A Numerical Study on Unsteady Natural Convection Flow with Temperature Dependent Thermal Conductivity past an Isothermal Vertical Cylinder

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ABSTRACT

This paper represents a numerical investigation of the effect of temperature dependent thermal conductivity on unsteady naturalconvection flow past a semi-infinite isothermal vertical cylinder immersed in air. The thermal conductivity is adopted to vary thetemperature. The governing non-dimensional boundary layer equations have been solved numerically employing an explicit finitedifference method. To interpret the influence of variable thermal conductivity on the velocity and temperature profiles aparametric study is accomplished. The numerical consequences disclose that the thermal conductivity has significant influence ontransient velocity and temperature profiles, average skin friction coefficient and average heat transfer rate. The conclusionindicates that when the thermal conductivity parameter increases the temperature and skin friction coefficient increases but thevelocity near the wall and Nusselt number decreases. We have also shown the effect of thermal conductivity variation parameteron isotherms and streamlines.

KEYWORDS: Heat transfer, Natural convection, Temperature dependent thermal conductivity, Vertical cylinder, Explicit finite difference method

I. INTRODUCTION

Among heat transfer problems natural convection heat transfer has always been of particular interest. Fluid motion is caused by natural means such as buoyancy due to density variations resulting from temperature distribution. Natural convection plays vital role in heat transfer in case of many applications such as electrical components transmission lines, heat exchangers and many other places. Many experiments have been performed during the last few decades and interesting results have been presented. Palani G. and Kim K [1] discussed the natural convection phenomenon in case of vertical cylinder and governing equations to determine heat transfer coefficient. L. Davidson et al. [2] developed the natural convection phenomenon in vertical shell and tube. Also it was shown that for larger inlet velocity, there is a large value of Nusselt number. L. J. Crane [3] studied the natural convection over the vertical cylinder for very large Prandtl number and discussed the effect of high Prandtl number on convection through vertical cylinder. The effect of curvature of the cylinder where the thickness of the boundary layer is considerable was studied by C. O. Popie [4].

In many studies done earlier on natural convection flow through vertical cylinder, the thermal conductivity were assumed to be constant. However these physical properties can change significantly with temperature and when the thermal conductivity is taken into account, the flow characteristics are substantially changed to the constant cases. Borah G. and Hazarika G.C. [5] have developed the effects of variable thermal conductivity on steady laminar free

III. NUMERICAL ANALYSIS OF THE PROBLEM

In order solve the nonlinear governing equations (7)-(9) along with (10) an explicit finite difference method has been employed. The finite difference equation corresponding to equations (7)-(9) get the equations (11) to (13) respectively

$$\frac{U(i,j)-U(i-1,j)}{\Delta X}+\frac{V(i,j)-V(i-1,j)}{\Delta R}+\frac{V(i,j)}{1+(j-1)\Delta R}=0 \tag{11}$$

$$\begin{aligned}&\frac{U'(i,j)-U(i,j)}{\Delta\tau}+U(i,j)\frac{U(i,j)-U(i-1,j)}{\Delta X}\\&+V(i,j)\frac{U(i,j+1)-U(i,j)}{\Delta R}=T(i,j)+\\&\left[\frac{U(i,j+1)-2U(i,j)+U(i,j-1)}{(\Delta R)^2}+\frac{1}{[1+(j-1)\Delta R]}\frac{U(i,j+1)-U(i,j)}{\Delta R}\right]\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{T'(i,j)-T(i,j)}{\Delta\tau}+U(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta X}+V(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta R}\\&=\frac{1}{\Pr}\left[\begin{aligned}&\frac{T(i,j+1)-2T(i,j)+T(i,j-1)}{(\Delta R)^2}+\frac{1}{[1+(j-1)\Delta R]}\frac{T(i,j+1)-T(i,j)}{\Delta R}\\&\left[1+\frac{\varepsilon R}{1+\varepsilon T}\frac{T(i,j)-T(i-1,j)}{\Delta R}\right]\end{aligned}\right]\end{aligned} \tag{13}$$

