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Using Pde Mathematical Model For Airodynamics Of Lifting Surfaces In Non-Uniform Motion

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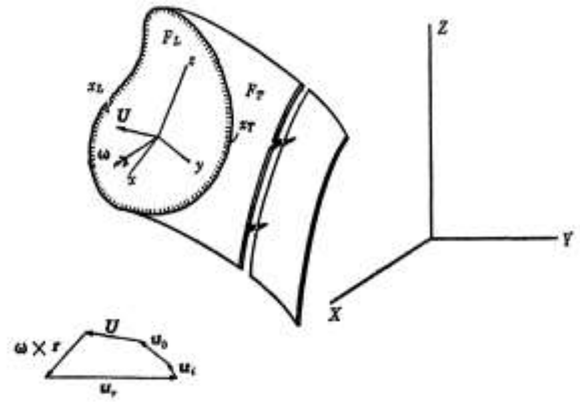
Abstract:
The main scope of this paper is to solve the Boundary value problems in airodynamics of lifting surfaces in non-uniform motion by using application of Partial differential equation. The known volume element reserved in to problem to solve the unknown aerodynamic parameters by partial differential equations. In earlier days these kinds of problems are solved by trial and error method. Using of Partial differential equation to obtain the results is very close to exact values.

1. INTRODUCTION

In this paper to discuss about aspects of the theory of lifting surfaces in non-uniform motion. Briefly, lifting surface theory is concerned with the motion of an impenetrable, deformable surface through an incompressible or compressible on-viscous fluid. In general the impenetrable surface is intended to represent approximately an airplane wing, a tail surface, or a propeller. The adjective lifting indicates the nature of the interaction desired between the impenetrable surface and the surrounding fluid.

Lifting surface theory as developed may be designated as a perturbation theory in the following sense. Because of the assumption of no viscosity there are evidently types of motion of an impenetrable surface which proceed without disturbing the surrounding fluid at all. One now asks for such motions which proceed *nearly* without producing any disturbances and one uses the assumption of small disturbances to simplify the differential equations and boundary conditions of the theory. In general this simplification leads to a linearized theory and it is this linearized theory which will here be discussed. The main reason for the considerable literature on the subject is the fact that the range of applicability of the linearized theory has been found adequate for many problems arising in engineering, and in particular in aeronautical engineering.



2. THE GENERAL PROBLEM

The actual problem of linearized lifting surface theory will be considered as an approximation to the following nonlinear problem. An impenetrable, deformable surface of given extent moves in a prescribed manner through a compressible perfect fluid. From part of the edge of the impenetrable surface emanates a surface of velocity discontinuity in such a manner that the fluid velocity remains finite along this part of the edge, henceforth called the *trailling* edge. Along the remainder of the edge called *leading* edge the fluid velocity will on account of the sharpness of the edge in general assume infinite values for an incompressible fluid. For a compressible fluid the assumption of a sharp leading edge will in general make impossible a continuous solution in the region exterior to the surfaces of discontinuity. We need not for the present purposes consider this difficulty as it disappears in the linearized form of the problem.

Let X, F, *Z* be the axes of a fixed frame of reference and let *x, y, z* be the axes of a frame of reference moving with the impenetrable surface (Fig. 1). Let *U(t)* be the velocity of the origin of the moving system with reference to the fixed system and let $\omega(t)$ be the angular velocity of the moving system with reference to the fixed system. Let u_r be the velocity of a fluid particle relative to the moving system and let *u* be the velocity of the same particle relative to the fixed system. The velocity vector *u* may be written in the form u_o+u_i, where U_o exists without being caused by the presence of the impenetrable surface and where u, is induced by the motion of the impenetrable surface. Correspondingly we have a pressure $p=p_o+p_i$ and a density $\rho=\rho_o+\rho_i$.

We then have the following kinematical relations involving velocity *u* and acceleration a,

(1) $u = u_r+U+\omega \times r$,

(2) $a = (\partial u/\partial t)+u_r.\nabla u+\omega\times u$.

The differential equations of the problem are of the following form ,

(3) $\rho a+\nabla p = 0$,

(4) $(\partial\rho/\partial t)+\nabla.(\rho u_r) = 0$,

(5) $p = f(\rho)$.

