

Evaluation of lake eutrophication status based on set pair analysis and confidence intervals

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Abstract: Because traditional assessing methods usually yield extreme results and are of poor resolution, a set pair analysis based on triangular fuzzy number is established. The measured value and the lake quality criteria are two interrelated sets put together to compose a set pair with respect to the problem of determination of the magnitude of lake eutrophication. Set pair analysis expression is based on the three-element connection number which is extended at different levels and at the same level to form a multi-element connection number in relation to I-V grades evaluation standard commonly used in lake assessment. The weight value was calculated by weighted average method and finally got the evaluation grade as a confidence interval based on confidence level. The case study shows that the method presents the evaluating grade as a confidence interval which has advantages of high resolution and information utilization.

Keywords: Set pair analysis; Triangular fuzzy number; Super-standard weight method; Confidence interval; Lake eutrophication.

1. Introduction

Lake eutrophication is defined as the pollution of the water environment by industrial and domestic sewage, the polluted water body carries a large amount of nutrients needed by organisms [1]. Nutrients like nitrogen, phosphorous flow into the lake causing algae and other micro-organisms to multiply rapidly, as a result reducing the dissolved oxygen concentration in the water and finally causing water quality deterioration[2], which consequently lead to the decrease in biodiversity [3].

Evaluation of lake eutrophication status is of great significance to sustainable development of water resources, the safety of drinking water and the protection of the natural ecological environment [4]. An accurate assessment of eutrophication is an important tool for effective management of water resources because it provides a scientific basis for decision-making in eutrophication prevention and sustainable management of water resources [5].

Due to inevitable fuzziness and randomness of eutrophication assessment it is difficult to come up with a unified evaluation method. As a result, main credible methods of assessing eutrophication status of lakes have been designed and used. At present, the most used methods are [6]: single factor evaluation method, comprehensive water quality evaluation method, fuzzy evaluation method, grey evaluation method, artificial neural network method, parameter method, biological index evaluation method, nutritional status evaluation etc. These evaluation methods used alone have some uncertainty problems [7]. Utilizing all the advantages and strengths of the lake evaluation methods aforementioned, main other theories, methods and evolutionary algorithms are combined for various improvements; this is a trend that connects different advantages of methods with each other to form new methods [8].

This article introduces a new method called “set pair analysis based on triangular fuzzy number” to evaluation of lake eutrophication. When the original data of a problem is uncertain its logical to express it by confidence interval where the real evaluation result is included in a given range and the boundary of unknown solution can be obtained through the operation of confidence interval number [9], given that lake eutrophication assessment is uncertain, it is vital to carry out the set pair analysis based on confidence

interval. The proposed method enriches the traditional methods for it doesn't only give a concrete grade of evaluation but also gives the evaluation grade as a confidence interval, along with the confidence level. Utilization of the proposed method in lake eutrophication status evaluation will help to reduce pointlessly rigid interpretation of lake eutrophication status evaluation results as we aim to achieve quality evaluation.

2. Methodology

The set pair analysis model based on triangular fuzzy number was established as follows;

1.1. Set pair analysis

Set pair analysis is based on the three-element connection number as **a**- represents identity degree, **b**- represents discrepancy degree, and **c**- represents contrary degree as shown below.

$$\mu = a + bi + cj \tag{1}$$

In its practical application, there are some problems that it is rough only to divide state-space of research object into three, which can't definitely describe these problems [10]. Therefore, it is necessary to expand basic formula of the connection degree on difference levels, and then on the same level to expand it to form a kind of multivariate connection number whose formula is as following [11]:

$$\mu = a + b_1i_1 + b_2i_2 + \dots + b_ni_n + c_1j_1 \tag{2}$$

In order to set up a five-element connection number in relation to I-V grades evaluation standard commonly used in lake assessment the **b**- discrepancy degree was expanded into **b₁**- partial identity in discrepancy degree, **b₂**- entirety discrepancy degree and **b₃**- partial contrary in discrepancy degree [12]. This forms a multivariate connection number of **a**- identity degree, **b₁**- partial identity in discrepancy degree, **b₂**- entirety discrepancy degree, **b₃**- partial contrary in discrepancy degree and **c**- contrary degree of set pair. As follows [13];

$$\mu = a + b_1i_1 + b_2i_2 + b_3i_3 + c_1j_1 \tag{3}$$

Where: $a, b_1, b_2, b_3, j_1, c \in [0, 1]$ and $a + b_1 + b_2 + b_3 + c = 1$.