To obtain the finite difference equations the region of the flow is divided into the grids or meshes of lines parallel to X and R is taken normal to the axis of the cylinder. Here we convection flow of an electrically conducting Newtonian fluid along a porous hot vertical plate in presence of heat source The effect of variable viscosity and thermal conductivity on the flow and heat transfer in a laminar liquid on a horizontal shrinking sheet is analyzed by khan et al [6].The thermal conductivity of fluid to be proportional to a linear function of temperature which was proposed by Charraudeau [7]. Hossain and Munir [8] studied the natural convection flow of a viscous fluid about a truncated cone with temperature dependent viscosity thermal conductivity. Numerically unsteady natural convection of air and the effect of variable viscosity over an isothermal vertical cylinder was developed by H. P. Rani et al. [9] and concluded that as the viscosity increases the temperature and the skin friction coefficient increases while the velocity near the wall and Nusselt number decreases. The effect of variable viscosity and thermal conductivity on the flow heat transfer of a stretching surface in a rotating micro-polar fluid with suction has been developed by Borthakur P.J and Hazarika G. C. [10]

In this paper, the effect of variable thermal conductivity on unsteady natural convection flow through an isothermal vertical cylinder is investigated. Actually less attention has been paid to the unsteady natural convection flow of a viscous incompressible fluid with variable thermal conductivity over a heated vertical cylinder. The governing equations are solved numerically by explicit finite difference method to obtain the transient velocity, temperature, coefficient of skin friction, heat transfer rate, isotherms and streamlines for different values of thermal conductivity parameter. It is clear from the results that, the temperature and skin friction coefficient increases as the thermal conductivity parameter increases but the velocity near the wall and Nusselt number decreases. The effect of thermal conductivity variation parameter on isotherms and streamlines are also shown in the study.

II. MATHEMATICAL ANALYSIS OF THE PROBLEM

Consider an unsteady two dimensional natural convection boundary layer flow of a viscous incompressible fluid past an isothermal semi-infinite vertical cylinder of radius r_0. As the cylinder is infinite in extent, the physical variables are functions of x and r. The cylindrical coordinates are chosen in such a way that x is taken vertically upward along the axis of the cylinder and the origin of axis is taken to be at the leading edge of the cylinder where the boundary layer thickness is zero. The radial coordinate is assumed to be perpendicular to the axis of the cylinder. The surrounding stationary fluid temperature is measured as the ambient temperature T^*_∞. Initially it is assumed that at time $t^*= 0$ the cylinder and the fluid are of the same temperature T^*_∞. When $t^*> 0$, the temperature of the cylinder is raised to T^*_w which is greater than the ambient temperature T^*_∞ and it gives rise to a buoyancy force. The effect of the thermal diffusivity is measured negligible in the momentum equation and the effects of thermal conductivity is taken into account in energy equation.

Under these assumptions the governing boundary layer equations for continuity, momentum and energy for the free convection flow over a vertical cylinder with Boussinesq's approximation are as follows

consider that the height of the cylinder is X_{max}=100 i.e. X varies from 0 to 100 and regard R_{max}=25 as corresponding to $R\to\infty$ i.e. R varies from 0 to 25. In the above equations (11) to (13) the subscripts i and j designate the grid points along the X and R coordinates, respectively, where $X = i\Delta X$ and $R = 1+(j-1)\Delta R$ .There are m=500 and n=500 grid spacing in the X and R directions respectively. It is assumed that ΔX and ΔR are constant mesh sizes along X and R directions respectively and taken as $\Delta X = 2$ $(0 \le x \le 100)$ and $\Delta R = 0.50$ $(0 \le r \le 25)$ respectively with the smaller time step $\Delta\tau = 0.001$

From the initial and boundary conditions given in equation (10), the values of velocity U,V and temperature T are known at time $\tau = 0$; then the values of U,V and T at the next time step can be evaluated. Generally, when the above variables are known at $\tau = n\Delta\tau$, the values of variables at $\tau=(n+1)\Delta\tau$ are calculated as follows. The finite difference equations (12) and (13) at every internal nodal point on a particular i -level constitute a tri-diagonal system of equations. Such a system of equation is solved by Thomas algorithm. At first the temperature T is calculated from equation (13) at every j nodal point on a particular i -level at the (n+1) time step. By making the use of these known values of T, the velocity U at the (n+1) time step is calculated from equation (12) in a similar way. Thus the values of T and U are known at a particular i -level. Then the velocity V is calculated from equation (11) explicitly. This process is repeated for the consecutive i -levels. Thus the values of U, V and T are known at all grid points in the rectangular region at the (n+1)th time step. This iterative procedure is repeated for many time steps until the steady state solution is reached.

