

## Robust Temperature Control of Chemical Batch Reactor using Sliding Mode Control

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**Abstract:** *In this paper a nonlinear model of chemical batch reactor is considered. In this two chemicals with specified concentrations are mixed and they are maintained at desired temperature. The tracking problem is considered. The modified sliding mode control is applied for the controller design and robust tracking of the desired temperature trajectory is achieved. Therefore objective of robust tracking with disturbance rejection with finite time convergence is achieved. The stability of the sliding surface is analysed with the lyapunov method. The results are compared with the classical sliding mode control and analysed.*

Keywords: Chemical batch reactor, sliding mode control, robust, temperature control, temperature trajectory.

### 1. Introduction

Chemical batch reactor is crucial unit operation in any chemical industry. The mixing process in the chemical batch reactor is highly controlled process with desired parameters like pressure, flow, temperature at their predetermined level. When the chemicals are mixed in the reactor, the chemical property of the end component highly depends on the individual components concentration and so on the temperature during the reaction. Therefore, temperature control is considered in this work. Also the desired temperature being non-stationary, i.e. exponentially increasing with time during the mixing process, robust tracking of the temperature is the major challenge. The temperature rises initially from some initial condition and then reaches the desired temperature in finite time. The temperature control of batch reactor has been achieved using input output linearization technique [1]. The product specifications can be met according to the market demand in the batch control. Unlike, the continuous process, batch processes does not work on step change of desired values rather they are trajectories which may be time dependent. Therefore tracking and perturbation rejections are main objectives, which can affect the product quality and in turn the efficiency of the batch process. Therefore to meet these demands the robust control is required.

The control of these types of processes which are

highly nonlinear, designing a controller is a major challenge. In [5] the temperature control is achieved by adaptive algorithm that does not depend on process model but information about the system response direction. In batch reactor process actually the heating and cooling effect have to be synchronized to control the temperature, therefore in [8] the heating and cooling actuators are controlled so that the temperature is maintained. Several control techniques to control the batch reactor temperature are used like [3] in which the nonlinear model predictive control is implemented and results are compared with PID controller. Recently the practical swarm optimization technique and sequential quadratic programming is used to control the temperature of batch reactor [9]. In [10] model predictive control with iterative learning algorithm is used for batch process control. The literature shows that there is a need of advance controllers to be implemented for such complex nonlinear systems rather than the conventional controllers. As these type of systems have uncertainties leading to undesired situations which may cause damage to personnel as well as the surroundings. The model based control gives a robust performance but they are sensitive to perturbation and can cause the undesired situation during the control regime. So variable structure control is one such control strategy that is robust, in the sense, it is insensitive to

perturbation once the states are on the sliding surface. This property of variable structure control has motivated to apply on the batch reactor.

Variable Structure Control Systems (VSCS) is studied for more than six to seven decades. There is a lot of literature on the sliding mode VSCS. Variable structure control first discussion was presented in English by V.A. Utkin in his famous paper in 1967 [4]. Before 1967, there was a lot of discussion of sliding mode control (SMC) in and around Russia. SMC is an extension of bang bang control in which the structure changes in a way so that the states reach the equilibrium point. Once on this regime the system states are robust to parametric uncertainties. The design of sliding surface consists of two phases. The first phase is designing the reaching phase and second the designing the sliding surface. The states once reach the sliding surface are insensitive to perturbations and uncertainties. Due to these properties sliding mode is a robust controller, which can be applied to the batch reactor problem.

The VSC applied to similar type of temperature control system was implemented in [6] which works on discrete time variable structure systems. In [2] the fictitious set point tracking concept is used along with VSC to control the temperature of batch reactor. In recent years batch reactor problem was considered in [7] which implemented SMC on the batch reactor. This is the main motivation of the work. The reaching law that is used, is modified in this work and better tracking is achieved that will be further explained in detail in the sections to come. The paper is arranged as follows section II model of the batch reactor is discussed, in section III problem formulation following section IV SMC theory. Section IV deriving the control law, section V results and discussions and section VI conclusion

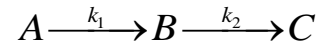
## 2. Batch Reactor

Dynamic model of batch reactor: Let us consider a batch reactor as in [11]. The dynamic equation of the batch reactor is given as

$$\begin{aligned}\dot{C}_A &= -k_1(T)C_A^2 \\ \dot{C}_B &= k_1(T)C_A^2 - k_2(T)C_B \\ \dot{T} &= \gamma_1 k_1(T)C_A^2 + \gamma_2 k_2(T)C_B + \alpha_1 + \alpha_2 T + (\beta_1 + \beta_2 T)u \quad (1)\end{aligned}$$

These equations represent the mass balance and energy balance equations of components A and B and the reactor temperature T of the batch reactor.