- i_1, i_2, i_3 -are the coefficient of the discrepancy degree where i_1 - is the coefficient of partial identity in discrepancy degree, i_2 - is the coefficient of entirety discrepancy degree, i_3 - is the coefficient of partial contrary in discrepancy degree, and are some uncertain values between -1 and 1, i.e. $i \in [-1, 1]$.
- j_1 - is the coefficient of contrary degree and is specified as -1.
- i_1, i_2, i_3, j -are regarded as the markers of the discrepancy degree and the contrary degree respectively.

1.2. Triangular fuzzy number theory

Given a fuzzy number \hat{A} in the real number field, and define a membership function: $\mu_{\hat{A}}(x): R \rightarrow [0, 1]$, $x \in R$, given that it attains the following triangular-type membership function [14]:

$$\mu_{\hat{A}}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases}$$

Then \hat{A} is called the Triangle Fuzzy Number (TFN) denoted by $\hat{A} = (a, b, c)$, where $a \leq b \leq c$, if $a = b = c$, \hat{A} is a crisp set.

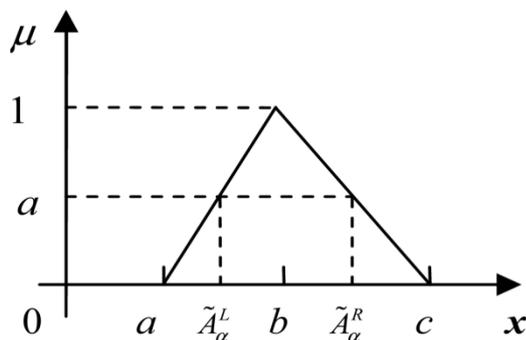


Figure 1 Triangular fuzzy interval under α -cut

Given TFN is $\hat{A} = (a, b, c)$, and confidence interval is a $\alpha \in [0, 1]$, confidence level interval can be attained as follows [15]

$$\hat{A}_\alpha = [\hat{A}_\alpha^L, \hat{A}_\alpha^R] = [(b - a)\alpha + a, -(c - b)\alpha + c]$$

For the fuzzy number in the real number field, α -cut set \hat{A} is the real number interval number $\hat{A}_\alpha = [\hat{A}_\alpha^L, \hat{A}_\alpha^R]$ which satisfies four arithmetic operation rules as follows [16]

$$\begin{aligned} [a, b] + [c, d] &= [a + c, b + d], \\ [a, b] - [c, d] &= [a - d, b - c], k \cdot [c, d] = [k \cdot c, k \cdot d] (k > 0), \\ [a, b] \cdot [c, d] &= [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}], \\ [a, b] \div [c, d] &= [a, b] \cdot \left[\frac{1}{d}, \frac{1}{c}\right] \\ &= \left[\min\left\{\frac{a}{d}, \frac{a}{c}, \frac{b}{d}, \frac{b}{c}\right\}, \max\left\{\frac{a}{d}, \frac{a}{c}, \frac{b}{d}, \frac{b}{c}\right\}\right] ([c, d] \neq 0). \end{aligned}$$

1.2.1. Determining the connection degree formula

In order to achieve a graded connection degree in a system analysis evaluation the following methods are adopted followed;

