Oversampled Graph Filter Banks For Acoustic Signal Processing

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Abstract--**Graph signal processing must explicitly consider the structure of the signal. To propose M channel filter bank for graph signal. Oversampled graph filter bank can be used to de-noise graph signals. Over sampling improves resolution, reduces noise and helps avoiding aliasing and phase distortion by relaxing anti-aliasing filter performance. Oversampled scheme that uses an oversampled wavelet transform in oversampled scheme. It provides better noiseless graph signal. Here noise reduction can be done by using Chebychev filter. Chevebychev filter minimize the error between the idealized and the actual filter characteristic over the range of the filter. Wavelet transform captures both frequency and time.**

Keywords--**Graph signal processing, oversampling, wavelet transform.**

I. INTRODUCTION

Different kinds of time–frequency localized waveforms have been subsequently used in signal processing. Oversampled filter banks whose number of channels is greater than their sub-sampling factor, because they can provide improved noise and erasure resistance [2]. Many signal processing techniques are based on transform methods, where the input data is represented in a new basis before analysis or processing [4]. One of the most successful types of transforms in use is wavelet transform.

In the first step, acoustic signal can be taken for oversampling process. A bandwidth-limited signal can be perfectly reconstructed, if sampled at or above the Nyquist rate, which is twice the highest frequency in the signal. Oversampling improves resolution, reduces noise and helps avoid aliasing and phase distortion by relaxing anti aliasing performance [1].

In the second step, wavelet transform is applied. Wavelet compression is a form of data compression well suited for audio compression [2]-[4]. Coefficients can then be compressed easily because of the information is statically concentrated in just a few coefficients. This principle is called transform coding. After that the coefficients are quantized and quantized values are entropy encoded and run length encoded. It captures both frequency and time.

The remaining part of this paper is organized as follows. Section II over sampled filter banks. Multi dimensional separable wavelets filter banks for arbitrary graphs in Section III. Section IV gives the experimental results and Section V presents the conclusion.

II. OVER SAMPLED FILTER BANKS

We consider a filter bank with M channels and sub sampling by the integer factor N in each channel. The filter bank is assumed to have perfect reconstruction with zero delay [15]-[17].

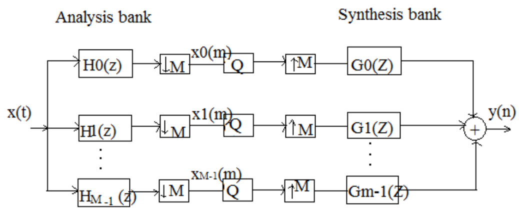

Fig.1 Block diagram of M-channel critically sampled filter bank

A subsequent down sampling, up sampling operation, discards and replaces with zeros the output coefficients on the set L in the low pass channel and on the set H in the high pass channel. Since L and H are disjoint and complementary subsets of vertex set V, the retained set of output coefficients is critically sampled.

Output at the low pass channel is, $Y_L=1/2(I_N-M)$

Output at the high pass channel is, $Y_H=1/2(I_N+M)$

Then the overall transfer function is,

$f=1/2\ G_0(I-M)H_0+ 1/2\ G_1(I+M)H_1$

$=1/2(G_0H_0+G1MH1) + 1/2\ (G1JH_1-G_0MH_0) =I_N$

In this case, $H_k=\sum \lambda_{i\in\sigma(£)} h_k(\lambda_i)P_{\lambda i}$ is a filter in the analysis section, $G_k==\sum \lambda_{i\in\sigma(£)} g_k(\lambda_i)P_{\lambda i}$ is a filter in the synthesis section. A necessary and sufficient condition for the perfect reconstruction in the two channel filter bank is,

$$\hat{G}0(\lambda)\hat{H}0(\lambda)+\hat{G}1(\lambda)\hat{H}1(\lambda) =2$$

$$\hat{G}0(\lambda)\hat{H}0(2-\lambda)-\hat{G}1(\lambda)\hat{H}1(2-\lambda)=0$$

The advantage of representing perfect reconstruction conditions is that the filter bank can be designed in the spectral domain of the graph by designing spectral kernels which satisfy [2].

