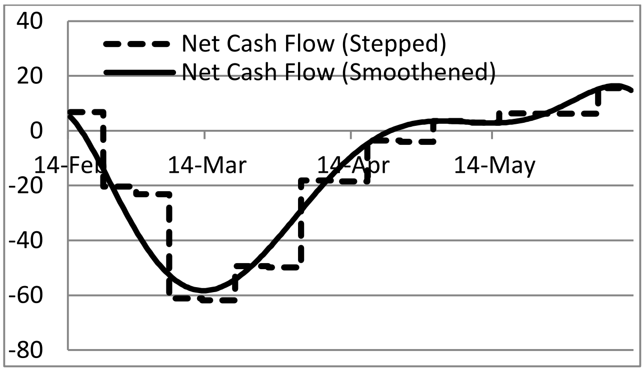


Figure 2 shows the errors in real net cash flows when they are smoothened out to fit a mathematical polynomial curve.

Figure 2: Net Cash Flow - Stepped and Smoothened



OBJECTIVES, SCOPE AND CONSTRAINTS OF THE PRESENT WORK

Objective of the present work is to propose a different mathematical approach based on step curves for forecasting of project cash flows. It will primarily need the following steps:

a) To identify different types of cash flow events and their related causative factors / incidents and establish time lags / leads between performance of the activity and related cash flows
b) Develop mathematical equations expressing the cash flows as step functions.
c) Adding different step functions to develop a consolidated cash flow for the project.

Since the objective is to establish the basic approach to resolving the problem at hand, actual development and testing of proposed mathematical model on real project will be carried out at a later stage.

DEVELOPMENT OF STEP CURVES FOR PROJECT CASH FLOW

1. **Different types of cash flow events, their related causative factors / incidents and timings**.

   Discrete non- periodic cash flows:

   - Advance against material and equipment orders: paid along with issue of confirmed order as % of order value or lump sum.
   - Mobilization advance to sub-contractors: paid along with issue of contract as % of contract value or lump sum.
   - Delivery payment against material and equipment delivery: paid against pro-forma invoice or delivery as % of contract value or lump sum on expiry of credit period.
   - Stage payment to sub-contractors: paid on completion of specified stage or milestone as per predefined value
   - Stage payment to equipment suppliers: paid on completion of specified part delivery as per predefined value
   - Final payment against material and equipment: paid on satisfactory completion and expiry of retention / warranty period, equivalent to the balance amount remaining unpaid.
   - Mobilization and other advances received from client: Received with the confirmed contract as % of contract value.
   - Final receipt: received on satisfactory completion and expiry of retention / warranty period, equivalent to the balance amount to be received.

   Periodic cash flows with specified intervals:

   - Payment to labour: Equivalent to the wages for the past period, usually weekly.
   - Payment to staff: Equivalent to monthly salaries, usually paid monthly
   - Running bills of sub-contractors: paid equivalent to the value of work completed during past period less retention amount, usually fortnightly or monthly.
   - Receipts against running bills submitted to the client: equivalent to the value of work completed during past period less retention amount, usually monthly.

2. **Mathematical equations for step curves**

   Consider the following function:

$$\boldsymbol{u(t-c) = u_c(t) = \begin{cases} 0 \; if \; t < c \\ 1 \; if \; t > c \end{cases}} \quad (1)$$

The function represents jump at *t* = *c* from zero to one at *t* = *c*.

Similarly, in a typical project situation where a contractor starts work with a cash surplus due to advance money received from the client. He later pays for material and labour at a specific time and attains a position of cash deficit (1st jump). He continues to remain in deficit till he receives the payment from the client against the bill for work completed and again attains a surplus position (2nd jump). Later, there would be further payments for labour and material that will reduce the surplus to a lower level (3rd jump). This can be expressed as discrete step functions as below:

$$\boldsymbol{f(t) = \begin{cases} 9 \; if \; t < 2 \\ -6 \; if \; 2 \le t < 6 \\ 25 \; if \; 6 \le t < 9 \\ 7 \; if \; 10 < t \end{cases}} \quad (2)$$

There are three jumps in the function: at t = 2, t = 6 and t = 9 hence there are three step functions, each one corresponding to one of the jumps.