3. In relation to the aim of connection degree, by adopting the probabilistic method calculate connection components, and then set up a connection degree formula [17].
4. Adopting the triangle membership function calculate connection components, and then to set up a connection degree formula [18]. The most important feature of the triangle membership function calculating membership degree is "dichotomy" as a result it is incompletely identical with the thought of "trichotomy" of set pair analysis. Hence, this is achieved by distributing connection components according to a specific proportion which accommodates the thought of "trichotomy" of set pair analysis [19] The connection degree formula is established as follows:

$$\mu_{mk} = \begin{cases} 1+0i_1+0i_2+0i_3+0j, c_{mk} \in [0, S_{m1}] \\ \left[\frac{1}{2} \frac{C_{mk} - S_{m2}}{S_{m1} - S_{m2}} + \frac{1}{2} i_1 - \frac{1}{4} \frac{S_{m1} - C_{mk}}{S_{m1} - S_{m2}} i_2 + 0i_3 + 0j \right], c_{mk} \in [S_{m1}, S_{m2}] \\ \left[0 + \frac{1}{2} \frac{C_{mk} - S_{m2}}{S_{m2} - S_{m3}} i_1 + \frac{1}{2} i_2 + \frac{1}{2} \frac{S_{m3} - C_{mk}}{S_{m2} - S_{m3}} i_3 + 0j \right], c_{mk} \in [S_{m2}, S_{m3}] \\ \left[0 + 0 + \frac{1}{2} \frac{C_{mk} - S_{m4}}{S_{m3} - S_{m4}} i_2 + \frac{1}{2} i_3 + \frac{1}{2} \frac{S_{m3} - C_{mk}}{S_{m3} - S_{m4}} j \right], c_{mk} \in [S_{m3}, S_{m4}] \\ 0+0i_1+0i_2+0i_3+1j, c_{mk} \in [S_{m4}, S_{m5}] \end{cases} \quad (4)$$

Where, m is the m -th evaluation index, $m = 1, 2, \dots, M$; k is the k -th evaluation sample, $k = 1, 2, \dots, K$; c_{mk} is the k -th evaluation sample value; $S_{m1}, S_{m2}, S_{m3}, S_{m4}$ and S_{m5} were each evaluated index threshold value of “Oligo-phytic”, “Oligo-meso”, “Meso-trophic”, “Meso-eutro”, and “Eutrophic”, respectively.

4.2.1.1. Determining the coefficient of the discrepancy degree based on triangular fuzzy number

The triangular fuzzy interval is based on the coefficient of discrepancy degree, according to the relation structure of identity, discrepancy and contrary. The identity degree “ a ” coefficient can be regarded as 1; the contrary degree “ c ” coefficient can be regarded as -1. At present, the value of i (coefficient of discrepancy degree) can be calculated by different methods (proportion method, mean method, probability method and function simulating method etc.). Based on the problem researched in the paper, the principle of equally sharing connection coefficient determines the value of the coefficients of discrepancy degree as $i_1 = 0.5$, $i_2 = 0$, $i_3 = -0.5$. Thereby, the coefficients of connection number forms a group of sequence: 1, 0.5, 0, -0.5, -1 which are continuous in $[-1, 1]$ [20]. As indicated by the principle of equally sharing the coefficients of connection have the minimum value i_{min} , the optimum value i_{opt} and the maximum value i_{max} which constructs the triangular fuzzy number (TFN) as follows; $\{TFN \tilde{A}(i_1) = i_{min}, i_{opt}, i_{max}\}$ [21]. For instance, the minimum value, optimum value and maximum value of the coefficient of the partial identity in discrepancy are 0, 0.5 and 1 respectively, which is used to construct the triangular fuzzy number TFN as $\tilde{A}(i_1) = (0, 0.5, 1)$ Applying the same method with the above we can construct TFN of the coefficient of entirety discrepancy as $\tilde{A}(i_2) = (-0.5, 0, 0.5)$ and the coefficient of partial contrary in discrepancy as $\tilde{A}(i_3) = (-1, -0.5, 0)$.