IV. RESSULTS AND DISCUSSION

We have obtained numerical solutions by solving the finite difference equations using explicit finite difference method. The velocity, temperature, coefficient of skin friction, rate of heat transfer in terms of Nusselt number, isotherms and streamlines have been carried out by assigning some arbitrarily chosen specific values to the physical parameters involved in the problem. Also for each feasible difference of wall and ambient temperature it can be said that the variation of the Prandtl number with temperature is not noticeable. Therefore, the non dimensionalized system of equations (7)-(9) along with (10) have been solved with a fixed value of Prandtl number. In the present numerical solution four values of ε are chosen 0.2, 0.4, 0.6, and 0.8 with a fixed value of Prandtl number Pr = 0.70. In case of isotherms and streamlines we have used another four values of ε 0.25, 0.50, 0.75 and 1.00. The figures computed from the numerical method of the problem have been displayed in Figs. (1-9).

The present velocity and temperature profiles are compared with the results of H. P. Rani et al. [9] for the steady state, isothermal and constant thermal conductivity with Pr = 0.7. The comparison results, which are shown in Fig. 1and Fig. 2 are found to be in good agreement.

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \quad (1)$$

$$\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=g\beta\left(T^*-T_\infty^*\right)+\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad (2)$$

$$\frac{\partial T^*}{\partial t^*}+u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\alpha r\frac{\partial T^*}{\partial r}\right) \quad (3)$$

with the following initial and boundary conditions

$$\begin{aligned} &t^*\le 0: u=0, v=0, T^*=T_\infty^* \quad \text{for all } x \text{ and } r\\ &t^*>0: u=0, v=0, T^*=T_\infty^* \quad \text{at } r=r_0\\ &u=0, v=0, T^*=T_\infty^* \quad \text{at } x=0\\ &u\to 0, v\to 0, T^*\to T_\infty^* \quad \text{as } r\to\infty \end{aligned} \quad (4)$$

To get the solution of the equation (1) to (3) along with (4) we want to make them non-dimensional. For this purpose we use the following non-dimensional quantities

$$X=Gr^{-1}\frac{x}{r_0},\ R=\frac{r}{r_0},\ U=Gr^{-1}\frac{ur}{\nu},\ V=\frac{vr_0}{\nu},$$
$$t=\frac{\nu t^*}{r_0^2}\ T=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*},\ Gr=\frac{g\beta r_0^3\left(T_w^*-T_\infty^*\right)}{\nu^2},\ \Pr=\frac{\nu}{\alpha} \quad (5)$$

We assume thermal conductivity of fluid to be proportional to a linear function of temperature as

$$\alpha(T)=\alpha_\infty(1+\varepsilon T) \quad (6)$$

where α_∞ is the thermal conductivity of ambient fluid, T is the dimensionless temperature and Ɛ is a scalar parameter which shows the influence of temperature on variable thermal conductivity.

By introducing the non-dimensional variables of (5) and using (6) into the equations (1) to (3) along with (4), we get the following no dimensional equations (7) to (9)

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial R}+\frac{V}{R}=0 \quad (7)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=T+\left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right) \quad (8)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{\Pr}\left[\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}\left(1+\frac{\varepsilon R}{1+\varepsilon T}\frac{\partial T}{\partial R}\right)\right] \quad (9)$$

The corresponding initial and boundary conditions in non-dimensional variables are reduced to the following form

$$\begin{aligned} &t\le 0: U=0, V=0, T=0 \quad \text{for all } X \text{ and } R\\ &t>0: U=0, V=0, T=1 \text{ at } R=1\\ &U=0, V=0, T=0 \text{ at } X=0\\ &U\to 0, V\to 0, T\to 0 \text{ as } R\to\infty \end{aligned} \quad (10)$$