It is assumed that the reaction takes place in the consecutive sequence as follows



While modelling (1) following assumptions were made

A.1. Density of the reaction liquid is kept constant

A.2. Mixing of the reacting mixture is perfect one

Where

$$k_1(T) = A_{10} \exp\left[\frac{-E_1}{R(273+T)}\right]$$

$$k_2(T) = A_{20} \exp\left[\frac{-E_2}{R(273+T)}\right]$$

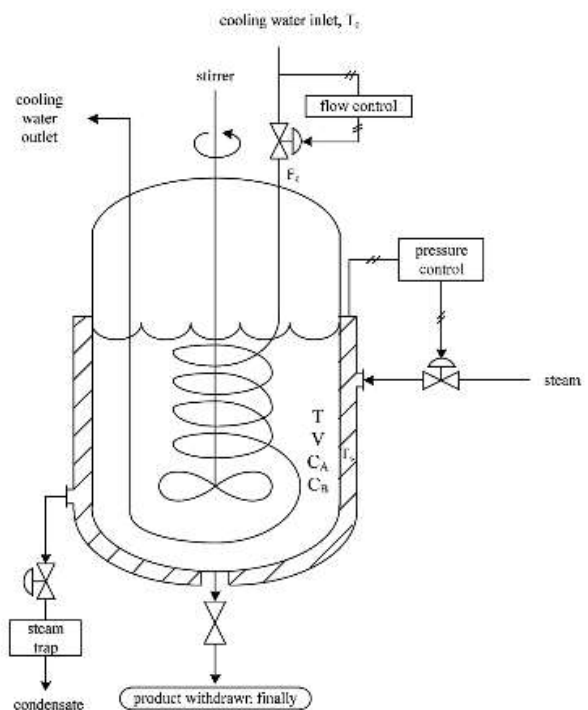


Figure 1: Schematic of Batch Reactor

The constants required in the above equations are given in the table below [11]

Table 1: Model Parameters

$A_{10} = 1.1 \text{ m}^3 \cdot \text{kmol}^{-1} \text{ s}^{-1}$	$\gamma_1 = 41.8^\circ \text{C} \cdot \text{s}^{-1}$
$A_{20} = 172.2 \text{ s}^{-1}$	$\gamma_2 = 83.6^\circ \text{C} \cdot \text{s}^{-1}$
$E_1 = 20900 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$	$\alpha_1 = 4.3145^\circ \text{C} \cdot \text{s}^{-1}$
$E_2 = 41800 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$	$\alpha_2 = -0.1099^\circ \text{C} \cdot \text{s}^{-1}$
$R = 8.3143 \text{ kJ} \cdot \text{kmol}^{-1} \cdot \text{K}^{-1}$	$\beta_1 = 1.4962^\circ \text{C} \cdot \text{s}^{-1}$
$T_c = 25^\circ \text{C}$	$\beta_2 = 0.0515^\circ \text{C} \cdot \text{s}^{-1}$

### 3. Problem Formulation

As discussed in section II batch reactor dynamic model is defined by the differential equations as in (1).

For designing the control law let us assign

$$[x_1 \quad x_2 \quad x_3] = [C_A \quad C_B \quad T] \quad (2)$$

Therefore, with respect to (1) and (2)

$$\dot{x}_1 = -k_1(x_3)x_1^2 \quad (3)$$

$$\dot{x}_2 = k_1(x_3)x_1^2 - k_2(x_3)x_2 \quad (4)$$

$$\dot{x}_3 = \gamma_1 k_1(x_3)x_1^2 + \gamma_2 k_2(x_3)x_2 + \alpha_1 + \alpha_2 x_3 + (\beta_1 + \beta_2 x_3)u \quad (5)$$

As discussed in [11], from the theory of batch reactor, the desired product is B and the batch cycle is one hour. To get maximum yield of component B the desired temperature should follow the following equation.

$$T_d(t) = 54 + 71 \exp(-2.5 \times 10^{-3} t) \quad (6)$$

Therefore, our problem is to design the control law to control the temperature in the batch reactor close to (6) so that the maximum yield of component B is achieved. Due to inherent highly nonlinear system it is a challenge to achieve (6). As seen from (6), the desired temperature trajectory is non-stationary and time variant. It is a challenge to control this type of temperature trajectory with the help of conventional controllers. Also in the presence of matched disturbance as well as with the process uncertainties, tuning the controller parameters becomes a tedious process. Therefore robust controller can achieve better tracking than the conventional controllers in the presence of uncertain conditions.