If the confidence level α is given, we can obtain the confidence interval under the confidence level α as follows [22].

$$\begin{aligned} \tilde{A} \alpha(i_1) &= [0.5\alpha, -0.5\alpha+1] \\ \tilde{A} \alpha(i_2) &= [0.5\alpha - 0.5, - (0.5\alpha - 0.5)] \\ \tilde{A} \alpha(i_3) &= [0.5\alpha - 1, - 0.5\alpha] \end{aligned} \quad (5)$$

If $\alpha=1$, then $\tilde{A} \alpha(i_1) = 0.5$, $\tilde{A} \alpha(i_2) = 0$, $\tilde{A} \alpha(i_3) = -0.5$ which is identical with the results of analysis technique [23]. The hierarchical connection number based on the triangular fuzzy intervals is obtained by using the connection degree formula in Eq. 4 and coefficients of the triangular fuzzy numbers in Eq. 5 in the hierarchical relation.

4.3. Determining the connection degree matrix

The connection degree value $[\mu_{mk}^L, \mu_{mk}^R]$ is used to establish a connection degree matrix R as follows.

$$R = \begin{bmatrix} \begin{bmatrix} \mu_{11}^L, \mu_{11}^R \end{bmatrix} & \begin{bmatrix} \mu_{12}^L, \mu_{12}^R \end{bmatrix} & \cdots & \begin{bmatrix} \mu_{1k}^L, \mu_{1k}^R \end{bmatrix} \\ \begin{bmatrix} \mu_{21}^L, \mu_{21}^R \end{bmatrix} & \begin{bmatrix} \mu_{22}^L, \mu_{22}^R \end{bmatrix} & \cdots & \begin{bmatrix} \mu_{2k}^L, \mu_{2k}^R \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} \mu_{m1}^L, \mu_{m1}^R \end{bmatrix} & \begin{bmatrix} \mu_{m2}^L, \mu_{m2}^R \end{bmatrix} & \vdots & \begin{bmatrix} \mu_{mk}^L, \mu_{mk}^R \end{bmatrix} \end{bmatrix} \quad (6)$$

4.4. Determining every evaluation index weight

In the weighted average method, categories of environmental factors are assigned a weight which determines how much that environmental factor counts towards your final grade by taking the averages of a collection of factors, some factors are assigned a greater “weight”, or importance than others. This method treats two components of weight equally, the first is the super-standard extant, and the second one is distinction among water quality levels. The super-standard of each index at each assessing object is computed; the bigger the measure of contamination, the greater the weight and contrasts among levels of water quality standard are accounted for, the more regrettable the level, the more prominent the weight [24].

Index weight is calculated as follows:

$$W_i = \frac{C_{mk}}{Y_j}, Y_j = \sum_{i=1}^s S_{mi} \quad (7)$$

Where, C_{mk} represents the measured value of index i and Y_j represents the arithmetic mean of index i in each grading representative value; S_{mi} -represents the typical value of index i in each grading standard.

With a specific end goal to make the compositional operation, the weight of each single factor should be normalized [25]: as shown below

$$W_m = \frac{W_i}{\sum_{j=1}^n W_j} \quad (8)$$

Where, W_m represents the normalized weight of the evaluation index i .

4.5. Determining the comprehensive assessment analysis

Embracing the comprehensive method for the expansion weighting completes the system comprehensive estimation [26] as shown below:

$$G = W \cdot R = [g_1, g_2, \dots, g_k]$$

$$\text{Where, } g_k = \begin{bmatrix} g_k^L, g_k^R \end{bmatrix} = \sum_{m=1}^m W_m \begin{bmatrix} u_{mk}^L, u_{mk}^R \end{bmatrix} \quad (9)$$

The result of comprehensive estimation g_k is also a confidence interval number, and $g_k \subset [1, -1]$