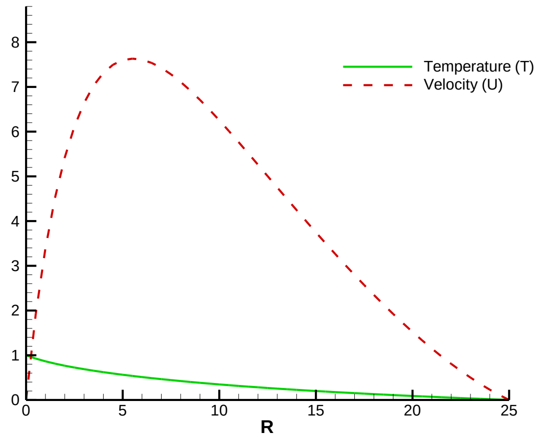


Fig.1. Comparison of the velocity and temperature profiles for Ɛ = 0.2

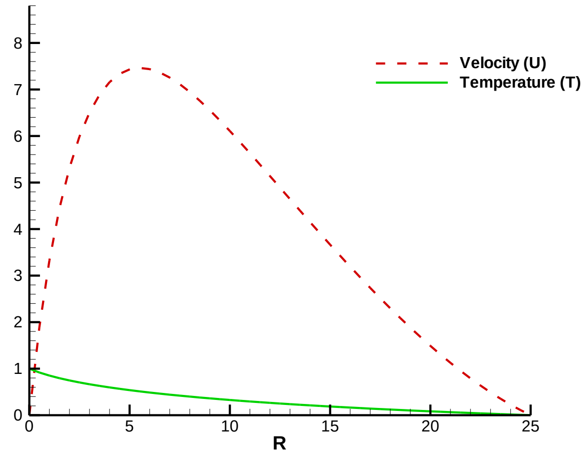


Fig.2. Comparison of the velocity and temperature profiles for Ɛ = -0.2

IV(a) VELOCITY

Fig. 3 and Fig. 4 illustrate the graphical representation of the velocity profiles at the temporal maximum and steady state against the radial coordinate R at X=1.0 for different values of thermal conductivity parameter Ɛ. It is noticed that the velocity profiles start with the value zero at the wall, reached their maximum close to the hot wall and then monotonically decrease to zero. It is clear that the time to reach the temporal maximum of velocity increases with the increasing thermal conductivity parameter Ɛ, while the time to reach the steady- state are almost the same for different Ɛ. It is observed that if we increase values of the thermal conductivity parameter Ɛ then it increases the velocity of the flow away from the wall. The location of the maximum velocity gets far away from the cylinder for higher values of Ɛ.

IV(c) AVERAGE SKIN FRICTION COEFFICIENT AND HEAT TRANSFER RATE

We have calculated average skin friction coefficient as

$$\overline{C_f} = (1+\varepsilon)\int_0^1 \left(\frac{\partial U}{\partial R}\right)_{R=1} dX \qquad (14)$$

The average heat transfer rate (Nusselt number) is expressed as

$$\overline{Nu} = -\int_0^1 \left(\frac{\partial T}{\partial R}\right)_{R=1} dX \qquad (15)$$

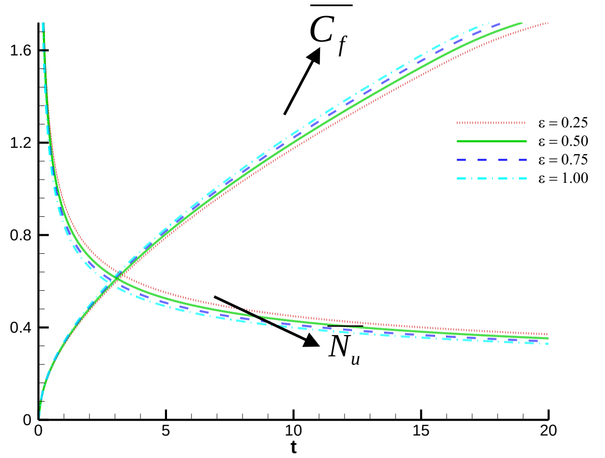


Fig. 7.Variation of the average skin friction and Nusselt number with respect to Ɛ.

In Fig.7, average skin friction coefficient and heat transfer rate for different values of thermal conductivity have been plotted against the time. It is observed from the figure that average skin friction coefficient and heat transfer rate decrease very fast initially but after a certain time these values become constant. The constant behavior of the Nusselt number and average skin friction guarantees the steady state. We notice that with the increase in thermal conductivity, the average skin friction increases near the wall. As we move upward along the cylinder the average skin friction starts to decrease. On the other hand with the increase in thermal conductivity, the nusselt number is also is also increasing. This fact explains that initially the heat transfer is performed mostly by the conduction with the large temperature difference between the wall and the fluid. With the increase of time, the free convection effect becomes more pronounced and as a result, the local Nusselt number generally increases, increasing the heat transfer rates.

