

Fuzzy Set Theory and Arithmetic Operations On Fuzzy N Umbers

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Abstract

In this paper we represent a fuzzy set theory and arithmetic operation on fuzzy numbers. The fuzzy set theory to derived operation on fuzzy set and fuzzy arithmetic operation. The produce with operation on closed interval.

Introduction

The development of fuzzy set theory, since its introduction in 1965 has been dramatic. The fuzzy set theory has pervaded almost all fields of study and its applications have percolated down to consumer goods level! Apart from this, it is being applied on a major scale in industries through intelligent robots for machine – building (cars, engines, turbines, ship, etc.) and controls and of course for military purposes.

Keywords: Fuzzy set, Fuzzy numbers, Arithmetic operation, Arithmetic intervals.

Preliminaries

Definition: Fuzzy set

If X is a collection of objects denoted generally by X, then a fuzzy set A in X is a set of order pairs.

$$\tilde{A} = \{ X, \mu_{\tilde{A}}(x) / x \in X \}$$

Where $\mu_{\tilde{A}}(x)$ is called membership function or grade of membership.

Example:

Let X is a ten natural numbers

$$X = \{2, 3, 6, 8, 9, 11, 12, 14, 16, 17\}$$

$$\tilde{A} = \{(2, 0.1), (3, 0.2), (6, 0.3), (8, 0.4), (9, 0.5), (11, 0.6), (12, 0.7), (14, 0.8), (16, 0.9), (17, 1)\}$$

Arithmetic Operations On Fuzzy Numbers

Definition:

Let A and B denote fuzzy numbers and let \forall denote any of the four arithmetic operations. Then we define a fuzzy set on R, $A * B$. by defining its *cut*, $\alpha_{(A*B)}$ as

$$\alpha_{(A*B)} = \alpha_A * \alpha_B \quad \forall \alpha \in (0, 1]$$

$$A * B = \bigcup_{\alpha \in [0, 1]} \alpha^{A*B}$$

Since, $\alpha_{(A*B)}$ is closed interval for each $\alpha \in [0, 1]$ and A, B are fuzzy numbers are also a fuzzy numbers.

Definition:

Let * denote any of the four basic arithmetic operators and let A, B denote fuzzy numbers. Then we define a fuzzy set on R, $A * B$ by the equation,

$$(A * B)(z) = \sup_{z=x*y} \min[A(x), B(y)] \quad \forall z \in R$$

NOTE:

- i. $(A+B)(z) = \sup_{z=x+y} \min[A(x), B(y)]$
- ii. $(A-B)(z) = \sup_{z=x-y} \min[A(x), B(y)]$
- iii. $(A.B)(z) = \sup_{z=x.y} \min[A(x), B(y)]$
- iv. $(A/B)(z) = \sup_{z=\frac{x}{y}} \min[A(x), B(y)]$

Arithmetic Operation On Intervals

Let * denote any of the four arithmetic operations on closed intervals,

+ → Addition

− → Subtraction

× → Multiplication

/ → Division

Then $[a, b] * [d, e] = \{ f * g \mid a \leq f \leq b, d \leq g \leq e \}$ is a general property of all arithmetic operations on closed intervals except that $[a, b]/[d, e]$ is not defined when $0 \in [d, e]$.

The result of an arithmetic operation on closed intervals is again a closed intervals.

$$l(x_\alpha) = A(x_n) = \alpha$$

$$l(x_\alpha) \geq \alpha$$

Similarly, we can prove that,

$$y_\alpha \in \alpha_A$$

α_A is closed interval

$$x_\alpha, y_\alpha \in \alpha_A$$

$$[x_\alpha, y_\alpha] \leq \alpha_A$$

A is a fuzzy number.

Definition:

The four arithmetic operation on closed intervals are defined as followed.

$$\text{i. } [a, b] + [d, e] = [a+d, b+c]$$

$$\text{ii. } [a, b] - [d, e] = [a-d, b-c]$$

$$\text{iii. } [a, b] * [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$$

$$\text{iv. } [a, b]/[d, e] = [a, b] * \left[\frac{1}{d}, \frac{1}{e}\right]$$

$$= \left[\min\left(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}\right), \max\left(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}\right) \right]$$

Example:

$$1) [a, b] + [d, e] = [a+d, b+c]$$

$$[1, 2] + [3, 4] = [1+3, 2+4]$$

$$= [4, 6]$$

$$2) [a, b] - [d, e] = [a-e, b-d]$$

$$[1, 2] - [3, 4] = [1-4, 2-3]$$

$$= [-3, -1]$$

$$3) [a, b] * [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$$

$$[1, 2] * [3, 4] = [\min(3, 4, 6, 8), \max(3, 4, 6, 8)]$$

$$= [3, 8]$$

$$4) [a, b] / [d, e] = \left[\min\left(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}\right), \max\left(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}\right) \right]$$

$$[1, 2] / [3, 4] = \left[\min\left(\frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}\right), \max\left(\frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}\right) \right]$$

$$= \left[\frac{1}{3}, \frac{2}{4}\right]$$

$$[1, 2] / [3, 4] = \left[\frac{1}{3}, \frac{1}{2}\right]$$

Conclusion

In the paper discussed some result in fuzzy set theory and fuzzy arithmetic number. Here provide the fuzzy numbers as well as explain and example. This results obtained by using fuzzy arithmetic are applicable for the control system. In applied of fuzzy set theory the field of engineering has undoubtedly been leader. Fuzzy set theory is also becoming important in computer engineering.

Reference

[1] Ganesh .M Introduction to Fuzzy Sets and Fuzzy Logic.

- [2] **George , Klir Boyuan.J** Fuzzy set and Fuzzy Logic Theory and Applications.[Eastern economy edition]
- [3] **George , Klir.J and Tina Folger.A** Fuzzy Sets, uncertainty and Information.
- [4] **Kaufmann.A , Gupta.M(1991)** Introduction to fuzzy arithmetic Theory and Applications, Van Nostrand Reinhold; Newyork.
- [5] **Mizumoto** – Fuzzy theory and its Applications. Science Publications.
- [6] **Palaniyappan. N 2005** – Fuzzy topology Second Edition , Narosa publishing house , on Allied publishers Ltd , New Delhi.
- [7] **Vasantha Kandasamy .W .B** - Elementary Fuzzy Matrix Theory and Fuzzy models for social scientists, University of Microfilm International, USA.
- [8] **Witold Pedrycz and Fernando Gomide** An Introduction to Fuzzy Sets Analysis and Design. [Eastern economy edition]
- [9] **Zimmermann. H. J** Fuzzy Set Theory and its applications of [Fourth edition].