4.6. Calculating the grade of lake eutrophication Status

As indicated by the connection frame of identity, discrepancy and contrary, the projection function of the grade y_k and results g_k of complete estimation are built up as shown below [27]:

$$y_k = f(g_k) = -2g_k + 3, g_k \in [-1, 1] \quad (10)$$

By using the projection function Eq. (10), the grade of comprehensive estimation can be determined. Let the confidence level of interval number be α and $y_k \in [1,5]$

5. Case study

The water eutrophication levels of Qinghai lake, Chao lake, Waihaizhong lake, Waihaibei lake, Wuhan east lake, Hangzhou west lake, Tianchi lake, Erhai lake, Hulun lake, Fuxian lake, Hongze lake, Tai lake, Dianchi lake, West lake and Ping lake are evaluated with five indices namely:

1. Total phosphorus (TP)
2. Total nitrogen (TN)
3. Chemical oxygen demand (COD_{Mn})
4. Secchi disk depth (SD)
5. Biomass (BIO)

Chosen by the characters of the eastern plain lakes and consulted the most broadly utilized standards in China [28].

The classification standard of lake eutrophication and evaluation indices of measured data from the lake are considered as a set pair, a five element connection number is Constructed in relation to the I – V grades evaluation of:

1. Oligotrophic (Grade I)
2. Oligo-meso (Grade II)
3. Meso-trophic (Grade III)
4. Meso-eutro (Grade IV)
5. Eutrophic (Grade V)

Adopting the triangular fuzzy number to get the evaluation results for a confidence interval, the weight vector of evaluation is obtained using weighted average and getting the final evaluating grade as a confidence interval. A case study is done to examine the feasible and effective of the proposed method.

(Table 1) shows the evaluation standard set $A=[s_{m1},s_{m2},\dots,s_{m5}]$. and (Table 2) shows the surveyed data of the evaluation index set $B=[c_{m1},c_{m2},\dots,c_{mK}]$ which is the monthly average value from the daily monitoring data of fifteen lakes in China.

Table 1 Standard values of the eutrophication of the lakes in China

TP ($mg.m^{-3}$)	TN ($mg.m^{-3}$)	COD_{Mn} ($mg.L^{-1}$)	BIO ($10^4.L^{-1}$)	SD (m)	Trophic state
<1	<0.02	<0.09	<4	> 37.00	Oligotrophic (Grade I)
4	0.06	0.36	15	12.00	Oligo-meso (Grade II)
23	0.31	1.80	50	2.40	Mesotrophic(Grade III)
110	1.20	7.10	100	0.55	Meso-eutro (Grade IV)
>660	>4.60	>27.10	>1000	>0.17	Eutrophic (Grade V)

Table 2 Surveyed data of evaluation water quality in China lakes

Lakes	TP ($mg.m^{-3}$)	COD_{Mn} ($mg.L^{-1}$)	SD (m)	TN ($mg.m^{-3}$)	BIO ($10^4.L^{-1}$)
Qinghai lake	20	1.40	4.50	0.22	14.60
Chao lake	30	8.26	0.25	1.67	25.30
Waihaizhong lake	40	3.53	0.92	0.87	916.00
Waihaibei lake	56	3.37	0.87	1.08	945.00
Wuhan east lake	105	10.70	0.40	2.00	1913.70
Hangzhou west lake	130	10.30	0.35	2.76	6920.00

Tianchi lake	23	4.05	1.20	0.62	51.63
Erhai lake	34	2.11	0.30	0.49	22.36
Hulun lake	80	8.29	0.50	0.13	11.60
Fuxian lake	20	1.61	7.03	0.21	19.00
Hongze lake	100	5.50	0.30	0.46	11.50
Tai lake	20	2.83	0.50	0.90	100.00
Dianchi lake	20	10.13	0.50	0.23	189.20
West lake	33	3.70	1.60	0.50	8.01
Ping lake	177	51.00	0.41	2.86	7060.00

Given that set pair is characterized as a pair that comprises of two interrelated sets, assembling set A and B to form set pair H concerning the problem W [29], by using the data in (Table 1) as set A and (Table 2) as Set B, then using confidence level α as 0.75 (75%), the connection degree values of the evaluation index sample values $\mu_{mk}L$, $\mu_{mk}R$ are obtained using Eq. (4) and the connection degree matrix R is constructed by Eq. (6). The weight vector W_m of evaluation is obtained using Eq. (7) and Eq. (8).