### 4. Preliminaries of sliding mode control

As discussed in introduction variable structure control sliding mode control is robust controller and has been applied to numerous nonlinear systems

[12], [13], [14], [15] etc. Last 5 decades there is rigorous research carried on sliding mode theory. The sliding mode control has a reaching phase and sliding surface.

Let (7) represent second order nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + b(x)u + d \end{aligned} \quad (7)$$

where  $x = [x_1 \quad x_2]^T$ , is the state vector  $f(x)$ ,  $b(x)$  nonlinear function in  $x$ ,  $u$  being the control input.  $d$  is the disturbance where  $d \leq d_{max}$ . Sliding surface is designed as

$$s = c_1 x_1 + c_2 x_2 \quad (8)$$

For the system to be on equilibrium or desired position the state trajectories should reach the sliding surface (8)

Reaching the sliding surface implies  $s = 0$ , Which implies that  $\dot{s} = 0$

$$s\dot{s} \leq -\eta|s| \quad (9)$$

Were  $\eta > 0$  ensures finite time convergence to  $s = 0$ , it is also called  $\eta$  reachability condition

It is clear that the states move from initial condition to the sliding surface based on this reaching law equation. It should also be noted that when the states are in this phase, they are sensitive to process uncertainty and disturbances, so it is clear that the reaching phase should be as small as possible. Many researchers are striving to minimize the reaching phase by increasing the gain during this phase, but it has been observed that due to it, there are large oscillations and system becomes extreme sensitive to the unmodelled dynamics [16],[17],[18],[19]. Therefore our aim is to implement the reaching law which has fast convergence, so that the temperature tracking of batch reactor is handled in efficient manner. The various reaching laws are proposed by Gao's, hung's and Bartoszewicz [16] in literature of sliding mode control.

a. Constant rate reaching law

$$\dot{s} = -k \text{sign}(s) \quad k > 0$$

The problem with constant rate reaching law is that the switching is directly proportional to the gain, which leads to chattering which is one of the major drawbacks of sliding mode control

b. Proportionate reaching law

$$\dot{s} = -(k_1 + k_2 |s|) \text{sign}(s) \quad k_1 > k_2 > 0$$

This satisfies the  $\eta$  reachability condition

c. Power rate reaching law

$$\dot{s} = -k |s|^\alpha \text{sign}(s) \quad k > 0, 0$$

$< \alpha < 1$

Once the states reach the sliding surface from the reaching phase they are insensitive to uncertainty and disturbances. Thus using this theory we will track the temperature of the batch reactor.

### 5. Design of Controller

To implement the SMC controller to the batch reactor let us design the control law. From (5) and (7) we get

$$f(x) = \gamma_1 k_1 (x_3) x_1^2 + \gamma_2 k_2 (x_3) x_2 + \alpha_1 + \alpha_2 x_3$$

$$b(x) = (\beta_1 + \beta_2 x_3)$$

(10)

The first step in SMC is designing the sliding surface. The designing of sliding surface, implies that  $s=0$ . Since it is a tracking problem, the error should tend to zero, as time tends to infinity, which also satisfies the condition.

Therefore, the sliding surface is chosen as

$$s = T_D - T$$

(11)

From the theory of SMC the control law  $u$  is composed of equivalent control and the discontinuous control

$$u = u_{equi} + u_{disc}$$

(12)

The equivalent control is the continuous control and the discontinuous control is the switching control.

