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Travelling Wave Solutions of the Perturbed Wadati-Segur-Ablowitz Equation by (G'/G)-Expansion Method

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Abstract:
The investigation of the exact solutions of NLPDEs plays an important role for the understanding of most nonlinear physical phenomena. Also, the exact solutions of this equations aid the numerical solvers to assess the correctness of their results. In this paper, (G'/G)-expansion method is presented to construct exact solutions of the Perturbed Wadati-Segur-Ablowitz equation. Obtained the exact solutions are expressed by the hyperbolic, the trigonometric and the rational functions. All calculations have been made with the aid of Maple program. It is shown that the proposed algorithm is elementary, effective and can be used for many PDEs in mathematical physics.

Keywords: The (G'/G) expansion method, Travelling wave solutions, Exact solutions, The Perturbed Wadati-Segur-Ablowitz equation.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are widely used to describe complex phenomena in many fields, such as physics, biology, chemistry, optical fibers, mechanics, etc. So, investigation of exact solutions of NLPDEs will help to understand these phenomena better [1]. Therefore, some researchers have used many powerful methods for obtaining exact solutions of nonlinear partial differential equations, such as inverse scattering method [2], Hirota's bilinear method [3], Backlund transformation [4], Painleve expansion [5], F-expansion method [24], sine-cosine method [6], homogenous balance method [7, 8], exp-function method [9, 10], improved (G'/G)-expansion method [11, 12, 13, 14, 15], ansatz method [16], the first integral method [17, 18, 19, 20], Kudryashov method, extended trial equation method [21], tanh function method [22, 23], auxiliary equation method [25], differential transform method [26], homotopy perturbation technique [27] and so on.

Recently, the (G'/G)-expansion method was introduced by the Chinese mathematicians Wang et al. [28]. This method is used to find travelling wave solutions of nonlinear partial differential equations. The main idea of this method is that the travelling wave solutions of nonlinear equations can be expressed by polynomial in (G'/G), where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$ where $\xi = x-ct$ and λ, μ and c are constants. The degree of the polynomial can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in the NLPDEs and the coefficients of the polynomial can be obtained by solving a set of algebraic equations resulting from the process of using the proposed method. This method has some pronounced merits over other methods like differential transform method, exp-function method, sine-cosine method, homotopy perturbation method, first integral methods, trial methods, F-expansion method. The solution procedure is direct and simple. The general solution has been obtained this method without approximation. The initial and boundary conditions have not been required. The availability of computer systems like Maple, Mathematica or Matlab facilitates the tedious algebraic calculations. The method is elementary and effective. We have noted that this method changes the given difficult problems which can be solved easily [12]. It will be seen that more travelling wave solutions of many

NLPDEs. More recently, Zhang et al. [29] applied (G'/G) -expansion method to improve Wang et al. s [28] work in order to solve variable coefficient and high-dimensional equations. Zhang et al. [30] devised an algorithm for using the method to solve nonlinear differential difference equations. This method is widely used by the references therein [28, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41].

The aim of this paper is to investigate travelling wave solutions of the Perturbed Wadati-Segur-Ablowitz equation by using the (G'/G) -expansion method. The Perturbed Wadati-Segur-Ablowitz equation is as follows:

$$iu_x + u_{tt} + 2\rho\left|u^2\right|u - \varepsilon u_{xt} = 0. \quad (1.1)$$

Wadati et al. introduced (1.1) to study certain instabilities of the modulated wave trains [42]. "This equation is apparently not integrable because it does not satisfy the Painleve property but it is a Hamiltonian analogue of Kuramoto-Sivashinsky equation which arises in dissipative systems [43]."

2. The (G'/G)-Expansion Method

In this section, we give main steps of the (G'/G) -expansion method [36, 37].

Step 1. Take into account a usual NPDE in:

$$W(\mathrm{u}, \mathrm{u}_t, \mathrm{u}_x, \mathrm{u}_{tt}, \mathrm{u}_{xx}, \mathrm{u}_{xt}, \ldots) = 0 \quad (2.1)$$

Then (2.2) transforms the nonlinear ordinary differential equation (NLODE) as

$$H(\mathrm{U}, \mathrm{U}', \mathrm{U}'', \mathrm{U}''', \ldots) = 0 \quad (2.2)$$

such that $u = u(\mathrm{x}, \mathrm{t}) = \mathrm{U}(\xi), \xi = \mathrm{x} - \mathrm{ct}$ and $U' = \dfrac{\partial U(\xi)}{\partial \xi}$. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

Step 2. Suppose that the solutions of (2.2) has a polynomial in (G'/G) as following form:

$$U(\xi) = \sum_{i=0}^{m} a_i\left(\frac{G'}{G}\right)^i = \alpha_m\left(\frac{G'}{G}\right)^m + \ldots \quad (2.3)$$

where a_i $(i = 0,1,2,\ldots,m)$ are constants with $a_m \neq 0$ to be determined later.