Equations (3) to (5) are to be solved in the space exterior to two surfaces $F_L= 0$ and $F_T = 0$, where F_L represents the given surface of pressure and velocity discontinuity and where *FT* represents a surface of velocity discontinuity, determination of which is part of the problem. On F_L we have the condition of no relative normal flow. On F_T we have the two conditions that the normal velocities of points of the surface are given by the corresponding velocities of the surrounding fluid and that the pressure is continuous across this surface. Thus

(6) $F_L(x,y,z,t) = 0;\ \partial F_L/\partial t+u_r.\nabla F_L = 0.$

(7) $F_T(x,y,z,t) = 0;\ \partial F_T/\partial t+u_{r\pm}.\nabla F_T =0, p_+ = p_-$,

where the subscripts + and -distinguish the two sides of F_T.

The surfaces F_L and F_T are connected along a line C_T which is part of the edge of $F_L=0$ and which is sufficiently described for the present purposes by the designation "trailing" edge. Along C_T we have the additional condition that *u* remains finite. In addition to the boundary condition (6) and (7) and the trailing edge conditions there are needed conditions at infinity. The form of these conditions will evidently depend on the form of the motion and on whether the fluid is compressible or incompressible.

For incompressible flow these conditions are roughly vanishing of all disturbances at an infinite distance from lifting surface and trailing surface. For compressible flow no such general statement can be made. In some cases all that is required is to superimpose on the conditions for imcompressible flow a condition stating that radiation energy is not created or reflected at infinity. In other cases the disturbances caused by the motion of the lifting surface cannot be required to vanish at infinity. General determination of these conditions is outside the scope of this report.

When the velocity distribution u_o which exists without the presence of the surface F_L is such that $\nabla\times u_o = 0$, equations (3) to (5) may be reduced to one scalar equation by means of the introduction of a velocity potential ϕ which, for incompressible flow, satisfies the Laplace equation but which for compressible flow is of a more general type.

3. THE LINEARIZED FORM OF THE GENERAL PROBLEM

Basic assumption for a linearized theory is that the lifting surface moves through the fluid *nearly* without disturbing the fluid such that powers and products of the quantities u_i ,p_i and ρ_i, and their derivatives may be neglected. The equation of motion (3) becomes the,

(8) $\partial u_i/\partial t-(U+\omega\times r).\nabla u_i+\omega\times u_i+u_i.\nabla u_o+u_o.\nabla u_i = -\frac{\nabla pi}{\rho_o}+\frac{\nabla p_o}{\rho_o}.\frac{\rho_i}{\rho_o}.$

The equation of continuity (4) becomes,

(9) $(\partial\rho_i/\partial t)+\nabla(\rho_o u_i)+\nabla[(u_o-U-\omega\times r)\rho_i] = 0.$

and the equation of change of state becomes,

(10) $p_i = f'(\rho_o)\rho_i$.

We shall in some of what follows write as abbreviations,

(11) $f'(\rho_o) = a^2_o$, $U+\omega\times r=V$.

The quantity a_o is the velocity of sound at a point of the undisturbed medium. Turning now to the boundary conditions (6) and (7) we begin by establishing the condition for the motion of a surface $F_{LP} = 0$ without any disturbance of the surrounding fluid. Equation (6) indicates that this condition is as follows,

(12) $\partial F_{LP}/\partial t + (u_o - V).\nabla F_{LP} = 0$.

We have chosen the subscript *P* to indicate this surface because we wish to refer to it henceforth as the projection of the lifting surface. As the actual lifting surface must deviate only slightly from this projection in order to move nearly without causing disturbances we may write,

(13) $F_L=F_{LP}+f_L=0$.

where f_L is small in the same sense as u_i,p_i and ρ_i are small. Then, considering (12), and except for quantities small of higher order, the boundary condition at the lifting surface becomes,

(14) $F_{LP} = 0$; $\partial f_L/\partial t+(u_o - V).\nabla f_L+u_i.\nabla F_{LP} = 0$.Note that in satisfying the boundary condition at the projection of the lifting surface rather than at the lifting surface itself we again depend on the perturbation properties of the solution to be obtained.

The next step is the determination of the form of the trailing surface of discontinuity F_T. As the equation of this surface is one of the unknowns of the problem we must, in order to have a linear problem, omit the term u_i in (7) and the shape of F_T is then such that the equation,

(15) $\partial F_T/\partial t+(u_o-V).\nabla F_T = 0$.