III. OVERSAMPLED GRAPH SIGNALS

In this section, we describe the actual oversampled position in the signal processing flow of spectral graph signal processing and explicitly show the redundancy of the oversampled graph filter bank.

A. TWO-CHANNEL FILTER BANK CONDITIONS FOR BIPARTITE GRAPHS

The nodes in H only retain the output of high pass channel and nodes in L retain the output of the low pass channel. In our proposed design, we also choose the synthesis filters G0 and G1 to be spectral filters with kernels G0($\boldsymbol{\lambda}$) and G1($\boldsymbol{\lambda}$) respectively. Then, by using the perfect reconstruction conditions it can be rewritten as:

$$T_{eq} = G0H0 + G1H1$$

$$= \sum_{\lambda \epsilon \sigma(G)} (G0(\lambda)H0(\lambda) + G1(\lambda)H1(\lambda)) P_{\boldsymbol{\lambda}}$$

$$T_{alias} = G1J\beta H1 - G0J\beta H0$$

$$= \sum_{\lambda\gamma \epsilon \sigma(G)} (G1(\lambda)hH(\gamma) + G0(\lambda)H0(\gamma))\, P_{\boldsymbol{\lambda}} J_{\beta} P\gamma$$

1) Aliasing cancellation: Using the spectral folding property of bipartite graphs, Talias f can be written as:

$$T_{alias}f = \sum_{\lambda \epsilon \sigma(G)} (G1(\lambda)H1(2-\gamma) + G0(\lambda)H0(2-\gamma)) P_{2-\boldsymbol{\lambda}} J_{\beta} f$$

$$= \sum_{\lambda \epsilon \sigma(G)} (G1(\lambda)H1(2-\gamma) + G0(\lambda)H0(2-\gamma) P_{\boldsymbol{\lambda}} J_{\beta} f^{2-\boldsymbol{\lambda}}$$

Since, $J_{\beta} f^{2-\boldsymbol{\lambda}}$is the aliasing term corresponding to $f^{\boldsymbol{\lambda}}$, Taliasf is the aliasing part of the reconstructed signal, and an alias-free reconstruction using spectral filters is possible if and only if for all $\boldsymbol{\lambda}$in (G),

$$G0(\boldsymbol{\lambda})H0(2-\boldsymbol{\lambda}) - G1(\boldsymbol{\lambda})H1(2-\boldsymbol{\lambda}) = 0$$

B. PERFECT RECONSTRUCTION

Perfect reconstruction means that the reconstructed signal $\hat{f}$is the same as the input signal f. Teq +Talias = I. Therefore assuming the filter banks cancel aliasing, the perfect reconstruction can be obtained if and only if Teq =c^2I for some scalar constant c. Thus, a necessary and sufficient condition for perfect reconstruction, using spectral filters, in bipartite graphs filter banks is that for all $\boldsymbol{\lambda}$ in $\boldsymbol{\sigma}$(G),

$$G0(\boldsymbol{\lambda})H0(\boldsymbol{\lambda}) + G1(\boldsymbol{\lambda})H1(\boldsymbol{\lambda}) = c2;$$

$$G0(\boldsymbol{\lambda})H0(2-\boldsymbol{\lambda}) - G1(\boldsymbol{\lambda})H1(2-\boldsymbol{\lambda}) = 0$$

IV. MULTI-DIMENSIONAL SEPARABLE WAVELET FILTER BANKS FOR ARBITRARY GRAPHS

At stage i with sets Hi and Li, Ei contains all the links in $E - U_{k=1}^{i-1}E_k$ that connect vertices in Li to vertices in Hi. Thus E1 will contain all edges between H1 and L1. Then, we will assign to E2all the links between node sin H2 and L2 that were not already in E1. This is also illustrated in Figure 2. Note that, by construction and B2 (H1), which each will be processed independently by one of the two filter banks at this second stage. Clearly, this guarantees invertibility of the decomposition. So it will be possible to recover the signals in B2 (L1) and B2 (H1) from the outputs of the 2nd stage of the decomposition. The same argument can be applied to the decompositions with more than two stages. That is, the output of a two-channel filter bank at level i leads to two sub graphs, one per channel, that are disconnected when considering the remaining edges $E - U_{k=1}^{i}$ The output of a Klevel decomposition leads to 2K disconnected sub graphs.