Taking Qinghai lake as an example, the connection degree values of the evaluation index sample values $\mu_{mk}L$, $\mu_{mk}R$ and the weight vector W_m of evaluation are shown in Table below.

Table 3 Connection degree values $\mu_{mk}L$, $\mu_{mk}R$ and weight vector W_m for indexes of Qinghai Lake

Evaluation index	$\mu_{mk}L$	$\mu_{mk}R$	weight vector	W_m
μ_{TP1}	-0.2951	-0.0461	W_{TP}	0.1267
μ_{COD1}	-0.2361	0.0139	W_{COD}	0.1941
μ_{SD1}	-0.0833	0.1667	W_{SD}	0.4364
μ_{TN1}	-0.1950	0.0550	W_{TN}	0.1797
μ_{BIO1}	0.1307	0.5928	W_{BIO}	0.0631

The comprehensive assessment is calculated by Eq. (9) and Eq. (10) using the connection degree values of the evaluation index sample values $\mu_{mk}L$, $\mu_{mk}R$ and the weight vector W_m of evaluation.

The grade of lake eutrophication assessment results are obtained and results shown in (Table 4). (Table 4) also shows results of another assessment method

Table 4 Comparison of the set pair analysis based on triangular fuzzy number method (SPA (TFN)) with the improved set pair analysis method (ISPA)

Lakes	Grades ISPA		Grades SPA(TFN)		
		Grades	$\mu_{mk}L$	$\mu_{mk}R$	Grades
1. Qinghai lake	3.0898	III	2.7662	3.2926	III
2. Chao lake	4.0923	IV	3.9767	4.4141	IV
3. Waihaizhong lake	4.2347	IV	4.2684	4.7680	IV
4. Waihaibei lake	4.3192	IV	4.4710	4.5318	IV
5. Wuhan east lake	5.0000	V	4.9556	4.9766	IV
6. Hangzhou west lake	5.0000	V	5.0000	5.0000	V
7. Tianchi lake	3.3114	III	3.2902	3.8314	III
8. Erhai	3.7421	III	3.2270	3.8150	III

9. Hulun lake	3.1447	III	4.4062	4.6318	IV
10. Fuxian lake	3.0540	III	2.6462	3.1822	III
11. Hongzhe lake	3.9003	IV	3.7676	4.2958	IV
12. Tai lake	4.0117	IV	3.7684	3.9086	IV
13. Dianchi lake	4.2804	IV	4.7564	4.8172	IV
14. West lake	3.9714	III	3.0228	3.8086	III
15. Ping lake	5.0000	V	5.0000	5.000	V

As seen from Table 4 above the set pair analysis based on triangular fuzzy number SPA (TFN) evaluation results are coherent to the results of the (ISPA) used in reference [28] which shows that the SPA (TFN) is feasible and reliable in lake evaluation assessment. The SPA (TFN) method doesn't only give a concrete grade of evaluation result but also gives the evaluation grade as a confidence interval, along with the confidence level, which has advantages of high resolution and information utilization.