**Proposition:** The control law proposed

$$u = b(x)^{-1} (\dot{T}_d - f(x) + k |s|^\alpha \text{sign}(s))$$

$k > 0, 0 < \alpha < 1$

(13)

for temperature tracking as per (6) of the batch reactor defined by (3), (4), (5) in the presence of  $d$  where  $d$  is bounded  $d \leq d_{max}$

Proof: from (5) and using (9), (10), (11), (12) we derive the following

$$\dot{x}_3 = f(x) + b(x)u + d$$

$$\dot{s} = \dot{T}_d - \dot{T}$$

$$\dot{s} = \dot{T}_d - \dot{x}_3$$

$$\dot{s} = \dot{T}_d - (f(x) + b(x)u + d) = -k |s|^\alpha \text{sign}(s)$$

(15)

Therefore from (14) we get the control law (13) The reaching law used to design the control law is power rate reaching law. The peculiarity of the Gao's power rate reaching law is it has fast convergence, as compared to conventional SMC.

#### 4.1 Stability Analysis

To prove the stability of sliding surface let us consider the most widely used Lyapunov function.

$$V = \frac{1}{2} s^2$$

$$\dot{V} = s \dot{s} \leq 0$$

$$(T_d - x_3)(\dot{T}_d - \dot{x}_3) \leq 0 \quad (16)$$

$$(T_d - x_3)(\dot{T}_d - (f(x) + b(x)u + d)) \leq 0$$

$$(T_d - x_3)(\dot{T}_d - (f(x) + b(x)(b(x)^{-1}(\dot{T}_d - f(x) + k |s|^\alpha \text{sign}(s) + d))) \leq 0$$

$$(T_d - x_3)(\dot{T}_d - f(x) - \dot{T}_d + f(x) - k |s|^\alpha \text{sign}(s) + d) \leq 0$$

$$(T_d - x_3)(-k |s|^\alpha \text{sign}(s) + d) \leq 0$$

)

Therefore  $k > 0$  for the sliding surface to be stable.

### 5. Results & Discussion

The simulation is done on the matlab Simulink. The disturbance used is  $d=0.5+\sin 2t$ . The purpose of using this type of disturbance is to make the system robust. From (15) it is clear that  $k$ , gain of the SMC should be large enough to reject the disturbance. In the batch reactor application, the desired temperature trajectory is time dependant one and the reactor temperature should faithfully track the desired temperature using the control law in (13). The Batch

reactor theory says that the control law is responsible for heating or cooling the element which means maximum heating if  $u=1$  and maximum cooling if  $u=0$ , therefore with these hard input constraints, the control law has to be kept under the condition,  $0 \leq u \leq 1$ [11].

The control law for conventional SMC is given as

$$u = b(x)^{-1}(\dot{T}_d - f(x) - d + k \text{sign}(s))$$

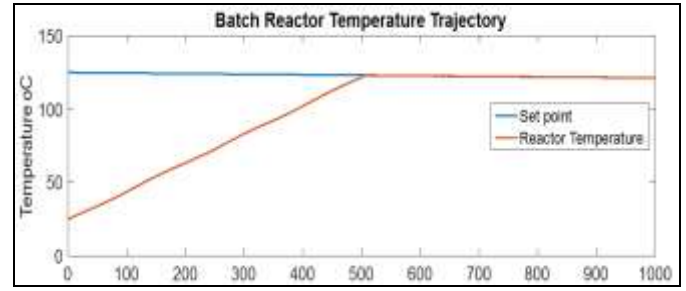
(16)

By applying (16) to the dynamic model of batch reactor, without hard input constraints, tuning the controller with  $k=10$ , the conventional SMC is giving good tracking in the presence of matched disturbance as shown in figure 2. The sliding surface for it is shown in figure 3. The control law for this controller plotted in figure 4.

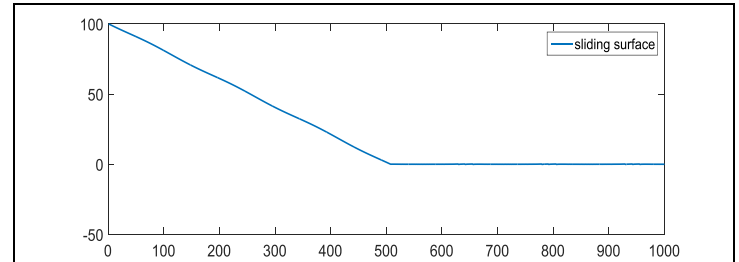
The conventional control law with hard input constraints, applied to the batch reactor model, is shown on figure 5. It has been observed that temperature tracking is achieved, but the convergence is delayed. Also the sliding surface is shown in figure 6, and the control law for it is shown in figure 7. To track the temperature in hard input constraints, the gain  $k$  of the controller is increased, with  $k=200$ , the tracking is achieved, but the control effort is such that high frequency oscillations are induced in it, which is an undesirable feature of the sliding mode control.