In addition to this we have the condition that at the trailing edge C_T the surface F_T is connected to the surface F_{LP}. The meaning of (IS) is that within the linearized theory the shape of the trailing surface of discontinuity is independent of the velocity distribution u_i induced by the motion of the lifting surface. Having the equation of F_T we then obtain from (17) the two conditions of continuous normal velocity and pressure across the surface in the form,

(16) $F_T=0$; $(u_{i+}-u_{i-}).\nabla F_T=0$,$p_+-p_- = 0$.

To the formulation of the problem as contained in equations (8) to (16) we must add the condition of finite u_i along C_T and appropriate conditions at infinity. We remark that previous formulations of this problem of nonsteady motion in their most general form are based on the assumptions $u_o = 0$, $\omega= 0$, $U=U_i$. Under this assumption, under the additional assumptions that $dU/dt= 0$ and $\partial F_{LP}/\partial t= 0$, Kuessner has obtained an integral equation for the pressure distribution at the lifting surface.

4.VELOCITY POTENTIAL FORMULATION OF THE LINEARIZED PROBLEM

In what follows we shall assume that the fluid is at rest except for the motion induced by the lifting surface, that is, we put,

(17) $u_o=0$, $\nabla p_o = 0$.

With these assumptions we have the existence of a velocity potential ϕ in terms of which,

(18) $u_i = \nabla\phi$.

Combination of (18), (17), and (8) gives for the pressure *pi* the following expression,

(19) $p_i / p_o = (U+\omega\times r).\nabla\phi - \partial\phi/\partial t\ \phi$.

Combination of (19), (11), (10), and (9) gives the following differential equation for ϕ,

(20) $\nabla^2\phi -(1/a_o^2)[\partial/\partial t-(U+\omega\times r).\nabla]^2\phi = 0$.

The boundary condition (14) becomes,

(21)

$$F_{LP}=0; \frac{\partial\phi}{\partial n}|\nabla F_{LP}| = (U+\omega\times r).\nabla f_L - \frac{\partial f_L}{\partial t}$$

$$\frac{\partial\phi}{\partial n}|\nabla F_{LP}| = (U+\omega\times r).\nabla f_L - \frac{\partial f_L}{\partial t}$$

The transition conditions (16) becomes,

(22)

$F_T=0$

$$\begin{cases}(\frac{\partial\phi}{\partial n})_+ = (\frac{\partial\phi}{\partial n})_-, \\ [\frac{\partial\phi}{\partial t} - (U+\omega\times r).\nabla\phi]_+ = [\frac{\partial\phi}{\partial t} - (U+\omega\times r).\nabla\phi]_-\end{cases}$$

and the trailing edge condition is that along C_T we have $\nabla\phi$ finite. The problem from here on is the solution of the mixed boundary value problem (20) to (22), with appropriate conditions at infinity.

The object of such solutions is primarily the determination of the pressures *pi* on both sides of the lifting surface. Solutions obtained so far are all for nearly plane lifting surfaces such that $F_{LP}\equiv z=0$. Predominant among these are solutions for the two-dimensional problem, which may be characterised by the requirement that $\partial\phi/\partial y=0$.

5. MOTION OF NEARLY PLANE LIFTING SURFACE IN INCOMPRESSIBLE FLOW

Further discussion will be carried out for this subclass of the general problem. Our object is to indicate the particular nature of the boundary

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value problem in question and to outline one of the possible methods of solution. We assume that the projection of the lifting surface lies in the *x*, y-plane and that the motion of the lifting surface is in the direction of negative *x*. We further assume incompressible flow.