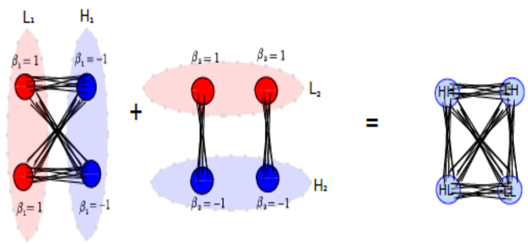

Fig. 2 Example for graph over sampling

SPECTRAL GRAPH WAVELET TRANSFORM

The transform will be determined by the choice of a kernel function G: $R^{+} \rightarrow R^{+}$. This kernel g should behave as a band-pass filter, i.e. it satisfies lim $x \rightarrow \infty$ G(0) = 0.

WAVELETS

The spectral graph wavelet transform is generated by wavelet operators that are Operator valued functions of the Laplacian. Spectral graph wavelet kernel *g*, the wavelet operator *Tg*= *g*(L)acts on the Fourier mode as,

$$\hat{T_g f}(l) = g(\boldsymbol{\lambda}_l)\hat{f}(l)$$

Inverse Fourier mode is,

$$(Tgf)(m) = \sum_{l=0}^{N-1} g(\lambda l)\ \hat{f}(l)x_l(m)$$

The wavelet operators at scale t is then defined by T_t^g=g(Tl).

In graph domain it can be expanded as,

$$\Psi_{t,n}(m) = \sum_{l=0}^{n-1} G(t\lambda)\chi_l^*(n)\chi_l(m)$$

The wavelet coefficients of a given function *f* are produced by taking the inner product with wavelets,

$$W_f(t,n) = \langle \psi_{t,n}, f \rangle$$

Wavelet coefficients are achieved as,

$$W_f(t,n) = \sum_{l=0}^{n-1} G(t\lambda)\chi_l(m)\hat{f}(l)$$

SCALING FUNCTION OF WAVELET TRANSFORM

Wavelets are defined by the wavelet function ψ (*t*) and scaling function φ (*t*) in the time domain. The wavelet function is in effect a band-pass filter and scaling it for each level halves its bandwidth. These spectral graph scaling functions have an analogous construction to the spectral graph wavelets. They will be determined by a single real valued function h: $R^+ \rightarrow R$, which acts as a low pass filter and satisfies h (0) >0 and h (x) →0 as x→ ∞. The scaling functions help ensure stable recovery of the original signal *f* from the wavelet coefficients when the scale parameter *t* is sampled at a discrete number of values.

V. EXPERIMENTAL RESULTS

Below figure 3 shows the output graph of the applied input signal. In this case the below graph is the relationship between frequency in hertz and amplitude in voltage.