By applying the proposed controller in (13), on the batch reactor model, the results are shown in figure 8. The convergence is faster than in figure 2. With gain of the controller  $k=10$  and tuning parameter  $\alpha=0.7$ , the convergence is achieved. The sliding surface is shown in figure 9. The control law corresponding to it is shown in figure 10, with reduced chattering as compared to conventional SMC.

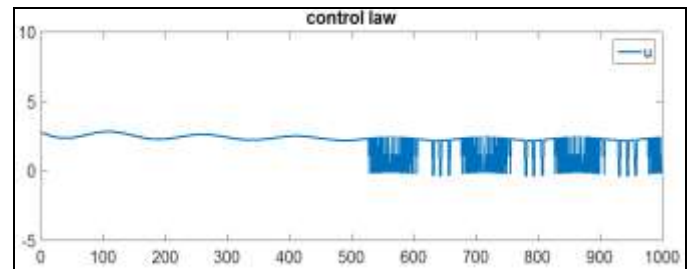
The proposed controller applied to the batch reactor model with hard input constraints, the results are obtained as in figure 11. The gain  $k=20$ , and  $\alpha=0.7$  which are almost similar as without input constraints. Since the gain is slightly increased, the chattering is the control law as observed in figure 13. The sliding surface for it is observed in figure 12.



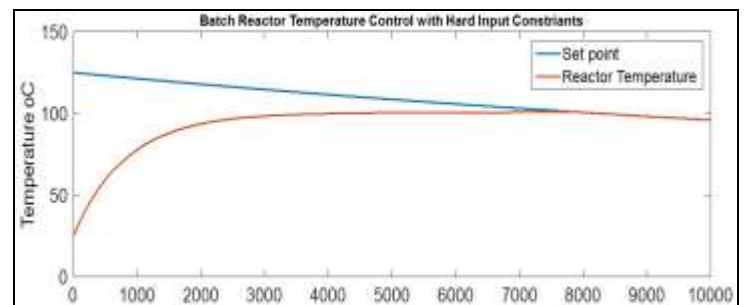
**Figure 2.** Conventional SMC without hard input constraints



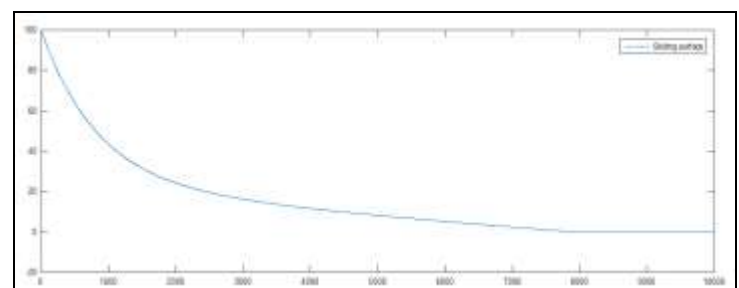
**Figure 3.** Sliding surface for conventional SMC without hard input constraints



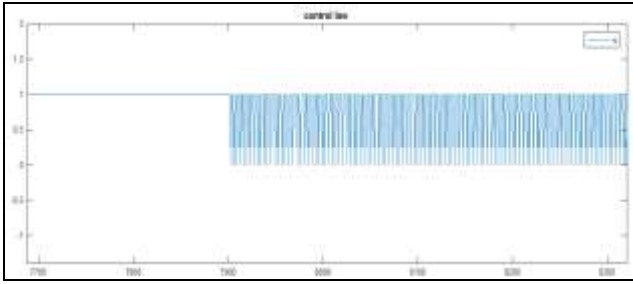
**Figure 4.** Control law for conventional SMC



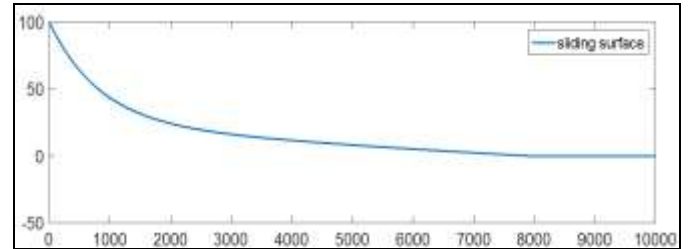
**Figure 5.** Conventional SMC with hard input constraints