We have then,

(23) $U = - Ui, \omega = 0, a_o = \infty.$

and, in accordance with (12) and (15),

(24)

$F_{LP}=z=0, F_T=z=0.$

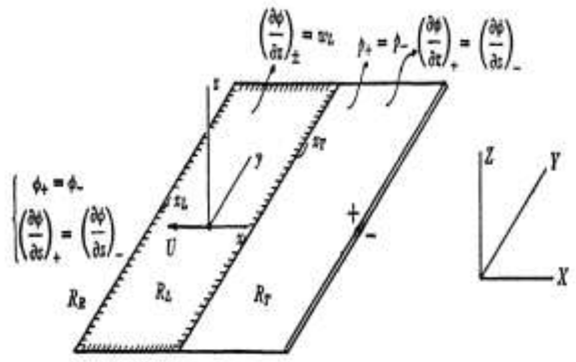

Fig. 2

We shall designate the region occupied by the projection of the lifting surface by R_L and the region occupied by the trailing surface of discontinuity by R_T (Fig. 2). From (20) follows that when $a_o=\infty$ the differential equation is, for steady as well as for nonsteady motion,

(25) $\nabla^2 \phi = 0.$

From (19) follows, for the pressure induced by the motion of the lifting surface,

(26) $$\frac{p_i}{\rho_o} = -\left(\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x}\right).$$

If the instantaneous distance of a point of the lifting surface from the *x*, y-plane is designated by *Z(x, y, t)* we have $F_L = F_L + f_L = z — Z(x, y\, y\, /\,) = 0$. Consequently the boundary condition (21) becomes,

(27) *xy* inside R_L:

$$\frac{\partial \phi}{\partial z} = \frac{\partial Z}{\partial t} + U\frac{\partial Z}{\partial x} = \omega_L.$$

The transition conditions (22) become,

(28) *x*, *y* inside R_T

$$\left\{\left(\frac{\partial \phi}{\partial z}\right)_+ = \left(\frac{\partial \phi}{\partial z}\right)_-, \left(\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x}\right)_+ = \left(\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x}\right)_-\right.$$

To equations (27) and (28) is to be added the condition of finite trailing edge velocities,

(29) $z = 0, x=x_T(y);$ $\nabla \phi$ finite and conditions at infinity which for incompressible flow may be taken in the form,

(30)

$$\begin{matrix} x = -\infty \\ z = \pm\infty \end{matrix} \left\{ \nabla\phi = 0, \qquad \frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} = 0. \right.$$

The above problem is to be understood as a boundary value problem for the exterior of an infinitely thin semi-infinite tube surrounding the regions R_L and R_T, in the sense that (27) holds for z=+0 and for *z* = — 0. It may be recalled that the main object is the determination of *pi+* and *pi-* in R_L, with *pi-—pi+* being the lift intensity produced by the motion of the impenetrable surface F_L. The form of the boundary conditions (27) to (30) is such that the problem for the exterior of the semi-infinite tube may be transformed by a symmetry consideration into a mixed boundary-value problem for one of the half spaces $z>0$ or $z<0$. This is done by observing that equations (27) and (30) are compatible with the assumption that ϕ is an odd function of *z*. If we define a region R_R as the *x*, y plane minus the regions R_L and R_T and take into account that ϕ is continuous except across R_L and R_T we may replace the boundary conditions (27) and (28) by the following system of conditions at z = 0:

(31)

$$\begin{cases} x, y, inside R_L : \frac{\partial \phi}{\partial z} = w_L, \\ x, y, inside R_T : \frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} = 0, \\ x, y, inside R_R : \phi = 0. \end{cases}$$

We then must determine ϕ in one of the half-spaces, say z>0, with the conditions (29) and (31) at the boundary *z* = 0 and with equations (30) giving the conditions at infinity. Let us remark that explicit solutions of the problem thus formulated are possible in the two-dimensional case by the use of elliptic cylinder coordinates, while for the circular plan form wing the use of spheroidal coordinates is appropriate. The problem would be of a standard nature if the conditions in R_T were the same as in R_R. The main difficulty of obtaining an explicit solution comes from the particular form of the boundary condition holding in R_T in as much as all that can be said on the basis of (31) about the values of ϕ in R_T is,

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(32) $\phi(x,y,0,t) = \Phi(x-\int U dt, y)$

where Φ is an arbitrary function of its two arguments. Compensating for this arbitrariness is, as will be seen, the finiteness condition (29).