IV(d) **STREAMLINES AND ISOTHERMS**

Figure 8 and Figure 9 illustrate the effect of thermal conductivity parameter Ɛ on the development of streamlines and isotherms profile which are plotted for Ɛ (= 0.25, 0.50, 0.75 and 1.00) and Prandtl number Pr = 0.70. It is observed that the values of stream are lower when the boundary layer thickness is highest shown in figure 8(a), but with the increase of thermal conductivity parameter Ɛ (when Ɛ= 0.50, Ɛ= 0.75 Ɛ = 1.00) increases the values of

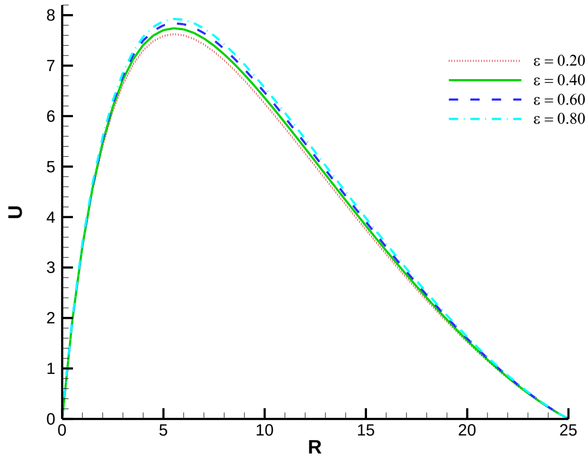


Fig.3. Variation of the steady state velocity profiles with respect to positive values of Ɛ at time step τ = 115.

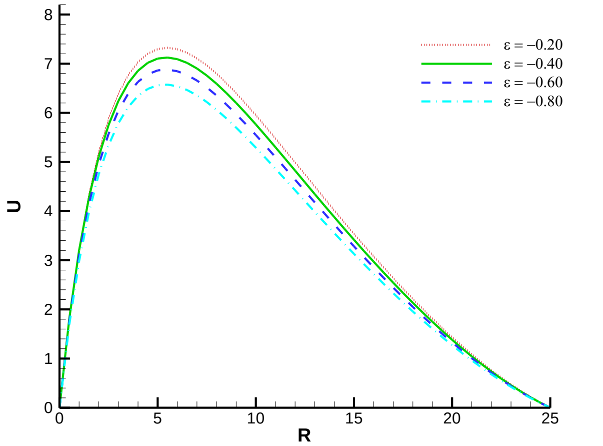


Fig.4. Variation of the steady state velocity profiles with respect to negative values of Ɛ at time step τ = 115.

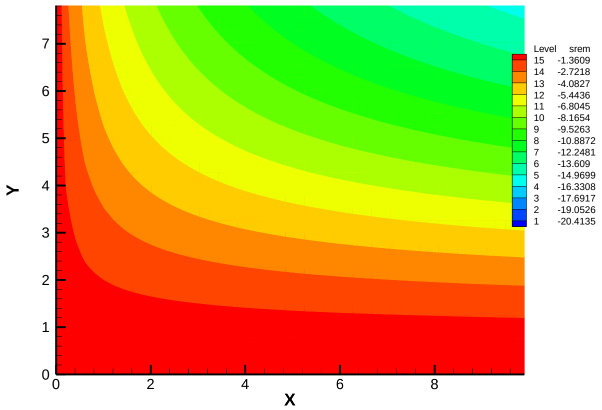
IV(b) TEMPERATURE

Fig. 5 and Fig. 6 depict the graphical representation of the simulated steady state temperature profiles against the radial coordinate *R* at *X*=1.0 for different Ɛ. It is observed that the temperature profiles start with the hot wall temperature (*T*=1) and then monotonically decrease to zero as the radial coordinate increases. Also it is noticed that temperature profiles increase with the increase of the thermal conductivity parameter. It is connected to the matter that with the increase in the thermal conductivity parameter the diffusivity of the fluid is increases, which permits higher velocity away from the hot wall.

The temperature profiles increase with increasing Ɛ, which is related with the fact that the increase in Ɛ causes the decrease in the peak velocity as shown in Fig. 3.and Fig. 4 However, two opposite effects of the increase in Ɛ on the fluid particle can be considered. The first effect decreases the velocity of the fluid due to increase in diffusivity where the second effects increase the velocity of the fluid particle due to increase in temperature as shown in Fig. 5 and Fig.

stream shown in figure 8(b), 8(c) and 8(d), also the momentum and boundary layer become thinner. From figure 9 it is clearly observed that the thermal conductivity of the fluid increased at the vicinity of the surface which proves that thermal conductivity of the fluid is strongly dependent on temperature. Finally it is concluded that for the effect of thermal conductivity parameter Ɛ the velocity of the flow and temperature of the fluid within the boundary layer increase.