Similarly, based on the values and timings of different cash flows as detailed above, individual step functions can be written for each of the cash flow. The discrete cash flows will be based on the terms of orders and contracts and the periodic cash flows will be based on the project schedules developed using any of the project management software with facility for resource assignments and costs.

3. Consolidation of the step functions:

The above set of step functions in equation (1) can be consolidated in a single equation as:

$$f(t) = 9 - 15u(t-2) + 31u(t-6) - 18u(t-9) \quad (3)$$

This consolidation has been worked out as follows:

When $t < 2$ all the step functions have a value of 0. So the only contributing term in the equation for $f(t)$ is 9, and on this region $f(t) = 9$. On the next interval, $2 \leq t < 6$ we want $f(t) = -6$. The first step function $u(t-2)$ is on, while the others are off. Since 9 term is still contributing, the coefficient of $u(t-2)$ will need to cause the sum of it and 9 equal to -6. Therefore, it must be 15. On the third interval, $6 \leq t < 9$ we have two 'on' step functions and the last one is off. The first two terms, the 9 and the $-15u(t-2)$ continue to contribute. Hence the coefficient of the newly contributing step function $u(t-6)$, will need to combine with these to give us our desired value $f(t) = 25$. Thus it should be 31. Lastly, we have the interval $9 < t$. Now all of the terms contribute, and the coefficient of $u(t-9)$, the step function corresponding to the final jump, should move us from our previous value of 25 to our new value, $f(t) = 7$. As a result, it must be -18. So the function written above has the right value on all intervals.

This way, it is possible to work out the net cash flows for the project by adding up the cumulative cash outflows and inflows at specific time interval either weekly or monthly.

CONCLUSION

Many researchers have developed the mathematical models for forecasting project cash flows assuming the cash flows behave in a similar fashion as project expenditures albeit with certain time lags. However in reality, the cash flows are step functions and behave much differently than the smooth expenditure curves. The paper has presented a feasible approach to development of mathematical model for cash flows using step functions. This lays a foundation for further work that needs to be done to develop the actual equations based on project data and validate the approach for practical use.

REFERENCES

[1] Kenley R. (2003) Financing Construction: Cash Flows and Cash Farming. Spon Press

[2] Kenley, R. and Wilson, O. D. (1986). 'A Construction Project Cash Flow model—an Idiographic Approach'. Construction Management and Economics 4: 213–232.

[3] Park H. K.., Han S. H. & Russell J. S. (2005), Cash Flow Forecasting Model for General Contractors using Moving Weights of Cost Categories. Journal of Management in Engineering, ASCE October 164 - 172

[4] Ra'ed Jarrah; Devdatta Kulkarni; and James T. O'Connor, (2007) Cash Flow Projections for Selected TxDoT Highway Projects: Journal Of Construction Engineering And Management © ASCE / Page 235-241

[5] Mihály Görög (2009). A comprehensive model for planning and controlling contractor cash-flow: International Journal of Project Management (27) 481–492

[6] Garden II G. W., Creese R. C.(2000); Cash Flow Analysis in Construction Projects. AACE International Transactions;, ABI/INFORM Global PageR12A

[7] Khosrowshahi, F.,(2001) Project cash flow forecasting: a mathematical approach. 17th Annual ARCOM Conference, Sept. 2001, University of Salford, Assn. of Researchers in Construction Management

[8] Min-Yuan Cheng 1, Andreas F.V. Roy, (2010) Evolutionary fuzzy decision model for cash flow prediction using time-dependent support vector machines: International Journal of Project Management

[9] O'Leary T. R.; Tucker S. Techniques and Tools for Project Cash Flow Prediction in Australian Construction Industry. Transactions of AACE International; 1996; ABI/INFORM Page CS61

[10] Ng Ghim Hwee, Robert L. K. Tiong, (2002) Model on cash flow forecasting and risk analysis for contracting firms: International Journal of Project Management (20) 351–363.

AUTHOR PROFILE

Vivek Datey received the B.E. (Mech) degree from Visvesvaraya Regional College of Engineering, Nagpur in 1973 and M.M.S. (Financial Management) from Jamnalal Bajaj Institute of Management Studies in 1975. He has over 35 years of experience in project planning and execution and consultancy with large and reputed engineering organizations. He also has been working as an educator in project management for more than ten years and has retired as Dy. Dean, Project Engineering & Management, National Institute of Construction Management & Research (NICMAR), Pune.