6. INTEGRAL EQUATION FORMULATION OF THE PROBLEM

As one is interested primarily in the values of *pi* in R_L it suggests itself to derive an integral equation for this quantity. This procedure, adopted by Birnbaum, Possio , Kuessner , and others at the suggestion of Prandtl, and known under the name acceleration potential method, has the advantage that it can be developed without explicit introduction of the trailing surface of discontinuity. It does however have the disadvantage of leading to an integral equation with a distinctly more complicated kernel than the corresponding integral equation for the values of $\partial\phi/\partial x$ in R_L which we propose to discuss here. The main advantage of the latter formulation is that it permits immediate recognition of the explicit solvability of the problem of non-steady motion in terms of the solution of the corresponding problem of steady motion. Setting as an abbreviation,

(33) $\gamma(x,y,t) = \partial\phi(x,y,0,t)/\partial x$.

we have the follwing representation for $\partial\phi/\partial x$ in terms of the boundary values γ,

(34) $$\frac{\partial\phi}{\partial x} = \frac{-1}{2\Pi}\iint \gamma(\xi,\eta,t)\frac{\partial}{\partial z}\left(\frac{1}{r}\right)d\xi d\eta$$

Where

(35) $r^2=(x-\xi)^2+(y-\eta)^2+z^2.$

Equation (34) may be converted into an expression for $\partial\phi/\partial x$ by appropriate differentiation and integration, of the following form,

(36) $$\frac{\partial\phi}{\partial z} = \frac{-1}{2\Pi}\iint \gamma(\xi,\eta,t)\left[\int_{-\infty}^{x}\frac{\partial^2}{\partial z^2}\left(\frac{1}{r'}\right)dx'\right]d\xi d\eta$$

$$= \frac{-1}{2\pi}\iint \gamma(\xi,\eta,t)\left\{\frac{x-\xi}{r^3} + \frac{\partial}{\partial y}\left[\frac{y-\eta}{(y-\eta)^2+z^2}\left(\frac{x-\xi}{r}+1\right)\right]\right\}d\xi d\eta.$$

Note that when $\partial\gamma/\partial\eta=0$ which corresponds to the assumption of two-dimensional flow the η-integration in (36) can be carried out with the result that,

(36') $$\frac{\partial\phi(x,z,t)}{\partial z} = -\frac{1}{\pi}\int\frac{\gamma(\xi,t)(x-\xi)}{(x-\xi)^2+z^2}d\xi$$

which equation can of course be obtained directly in a simpler manner. To separate two-dimensional from three-dimensional effects the following transformation is useful. We write (omitting for brevity the t in γ),

(37) $\gamma(\xi,\eta) = \gamma(\xi,y) + [\gamma(\xi,\eta) - \gamma(\xi,y)]$

Appropriate integration by parts then gives the following relation,

(38) $$\frac{\partial\phi}{\partial z} = -\frac{1}{\pi}\int\frac{\gamma(\xi,y)(x-\xi)}{(x-\xi)^2+z^2}d\xi = -\frac{1}{2\pi}\iint\frac{\partial\gamma}{\partial\eta}\left\{\frac{y-\eta}{(y-\eta)^2+z^2}\left(\frac{x-\xi}{r}+1\right) + \frac{(x-\xi)(y-\eta)}{(x-\xi)^2+z^2}\left(\frac{1}{r}-\frac{1}{|y-\eta|}\right)\right\}d\xi d\eta.$$

We must now in equation (38) let z tend to zero and substitute the boundary conditions (31). It is advantageous that *(38)* is in such a form that, as can be proved, the two limiting processes of integration and of letting z tend to zero can be interchanged, provided the integrals are defined where appropriate as Cauchy principal values. Thus from (31) and (38),

(39) $$\omega_L(x,y,t) = -\frac{1}{\pi}\int_{x_L(y)}^{x_T}\frac{\gamma(\xi,y,t)}{x-\xi}d\xi - \frac{1}{\pi}\int_{x_T(y)}^{\infty}\frac{\gamma_T(\xi,y,t)}{x-\xi}d\xi$$

$$-\frac{1}{2\pi}\iint_{R_L}\frac{\partial\gamma}{\partial\eta}\left\{\frac{1}{y-\eta}+K(x-\xi,y-\eta)\right\}d\xi d\eta$$

$$-\frac{1}{2\pi}\iint_{R_T}\frac{\partial\gamma_T}{\partial\eta}\left\{\frac{1}{y-\eta}+K(x-\xi,y-\eta)\right\}d\xi d\eta$$