6. Close to the cylinder wall the temperature is relatively high, as a results, the first effect will be dominant and the velocity decreases as Ɛ increases (Fig. 3 and Fig. 4). On the other hand, away from the cylinder wall, where the temperature T is relatively low, the second effect will be dominant and the velocity increases as Ɛ increases (Fig. 3 and Fig. 4).



(a)

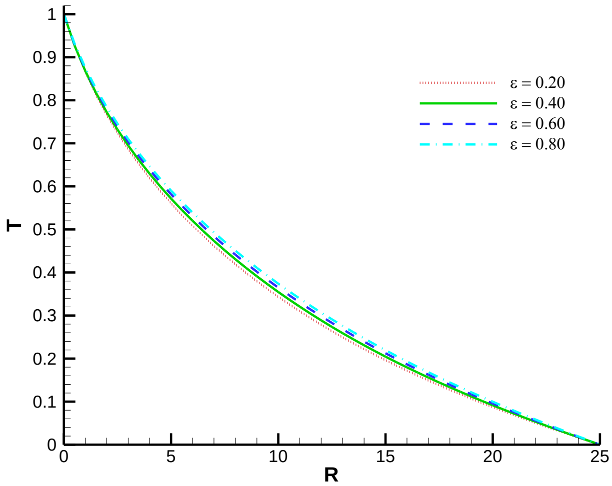
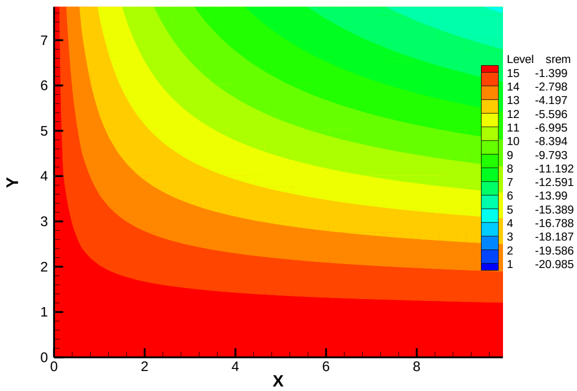


Fig.5. Variation of steady state temperature profiles with respect to positive values of Ɛ at time step τ = 115.



(b)

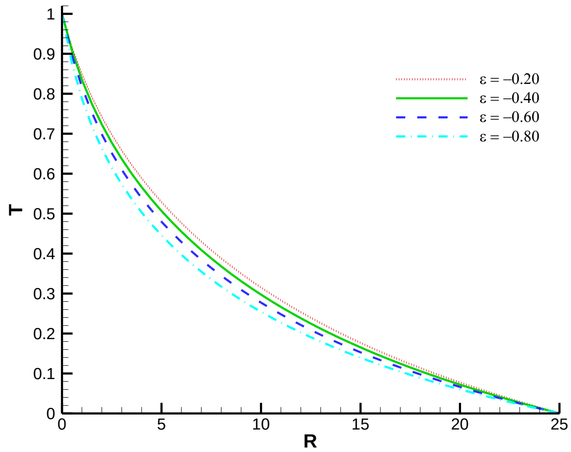
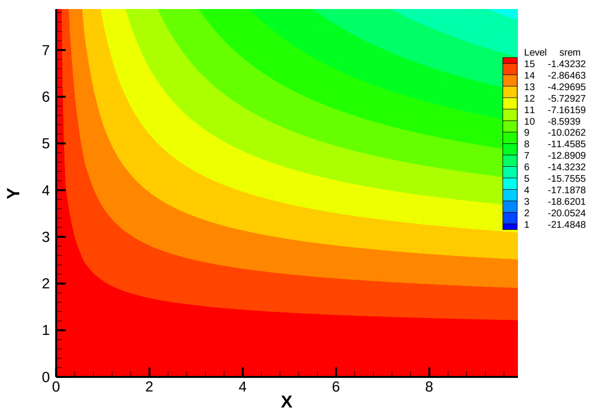