When the original data of a problem is uncertain, the evaluation grade can be expressed by confidence interval in which the real evaluation result is included in a given range and the boundary of unknown solution can be obtained through the operation of confidence interval number. The lake eutrophication assessment results of SPA (TFN) method is calculated at different confidence levels, confidence levels ranging from 75% to 100% using an interval of 5%. (75%, 80%, 85%, 90%, 95% and 100%) and the results shown in (Table 5) below

Table 5 The assessment results at different levels of confidence

Confidence level	75%	80%	85%	90%	95%	100%
Qinghai lake	[2.8408, 3.3389]	[2.8906, 3.2891]	[2.9404, 3.2393]	[2.9902, 3.1895]	[3.0400, 3.1397]	3.0898
Chao lake	[3.8092, 4.3753]	[3.8546, 4.3446]	[3.8999, 4.3138]	[3.9453, 4.2831]	[3.9907, 4.2523]	[4.0360, 4.2216]
Waihaizhong lake	[3.9535, 4.5158]	[3.9990, 4.4851]	[4.0445, 4.4543]	[4.0899, 4.4236]	[4.1354, 4.3929]	[4.1809, 4.3622]
Waihaibei lake	[4.0409, 4.5975]	[4.0862, 4.5691]	[4.1315, 4.5406]	[4.1768, 4.5121]	[4.2221, 4.4837]	[4.2674, 4.4552]
Wuhan east lake	[4.4898, 4.9127]	[4.5257, 4.8965]	[4.5617, 4.8802]	[4.5976, 4.8640]	[4.6336, 4.8478]	[4.6695, 4.8315]
Hangzhou west lake	[4.5477, 5.0249]	[4.5818, 5.0108]	[4.6159, 4.9967]	[4.6500, 4.9826]	[4.6841, 4.9685]	[4.7182, 4.9545]
Tianchi lake	[3.5408, 4.0045]	[3.5908, 3.9617]	[3.6408, 3.9190]	[3.6908, 3.8763]	[3.7408, 3.8335]	3.7908
Erhai lake	[3.4517, 4.0157]	[3.4984, 3.9760]	[3.5451, 3.9362]	[3.5919, 3.8964]	[3.6386, 3.8566]	[3.6853, 3.8169]
Hulun lake	[3.4068, 3.8537]	[3.4543, 3.8194]	[3.5018, 3.7852]	[3.5493, 3.7510]	[3.5968, 3.7168]	[3.6443, 3.6825]
Fuxian lake	[2.8384, 3.3384]	[2.8884, 3.2884]	[2.9384, 3.2384]	[2.9884, 3.1884]	[3.0384, 3.1384]	3.0884
Hongze lake	[3.6626, 4.1925]	[3.7093, 4.1595]	[3.7560, 4.1266]	[3.8027, 4.0936]	[3.8494, 4.0607]	[3.8961, 4.0277]
Tai lake	[3.8063, 4.2546]	[3.8557, 4.2196]	[3.9050, 4.1845]	[3.9543, 4.1495]	[4.0037, 4.1144]	[4.0530, 4.0793]
Dianchi lake	[3.8022, 4.2846]	[3.8503, 4.2515]	[3.8984, 4.2184]	[3.9465, 4.1853]	[3.9946, 4.1522]	[4.0427, 4.1191]
West lake	[3.1633, 3.6035]	[3.2102, 3.5623]	[3.2570, 3.5211]	[3.3038, 3.4799]	[3.3506, 3.4387]	3.3975
Ping lake	[4.6723, 4.7978]	[4.6974, 4.7978]	[4.7225, 4.7978]	[4.7476, 4.7978]	[4.7727, 4.7978]	[4.7978, 4.7978]

	5.0441]	5.0340]	5.0239]	5.0138]	5.0036]	4.9935]
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From (Table 5) above it is obvious that the evaluation level with higher confidence level is obviously smaller than that with lower confidence level. When the confidence level is 100% even the minimum and maximum values of some confidence intervals are equal and the evaluation level is one point while other rating levels are not. The difference between the maximum value and the minimum value of the confidence interval is also small, and the evaluation level varies almost within a small range. At this point, the evaluation level accuracy is highest. However, with the increase of confidence level, the evaluation range of confidence interval becomes smaller and smaller, the higher the accuracy, the lower the fuzziness. Taking Qinghai Lake as an example, when the confidence levels were 75%, 80%, 85%, 90%, 95% and 100%, the evaluation levels of Qinghai Lake were [2.8408, 3.3389], [2.8906, 3.2891], [2.9404, 3.2393], [2.9902, 3.1895], [3.0400, 3.1397], 3.0898 respectively. As seen in (Fig. 2) below.