In (39) x_L and x_T indicate the coordinates of the leading and trailing edge, *yr* is still to be determined as far as possible from the boundary conditions and K is of the following form,

(40) $$K = \frac{((x-\xi)^2+(y-\eta)^2)^{1/2}-|y-\eta|}{(y-\eta)(x-\xi)}$$

We shall from now on assume for simplicity's sake that the region R_L is the rectangle $|x| \le b, |y| \le sb$ and that the velocity U which occurs in (31) is constant. We further introduce dimensionless variables,

(41) $x' = x/b,\ y' = y/b,\ t' = \omega t.$

and a dimensionless parameter k of the form,

(42) $$k = \frac{\omega b}{U}$$

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For the case of simple harmonic motion k is referred to as the "reduced frequency" of the motion. We may again for simplicity's sake omit in what follows the primes designating the dimensionless variables. We then have from (32) that,

$$(43) \quad \gamma_T(\xi, y, t) = \frac{\partial\Phi(\xi - t/k, y)}{\partial(\xi - t/k)}$$

Furthermore on account of the finiteness condition (29),

$$(44) \quad \Phi(1 - t/k, y) = \int_{-1}^{1} \gamma(x, y, t)dx \equiv \Gamma(t, y) \; .$$

Then,

$$(45) \quad \Phi(\xi - t/k, y) = \Gamma(t - k(\xi - 1), y) \; .$$

and with,

$$(46) \quad t - k(\xi - 1) = \tau, \quad d(k\xi - t) = -d\tau$$

we have from (43),

$$(47) \quad \gamma_T(\xi, y, t) = -k\frac{\partial\Gamma(\tau, y)}{\partial\tau}$$

We introduce (46) and (47) into (39) and obtain the following form of the integral equation of the problem,

For uniform motion we have $\partial\Gamma/\partial\tau = 0$ and the second and fourth integrals in (48) are absent. The form of equation (48) indicates clearly the manner in which for non-uniform motion the values of the solution $\gamma(x, y, t)$ depend on the past history of the motion through the cumulative effect of successive changes of,

$$(49) \quad \Gamma(\tau, y) = \int_{-1}^{1} \gamma(x, y, \tau)dx$$

The quantity Γ is one-half of what is usually referred to as the circulation intensity at a station y = constant of the lifting surface.

Equation (48) is now to be solved for γ, in terms of ω_L and Γ. Once this is done Γ is found by integration in terms of ω_L and therewith γ is expressed in terms of ω_L only. The pressure p_i at the lifting surface is then, according to (26), of the form,

$$(50) \quad \frac{p_i}{\rho_o U} = -\left[k\frac{\partial}{\partial t}\left(\int_{-1}^{x} \gamma(\xi, y, t)d\xi\right) + \gamma(x, y, t)\right]$$

The advantage of (48) compared with the corresponding equation for the values of p_i lies in the form of the kernel l/(x—ξ) in the first term.

This permits solution of (48) in a manner analogous to what is done for the problem of uniform motion.

7.SOLUTION OF THE TWO-DIMENSIONAL PROBLEM

In what follows we wish to describe briefly one of the possible methods of solution of this problem, namely that by L. Schwarz .We shall subsequently indicate how to utilize this method for an approximate solution of the three-dimensional problem. Introducing the (unessential)restriction of simple harmonic motion we set,

$$(51) \quad \omega_L = \varpi_L e^{it}, \quad \gamma = \bar{\gamma} e^{it}, \quad \Gamma = \bar{\Gamma} e^{it},$$

where the barred quantities are functions of the space coordinates at most. Equation (48) can then be written in the following form,

$$(52) \quad \varpi_L(x) = -\frac{1}{\pi}\int_{-1}^{1} \frac{\bar{\gamma}(\xi)d\xi}{x - \xi} + \frac{ik}{\pi}\bar{\Gamma}\int_{1}^{\infty} \frac{e^{ik(1-\xi)}}{x - \xi}d\xi$$

Equation (52) is solved by means of a pair of inversion formulas of the form,

$$(53a) \quad g(x) = -\frac{1}{\pi}\int_{-1}^{1} \frac{f(\xi)}{x - \xi}d\xi, \; f(1) \; finite \; ,$$

$$(53b) \quad f(x) = \frac{1}{\pi}\left(\frac{1-x}{1+x}\right)^{1/2}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\frac{g(\xi)}{x - \xi}d\xi$$

which may be considered as a result of two-dimensional potential theory, as discussed most fully by H. Söhngen.Application of (53) to (52) leads to the following expression for $\bar{\gamma}$.