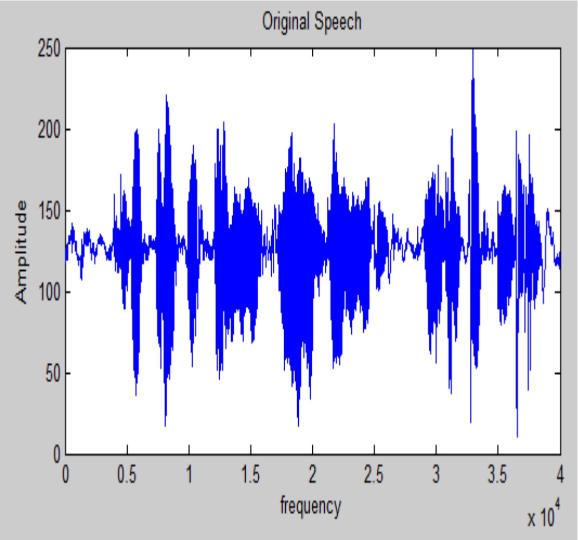

Fig. **3** Input signal

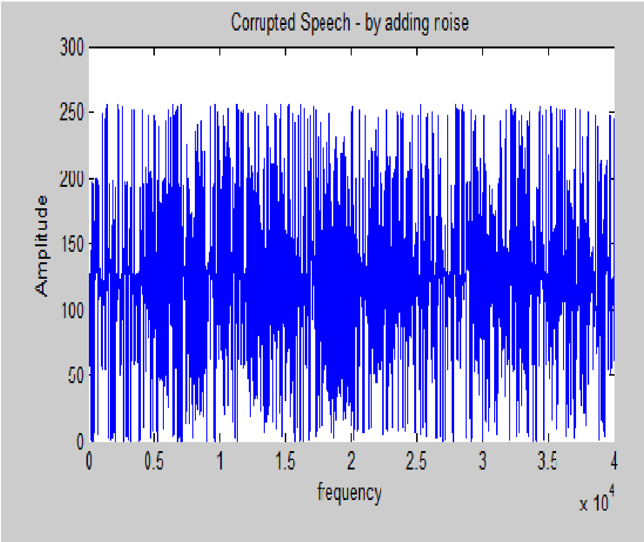

Fig. 4 Input signal after addition of noise

Spectrograms can be used to analyze the results of passing a test signal through a signal processor such as a filter in order to check its performance. A spectrogram is a visual representation of the spectrum of frequencies in a sound or other signal as they vary with time or some other variable. Spectrograms are sometimes called spectral waterfalls, voice prints. Spectrograms can be used to identify spoken words phonetically and to analyze the various calls of animals. They are used extensively in the development of the fields of music, sonar, radar, and speech processing, seismology, etc. Spectrogram of the given input signal is shown below. Spectrogram usually created by filter bank that results from a series of band pass filters.

DENOISING

De-noising removes white Gaussian noise from the acoustic signal. Over sampled filter bank is combined with the spectral graph wavelet transform by using more number of channels. This operation is established by using matlab. Over sampled graph filter banks consider all the edges in the bi partition filter bank output.

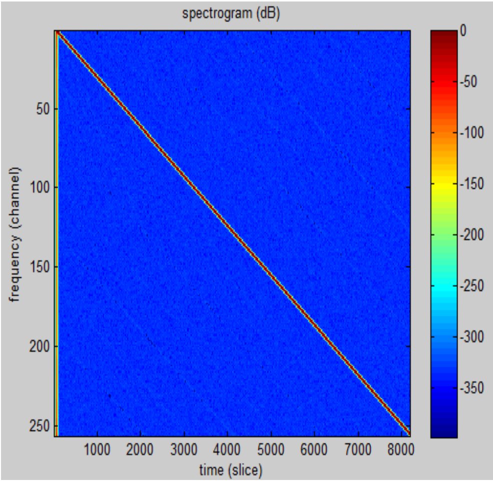

Fig. 5 Spectrum of the input signal

Below figure 6 shows the resultant output after the over sampling and wavelet transform. In this case noise is removed by using chebychev filter.

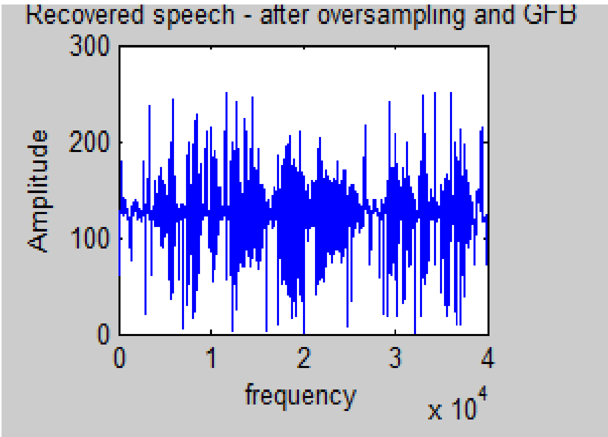

Fig. 6 Final result

V. CONCLUSION

We have proposed the construction of critically sampled wavelet filter banks for analyzing graph-signals defined on any arbitrary finite weighted graph. It can be successfully applied in many different places were signal extraction, de-noising and many real time applications.

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