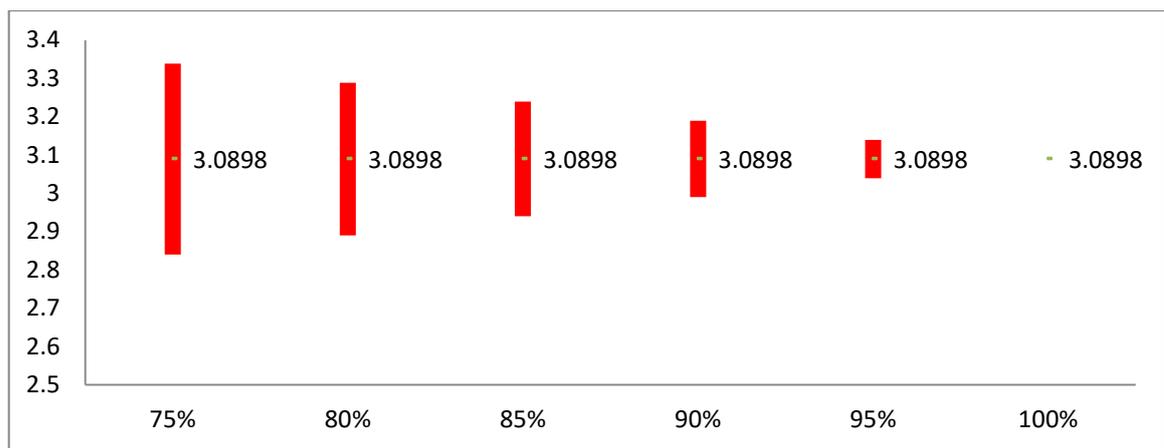


Figure 2. The results of Qinghai Lake at different levels of confidence

(Fig.2) shows that the higher the confidence level, the smaller the rating range, when the confidence level is 100%, the minimum and maximum values of the confidence interval are usually equal, and the evaluation level is one point value.

The SPA (TFN) allows a more flexible and subtle different way to deal with examination of research information in lake eutrophication evaluation. The new method doesn't only allow specialists to test hypotheses about their data but likewise more useful about such imperative highlights as sample size and the precision of point estimates of gathering contrasts and associations. Confidence intervals are useful in the interpretation of studies with small sample sizes, permitting specialists to draw more meaningful conclusions about the significance of such studies. Expanded utilization of SPA (TFN) by lake water resource managers alongside enhanced comprehension of confidence intervals with respect to lake eutrophication status evaluation will help to keep away from pointlessly rigid interpretation of lake eutrophication status evaluation results as we advance towards confirm based practice

6. Conclusion

1) The set pair analysis based on triangular fuzzy numbers enriches the traditional set pair analysis method, which is relatively simple and accurate, and has a wide application in the evaluation of lake eutrophication. The traditional evaluation method can only get a rough assessment of the evaluation grade, and can't further distinguish between the same grade. The proposed model cannot only give the specific evaluation grade, but also obtain a confidence interval.

2) The set pair analysis based on triangular fuzzy intervals the confidence intervals of results are different when the confidence level α takes different values between 0 and 1. The results are not a confidence interval but a precise value while $\alpha=1$.

3) The super-standard weight method takes into account the portions of water environment factors that may have uneven representation, and account for them by making the final assessment result reflect a more balanced and equal interpretation of the monitor data accurately.

4) Set pair analysis based on triangular fuzzy intervals is more feasible and informative, compared to traditional methods the proposed method is easier to interpret, understand and provides a strong scientific basis for decision making in Sustainable management of lake.

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