$$(54) \quad \bar{\gamma}(x) = \frac{1}{\pi}\left(\frac{1-x}{1+x}\right)^{1/2}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\frac{g(\xi)}{x - \xi}d\xi$$

$$+ik\bar{\Gamma}\int_{1}^{\infty}\left(\frac{\xi+1}{\xi-1}\right)^{1/2}\frac{e^{ik(1-\xi)}}{x - \xi}d\xi \quad \}$$

The main difficulty from here on is the calculation of the pressure p_i which according to (50) is given by,

$$(55) \quad \frac{p_i}{\rho_o U} = -\left[ik\int_{-1}^{x}\bar{\gamma}(\xi)d\xi + \bar{\gamma}(x)\right]$$

Before listing the result of this lengthy and somewhat devious calculation we may indicate the nature of the equation for $\bar{\Gamma}$ which occurs in (54). If we integrate both sides of (54) as follows:

$$(56) \quad \int_{-1}^{1}\bar{\gamma}dx = \frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\left[\int_{-1}^{1}\left(\frac{1-x}{1+x}\right)^{1/2}\frac{dx}{x - \xi}\right]\varpi_L(\xi)d\xi$$

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$$+\frac{ik}{\pi}\bar{\Gamma}\int_1^{\infty}\left(\frac{\xi+1}{\xi-1}\right)^{1/2}\left[\int_{-1}^{1}\left(\frac{1-x}{1+x}\right)^{1/2}\frac{dx}{x-\xi}\right]e^{ik(1-\xi)d\xi}$$

and take account of the formulas,

(57a) $$\int_{-1}^{1}\left(\frac{1-x}{1+x}\right)^{1/2}\frac{dx}{x-\xi}=-\pi \quad , \quad |\xi|<1,$$

(57b) $$\int_{-1}^{1}\left(\frac{1-x}{1+x}\right)^{1/2}\frac{dx}{x-\xi}=-\pi\left(1-\left(\frac{\xi-1}{\xi+1}\right)^{1/2}\right), \quad 1\le\xi,$$

then equation (56) can be written one of the forms,

(58) $$\bar{\Gamma}=-\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\varpi_L(\xi)d\xi-ike^{ik}\bar{\Gamma}\int_1^{\infty}\left(\left(\frac{\xi+1}{\xi-1}\right)^{1/2}-1\right)e^{-ik\xi d\xi}$$

(59) $$\bar{\Gamma}=\frac{-\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\varpi_L(\xi)d\xi}{1+ike^{ik}\int_1^{\infty}\left(\left(\frac{\xi+1}{\xi-1}\right)^{1/2}-1\right)e^{-ik\xi d\xi}}$$

It is at this stage that a combination of Bessel functions makes its appearance in the theory. The integral in the denominator of (59) is expressible in terms of Hankel functions, as follows ,

(60) $$\int_1^{\infty}\left(1-\left(\frac{\xi+1}{\xi-1}\right)^{1/2}\right)e^{-ik\xi d\xi}=\frac{\pi}{2}\left[H_1^{(2)}(k)+iH_o^{(2)}(k)\right]+\frac{e^{-ik}}{ik}$$

In view of (60) F can be written in the alternate form,

(61) $$\bar{\Gamma}=\frac{\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\varpi_L(\xi)d\xi}{2^{-1}ike^{ik}\left[H_1^{(2)}(k)+iH^{(2)}{}_o(k)\right]}$$

Our purpose in outlining in some detail the steps leading from (52) to (61) has been to indicate the nature of some of the more simple transformations in the calculation of the pressure distribution on oscillating airfoils. Considerable care is necessary to arrange the analysis in such a manner that advantage is taken of all possible simplifications. In this way there is found the following expression for *pi* of equation (55),

(62) $$\frac{\bar{p}_i}{\rho_o U}=\frac{1}{\pi}\int_{-1}^{1}\left(\left(\frac{1-x}{1+x}\right)^{1/2}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\frac{1}{x-\xi}-ik\wedge\right)\varpi_L(\xi)d\xi$$

$$+\frac{Ck-1}{\pi}\left(\frac{1-x}{1+x}\right)^{1/2}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\varpi_L(\xi)d\xi$$