Step 3. Determine the positive integer m by balancing between the highest order derivatives and highest order nonlinear terms appearing in (2.2). $G = (G'/G)$ satisfies the second order LODE in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \quad (2.4)$$

where λ and μ are constants to be determined later.

Step 4. By substituting (2.3) together with (2.4) into (2.2), we get an algebraic equation involving powers of (G'/G). Equating the coefficients of each power of $(G'/G)$to gives a system of algebraic equations for a_i $(i = 0,1,2,\ldots,m)$, λ , μ and c. By solving this system with aid of Maple or Mathematica, we get unknown constants.

Step 5. The constants a_m , λ , μ c and the general solutions of (2.4) into (2.3) we have travelling wave solutions of the (2.1). Solutions of (2.4) depending on whether $\lambda^2 - 4\mu > 0, \lambda^2 - 4\mu < 0, \lambda^2 - 4\mu = 0,$

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}\right) - \frac{\lambda}{2}, & \lambda^2 - 4\mu > 0 \\ \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi - C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}{C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi + C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}\right) - \frac{\lambda}{2}, & \lambda^2 - 4\mu < 0 \\ \left(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\right), & \lambda^2 - 4\mu = 0 \end{cases} \quad (2.5)$$

where C_1 and C_2 are arbitrary constants.

3. Applications to the Perturbed Wadati-Segur-Ablowitz Equation

In this section, we apply the $(\frac{G'}{G})$ -expansion method to construct the travelling wave solutions for the erturbed Wadati-Segur-Ablowitz equation.

3.1. The Perturbed Wadati-Segur-Ablowitz equation

Let us consider, the Perturbed Wadati-Segur-Ablowitz equation:

$$iu_x + u_{tt} + 2\rho\left|u^2\right|u - \varepsilon u_{xt} = 0. \quad (3.1)$$

Let

$$u(\mathrm{x}, \mathrm{t}) = V(\xi)\mathrm{e}^{i\eta}, \xi = x - ct, \eta = c(\mathrm{x} - \sigma\,\mathrm{t}). \quad (3.2)$$

where v,c and σ are constants. After the transform (3.2) and separation of real parts and imaginary parts, we get:

$$1 + 2vc\sigma - cV - c^2\sigma^2 V - \varepsilon c^2\sigma V + 2\rho V^3 = 0 \quad (3.3)$$

and

$$v^2V'' + \varepsilon vV' - cV - c^2\sigma^2 V - \varepsilon c^2\sigma V + 2\rho V^3 = 0 \qquad (3.4)$$

Balancing (3.4) we get $m = 1$, so the solution of (3.1) is of the form:

$$V(\xi) = a_0 + a_1\left(\frac{G'(\xi)}{G(\xi)}\right),\ a_1 \neq 0 \qquad (3.5)$$

Substituting Eqs (2.3) and (2.4) into (3.4), collection the coefficients of $(\frac{G'}{G})^m (\mathrm{m} = 0,1,2,3)$ and set it zero we obtain the system

$$\begin{aligned}
(\frac{G'}{G})^0 &: -c^2\sigma^2 a_0 + v^2\lambda\mu a_1 + \epsilon v\lambda\mu a_1 + 2\rho a_0^3 - ca_0 - \epsilon c^2\sigma a_0 \\
(\frac{G'}{G})^1 &: v^2\lambda^2 a_1 + \epsilon v\lambda^2 a_1 - ca_1 + 6\rho a_0^2 a_1 - c^2\sigma^2 a1 + 2\epsilon v a_1\mu - \epsilon c^2\sigma a_1 + 2v^2\mu a_1 \\
(\frac{G'}{G})^2 &: 3\epsilon v\lambda a_1 + 3v^2\lambda a_1 + 6\rho a_0 a_1^2 \qquad (3.6) \\
(\frac{G'}{G})^3 &: 2\epsilon v a_1 + 2v^2 a_1 + 2\rho a_1^3
\end{aligned}$$