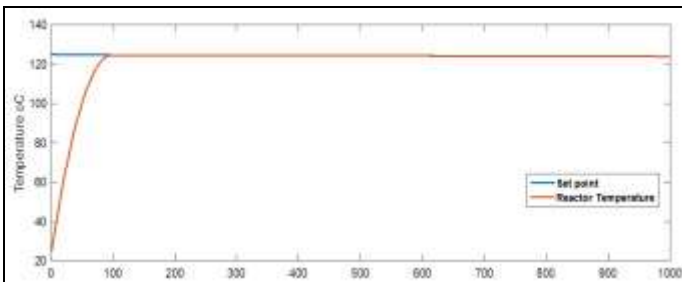
**Figure 6.** Sliding surface for conventional SMC with hard input constraints



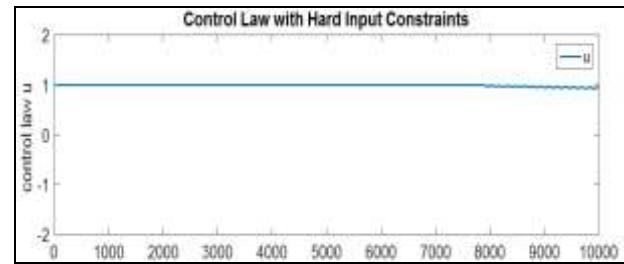
**Figure 7.** Control law for conventional SMC



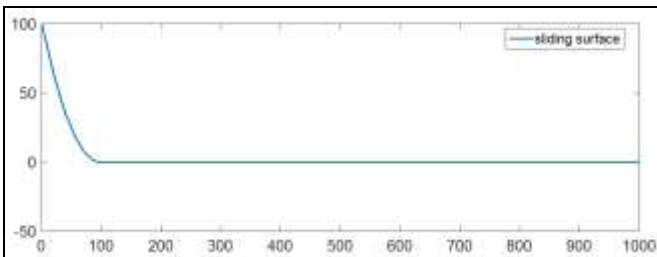
**Figure 12.** Sliding surface for hard input constraints for proposed SMC



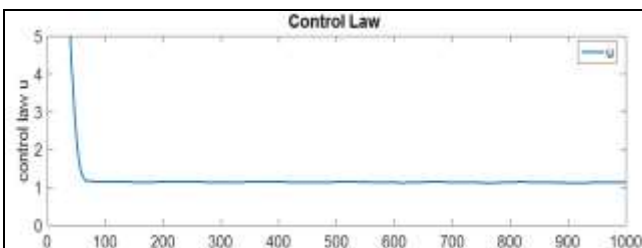
**Figure 8.** Temperature tracking without input constraints for proposed SMC



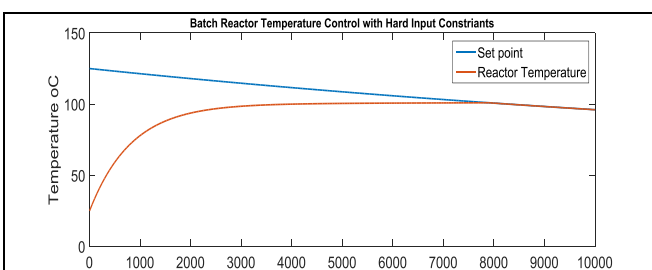
**Figure 13.** Control law for proposed SMC



**Figure 9.** Sliding surface without input constraints for proposed SMC



**Figure 10.** Control law for proposed SMC



**Figure 11.** Temperature tracking with hard input constraints for proposed SMC

The simulation results show the effect of switching gain, wherein the chattering can be reduced, if this gain is kept low. To reject the disturbance the switching gain should be large enough of disturbance, to reject the disturbance. Therefore optimum selection of the gain is one of the tuning factors in SMC. With the help of modification in reaching law, the tracking problem is solved with such hard input constraints in an efficient manner.

## 5. Conclusion

Thus, implementing SMC to the batch reactor model, the temperature tracking is achieved. There are many problems being solved with the fixed set point. To deal with time dependent set point tracking, non-conventional robust controller like SMC is a good choice as seen from the results. The modification in the reaching law has reduced the chattering which is undesired feature of the SMC. The work can be extended by applying variable gain controllers. The research in the SMC has evolved such that, the higher order sliding modes being used by which the chattering is further reduced.

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