In (62) the function A is given by,

(63) $$\wedge(x,\xi)=\frac{1}{2}In\frac{1-x\xi+(1-x^2)^{1/2}(1-\xi^2)^{1/2}}{1-x\xi-(1-x^2)^{1/2}(1-\xi^2)^{1/2}}$$

and the function *C*, first introduced by Theodorsen , is of the form,

(64) $$C(k)=\frac{H_1^{(2)}(k)}{H^{(2)}(k)+iH_o^{(2)}(k)}.$$

The results outlined in this section find their main application in the analysis of airplane flutter. For this purpose explicit expressions have been obtained by Cicala , Kuessner , Theodorsen , and others for lift and moment amplitudes $\bar{L}$ and $\bar{M}_a$ defined by,

(65a) $$\bar{L}=2b\int_{-1}^{1}p_i dx,$$

$$\bar{M}_a=2b^2\int_{-1}^{1}(x-a)p_i dx.$$

and for control-surface hinge moments $\bar{M}_c$ defined by,

(65c) $$\bar{M}_c=2b^2\int_0^{1}(x-c)p_i dx.$$

For various appropriate forms of ϖ_L Plots of representative pressure distributions for various values of the reduced frequency *k* and for some of the more important forms of ω_L may be found in a recent paper by Postel and Leppert .

We may conclude this section with some remarks concerning the solution of the problem for non-oscillatory motion.It may readily be seen that the results for simple harmonic motion may be used for Laplace transform analysis by replacing wherever it occurs *ik* by — *q* where upon equations (61) and (62) become relations between Laplace transforms. For applications of the Laplace transformmethod in this field reference may be made to work by I. E. Garrick and W. R. Sears. Another form of the results consists in integro-differential equations for Γ, L and M_a, without any assumption concerning the form of the solution. The nature of these results may be seen from the simplest of them, the equation determining Γ. Omitting all but the first two of the integrals on the right of (48) we may obtain the following relation.

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$$(66)\int_{-1}^{1}\left(\tfrac{1+x}{1-x}\right)^{1/2}\omega_L(x,t)dx=-\tfrac{1}{\pi}\int_{-1}^{1}\gamma(\xi,t)\left\{\int_{-1}^{1}\left(\tfrac{1+x}{1-x}\right)^{1/2}\tfrac{dx}{x-\xi}\right\}d\xi+$$
$$\frac{1}{\pi}\int_{-\infty}^{t}\frac{d\Gamma}{d\tau}\left\{\int_{-1}^{1}\left(\frac{1+x}{1-x}\right)^{1/2}\frac{dx}{x-1-(t-\tau)/k}\right\}d\tau$$

After evaluating the inner integrals on the right of (66) we are left with an equation of the form,

$$(67)\,\frac{e^{ik(1-x)}}{2\pi}\int_{s}^{s}\frac{d\bar{\Gamma}}{d\eta}\left\{\frac{1}{y-\eta}-ikF\left[k(y-\eta)\right]\right\}d\eta$$

$$\Gamma(t)+\int_{-\infty}^{t}\frac{d\Gamma}{d\tau}\left[\left(\frac{t-\tau+2k}{t-\tau}\right)^{1/2}-1\right]d\tau=-\int_{-1}^{1}\left(\frac{1+x}{1-x}\right)^{1/2}\omega_L(x,t)dx.$$

A formal solution of (67) by Fourier series or Laplace transform methods is again readily obtained. A special case of such a solution is given by (61). Practical applications, especially of the Laplace transform solution, are however not a simple matter, the reason for this being the occurrence of Hankel function combinations in the denominator of the transforms to be evaluated, and the possibility of solving (67) directly by machine methods would be of considerable advantage.

CONCLUSION

Conclusion from the study is observed that, the methodology or the application of partial differential equation is suitable for predict the values which is close to exact solution of boundary value problems in aircrafts of lifting surfaces in non-uniform motion. This partial differential equation methodology is very compactable to apply in aerodynamic problems.Future work of this study is to implement the same methodology for the uniform motion, and it can include with the effect of flows.

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