Solving this system (3.6) by MAPLE gives

Case1:

$$\begin{aligned}
a_0 &= \pm\frac{\sqrt{16}\sqrt{\frac{1}{\rho}}\epsilon\lambda}{16}, \\
a_1 &= \pm\frac{\epsilon\sqrt{16}}{8\rho\sqrt{\frac{1}{\rho}}}, \\
c &= \frac{2}{\epsilon^2}, v = -\frac{\epsilon}{2} \\
\sigma &= \frac{(-2 \pm \frac{\sqrt{-16-8\epsilon^4\mu+2\epsilon^4\lambda^2}}{2})\epsilon}{4}, \qquad (3.7)
\end{aligned}$$

Case2: If we take $k_1 = \frac{\varepsilon v + v^2}{4\rho}$

$$\begin{aligned}
a_0 &= \pm\sqrt{-k_1}\lambda, a_1 = \mp\frac{2k_1}{\sqrt{-k_1}}, v = v, \\
c &= \pm(\pm 4v^2 \pm 4\epsilon v + (16v^4 + 32\epsilon v^3 + 20\epsilon^2v^2 + 24\epsilon^3v^5\lambda^2 + 8\epsilon^2v^6\lambda^2 + 26\epsilon^4v^4\lambda^2 + 12\epsilon^5v^3\lambda^2 \\
&\quad -32\epsilon^2v^6\mu - 96\epsilon^3v^5\mu - 104\epsilon^4v^4\mu - 48\epsilon^5v^3\mu + 2\epsilon^6v^2\lambda^2 + 4\epsilon^3v - 8\epsilon^6v^2\mu)^{\frac{1}{2}})/(2\epsilon^2v(\epsilon + v)), \\
\sigma &= \mp 2(1 \pm (\pm 4v^2 \pm 4\epsilon v + (16v^4 + 32\epsilon v^3 + 20\epsilon^2v^2 + 24\epsilon^3v^5\lambda^2 + 8\epsilon^2v^6\lambda^2 + 26\epsilon^4v^4\lambda^2 + 12\epsilon^5v^3\lambda^2 \\
&\quad -32\epsilon^2v^6\mu - 96\epsilon^3v^5\mu - 104\epsilon^4v^4\mu - 48\epsilon^5v^3\mu + 2\epsilon^6v^2\lambda^2 + 4\epsilon^3v - 8\epsilon^6v^2\mu)^{1/2})/(2\epsilon(\epsilon + v)))\epsilon^2v(\epsilon + v)/ \\
&\quad ((\pm 4v^2 \pm 4\epsilon v + (16v^4 + 32\epsilon v^3 + 20\epsilon^2v^2 + 24\epsilon^3v^5\lambda^2 + 8\epsilon^2v^6\lambda^2 + 26\epsilon^4v^4\lambda^2 + 12\epsilon^5v^3\lambda^2 - \\
&\quad -32\epsilon^2v^6\mu - 96\epsilon^3v^5\mu - 104\epsilon^4v^4\mu - 48\epsilon^5v^3\mu + 2\epsilon^6v^2\lambda^2 + 4\epsilon^3v - 8\epsilon^6v^2\mu)^{1/2})(2v + \epsilon)) \qquad (3.8)
\end{aligned}$$

Substituting the solution sets (3.7,3.8) and the corresponding solutions of (2.4) into (3.5), we have the solutions of (3.1) as follows:

Case1: When $\lambda^2 - 4\mu > 0$, we have the hyperbolic type

$$V_1(\xi) = \pm\frac{\epsilon\sqrt{\lambda^2 - 4\mu}}{4\sqrt{\rho}} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right), \qquad (3.9)$$

and (3.4)

$$u_1(x,t) = \left[\pm\frac{\epsilon\sqrt{\lambda^2 - 4\mu}}{4\sqrt{\rho}} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right)\right] \times e^{ic(x-\sigma t)} \qquad (3.10)$$

where $\xi = x + \frac{\varepsilon}{2}t.$

When $\lambda^2 - 4\mu < 0$, we have the trigonometric type

$$V_2(\xi) = \pm\frac{\epsilon\sqrt{4\mu - \lambda^2}}{4\sqrt{\rho}} \times \left(\frac{C_1\cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi - C_2\sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi}{C_1\sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi + C_2\cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi}\right),$$