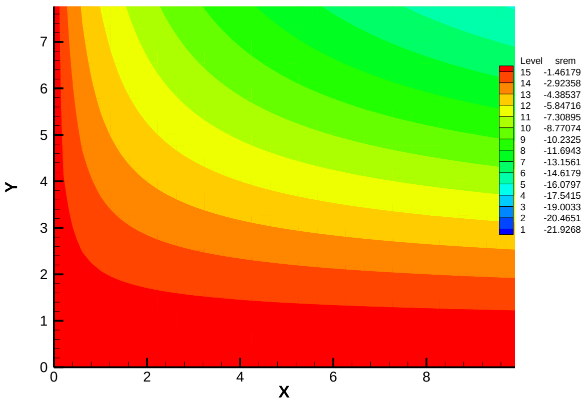
Fig.6.Variation of steady state temperature profiles with respect to negative values of Ɛ at time step τ = 115.



(c)

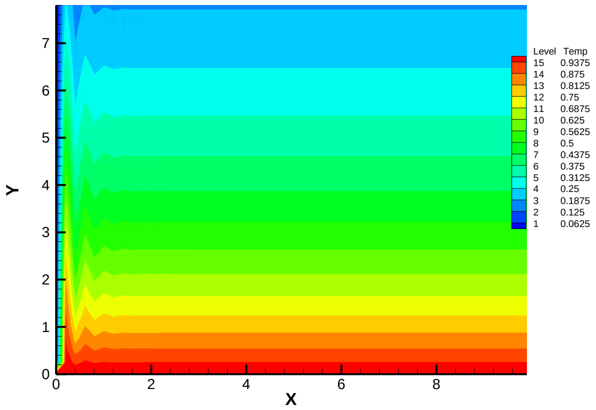
V. CONCLUTION

Numerical study for the unsteady natural convection of air with temperature dependent thermal conductivity along a semi infinite vertical cylinder has been investigated. The diffusivity of the fluid is assumed to be temperature dependent, while the Prandtl number is kept constant. An explicit finite difference method is used to solve the dimensionless governing equations. The computations are carried out to study the influence of the thermal conductivity parameter Ɛ on velocity, temperature, skin friction coefficient and heat transfer rate, streamlines and isotherms.

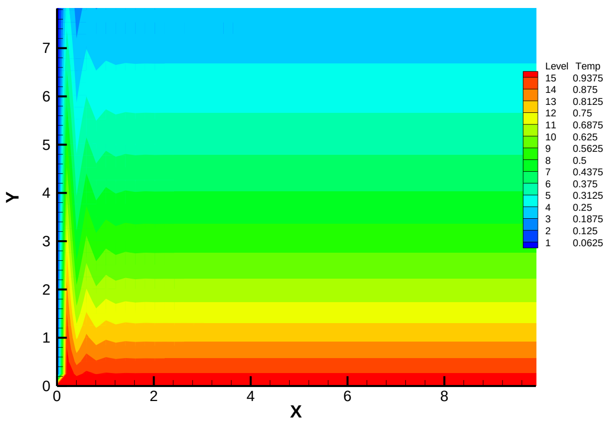


(d)

Fig.8 (a), (b) ,(c), (d) represent the streamlines with respect to Ɛ=0.25, Ɛ=0.50, Ɛ=0.75 and Ɛ=1.00 respectively.



(a)



(b)

Generally less attention has been paid to the unsteady natural convection flow of a viscous incompressible fluid with variable diffusivity over a heated vertical cylinder. The aim of the present work is to investigate the diffusivity effects on the free convective flow of air past a semi infinite vertical cylinder. From the present numerical analysis the following observations are established.

Velocity profiles near the wall decrease with the increase of Ɛ, while the temperature profiles increase. The time which is taken to reach the temporal maximum of the velocity increases with the increase of Ɛ. Initially, the unsteady behavior of the temperature with the variable thermal conductivity coincides with that of fluid with constant properties. Then the temperature with the variable thermal conductivity deviates from that with constant properties and reached the steady state asymptotically. When the thermal conductivity parameter is larger, lower velocity near the isothermal cylinder wall and higher velocity in a region away from the wall are observed, which gives the lower average Nusselt number. The increase in the thermal conductivity parameter leads to the decrease in the average heat transfer rate and to the increase in the average skin friction. With the increase of thermal conductivity parameter Ɛ increases the values of stream and the temperature distribution reduces slightly for large values of Ɛ. As a result the momentum and thermal boundary layer become thin.