and

$$u_2(x,t) = \left[\pm\frac{\epsilon\sqrt{4\mu - \lambda^2}}{4\sqrt{\rho}} \times \left(\frac{C_1\cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi - C_2\sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi}{C_1\sin\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi + C_2\cos\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\right)\xi}\right)\right] \times e^{ic(x-\sigma t)} \qquad (3.11)$$

where $\xi = x + \frac{\varepsilon}{2}t.$

When $\lambda^2 - 4\mu = 0$, we have the rational type

$$V_3(\xi) = \pm\frac{\epsilon C_2}{2\sqrt{\rho}(C_1 + C_2\xi)}$$

and

$$u_3(x,t) = \left[\pm\frac{\epsilon C_2}{2\sqrt{\rho}(C_1 + C_2\xi)}\right] \times e^{ic(x-\sigma t)}$$

(3.12)

Case2: When $\lambda^2 - 4\mu > 0, k_1 < 0$, we have the hyperbolic type

$$V_4(\xi) = \mp\frac{k_1}{\sqrt{-k_1}}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right)$$

and

$$u_4(x,t) = \left[\mp\frac{k_1}{\sqrt{-k_1}}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right)\right] \times e^{ic(x-\sigma t)}$$

where $\xi = x - vt.$

When $k_1 > 0$

$$V_5(\xi) = \pm i\sqrt{k_1}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right),$$

$$u_5(x,t) = \left[\pm i\sqrt{k_1}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + C_2\sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right)\right] \times e^{ic(x-\sigma t)}$$

When $\lambda^2 - 4\mu < 0$, $k_1 < 0$;we have the trigonometric type

$$V_6(\xi) = \mp \frac{k_1}{\sqrt{-k_1}} \sqrt{4\mu - \lambda^2} \times \left(\frac{C_1 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi - C_2 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi}{C_1 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi + C_2 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi} \right),$$

and

$$u_6(x,t) = \left[\mp \frac{k_1}{\sqrt{-k_1}} \sqrt{4\mu - \lambda^2} \times \left(\frac{C_1 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi - C_2 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi}{C_1 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi + C_2 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi} \right) \right] \times e^{ic(x-\sigma t)}$$

where $\xi = x - vt$.

When $k_1 > 0$

$$V_7(\xi) = \pm i\sqrt{k_1} \sqrt{4\mu - \lambda^2} \times \left(\frac{C_1 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi - C_2 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi}{C_1 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi + C_2 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi} \right),$$

and

$$u_7(x,t) = \left[\pm i\sqrt{k_1} \sqrt{4\mu - \lambda^2} \times \left(\frac{C_1 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi - C_2 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi}{C_1 \sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi + C_2 \cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)\xi} \right) \right] \times e^{ic(x-\sigma t)}$$

When $\lambda^2 - 4\mu < 0$, $k_1 < 0$, we have the rational type

$$V_8(\xi) = \pm \frac{2k_1 C_2}{\sqrt{-k_1}(C_1 + C_2\xi)}$$

and

$$u_8(x,t) = \left[\pm \frac{2k_1 C_2}{\sqrt{-k_1}(C_1 + C_2\xi)} \right] \times e^{ic(x-\sigma t)}$$

where $\xi = x - vt$.

When $k_1 > 0$

$$V_9(\xi) = \pm \frac{2i\sqrt{k_1} C_2}{(C_1 + C_2\xi)},$$

$$u_9(x,t) = \left[\pm \frac{2i\sqrt{k_1} C_2}{(C_1 + C_2\xi)} \right] \times e^{ic(x-\sigma t)}$$

where $\xi = x - vt$.

4. Conclusion

In this paper, the (G'/G)-expansion method has been successfully used to obtain travelling wave solutions for the Perturbed Wadati-Segur-Ablowitz equation. The obtained solutions may be important to expose most complex physical phenomena or to find new phenomena.This method has more advantages since it is too difficult to solve complex nonlinear partial differential equations by traditional methods. The procedure is direct and simple that the general solutions of the second order LODE have been well known. The availability of computer systems like Maple or Mathematica facilitates the tedious algebraic calculations. The obtained solutions in this study will be very useful in various physical situations where this equation arise.

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