The results of our study are compared with the results of H. P. Rani et al. [9] for the steady state, isothermal and constant viscosity with Pr = 0.7. The comparison results are found to be in good agreement. Additionally we have also showed the effect of thermal conductivity parameter Ɛ on streamlines and isotherms where we found that with the increase thermal conductivity parameter Ɛ increase the velocity and temperature of the fluid causing the increase in thermal boundary layer thickness.

VI. REFERENCES

[1] Palani G. and Kim K. "Numerical study on a vertical plate with variable viscosity and thermal conductivity", *Arch Appl. Mech.*, vol. 80, pp. 711-725, 2010.

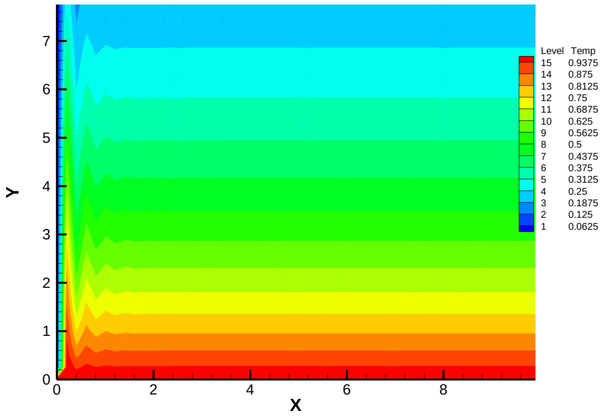
[2] D.G. Bahraini, L. Davidson, R. Karlsson, "Natural convection Heat Transfer In Vertical Shell And Tube", Chalmers University Of Technology, Goteborg, Sweden, December 2002.

[3] L.J. Crane, "Natural convection On Vertical Cylinder at Very High Prandtl Numbers", *Journal of Engineering Mathematics*, vol. 10(2), pp. 115-124,1976

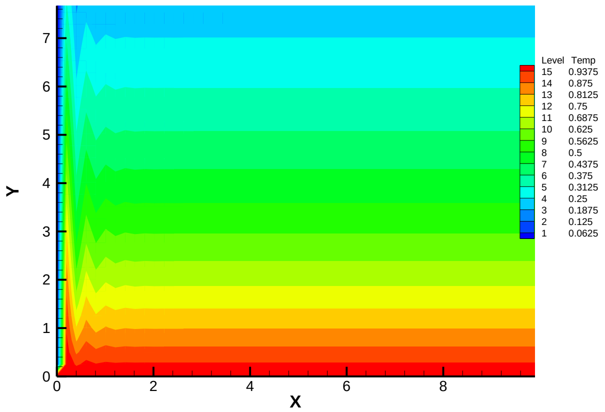
[4] C.O. Popiel, "Free convection heat transfer from vertical slender cylinder", *International Journal of Heat Transfer Engineering*, vol. 29(6) , pp. 521-536, 2008.

[5] Borah G., Hazarika G. C., "Effects of thermal conductivity on steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source ", *International Journal of Physical Science*, vol. 22, pp. 183–190, 2010.

[6] Y. Khan Q. Wu, "The effects of variable thermal conductivity on a thin film over a shrinking sheet", *Computer and Mathematics with Application*, vol. 61, pp. 3389–3399, 2011.



(c)



(d)

Fig.9 (a), (b) ,(c), (d) represent the isotherm lines with respect to Ɛ=0.25, Ɛ=0.50, Ɛ=0.75 and Ɛ=1.00 respectively.

[7] J. Charraudeau, "Influence De Gradients De Properties Physics En Convection Force Application Au Cas Du Tube," *International Journal of Heat Mass Transfer*, vol. 18, no. 8, pp. 87–95, 1975.

[8] M. S. Munir, M. A. Hossain, "Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone", *International Journal of Thermal Science*, vol. 40, pp. 437–443, 2001.

[9] H. P. Rani, Chang Y. Kim, "A numerical study on unsteady natural convection of air with variable viscosity over an isothermal vertical cylinder", *Korean J. Chem. Eng*, vol. 27, pp.759-765, 2010.

[10] Borthakur P.J and Hazarika G. C.,"Effect of variable viscosity and thermal conductivity on flow and heat transfer of a stretching surface of a rotating micro-polar fluid with suction and blowing", *International Journal of Pure and Applied Science*, vol. 25, pp. 361–370, 2006.