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Soft Multiset and its Application in Information System

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Abstract

The theory of soft set and multiset are vital mathematical tools used in handling uncertainties about vague concepts. In 1999, Molodtsov proposed the theory of soft set as a general frame work for dealing with indefinite, abstract idea. This paper we shall present the application of soft multiset in information system and show that every soft multiset is a multi value information system.

1. Introduction

Most of the challenges encountered in engineering, medical sciences, economics and social sciences have various uncertain features. Although a number of mathematical tools like probability theory, fuzzy sets [1], rough sets [2] and interval mathematics [3] are well known and effective models for dealing with uncertainties. However, each of them has distinguished advantages as well as certain limitations. One major weakness shared by these theories is possibly the inadequacy of parameterization tools as pointed out by Molodtsov [4].

The origin of soft set theory could be traced to the work of Pawlak [5] in 1993 titled Hard and Soft set in proceeding the international EWorkshop on rough sets and knowledge discovery at Banfff. His notion of soft sets is a unified view of standard rough and fuzzy sets. This might have motivated D. Molodtsov's work in 1999 titled Soft set theory: first result. There in, the basic notions of the theory of soft sets and some of its possible applications were presented. This theory to some extent is free from the inadequacy of the parameterization tools of other classical set theory.

Recently, soft set theory has been developed rapidly and focused by some scholars in theory and practice. Based on the work of Molodtsov, Maji et al. [6] defined equality of two soft sets, subsets and super set of soft set, complement of a soft set, null soft set and absolute soft set with examples. They also defined binary operations such as AND, OR and the operation of union, intersection and De Morgan's law. Aktas and Cagman [7] introduced the basic properties of soft sets to the related concepts of fuzzy sets as well as rough sets and they gave a definition of soft group and derived the basic properties by using Molodtsov's definition of the soft sets. Liu and Yan [8] discussed the algebraic structure of fuzzy soft sets and gave the definition of fuzzy soft group. In their paper, they defined operations on fuzzy soft group and prove some results on them as well; they also presented fuzzy normal soft subgroup and fuzzy soft homomorphism and discussed their properties.

Feng et al. [9] have investigated the problem of combining soft sets with fuzzy sets and rough sets. M. I. Ali [10] discussed the concept of an approximation space associated with the soft set is defined.

Tutut Herawan [11] presented the notion of multi soft sets representing a multi valued information system in to binary valued information systems. The concept of topology on soft set is studied by researchers [12, 13, 14]. Y. Jiang et al. [15] present an adjustable approach to intuitionistic fuzzy soft set. Using rough set theory, Z. Zhang [16] proposes a new approach to intuitionistic fuzzy soft set..

Alkhazaleh et al. [17] extended the theory of soft set to soft multiset. They defined approximate value set, equal soft multiset, not set of soft multiset, complement of soft multiset, semi null soft multiset and absolute soft multiset. Union and intersection of soft multiset were also defined.

Ibrahim and Balami [19] presented soft multiset and its application in decision making problems. They pointed out that, the work of Alkhazaleh et al. [17] does not actually convey the notion of soft multiset. In [18] the concept of multiset was captured but with restriction. After pointing out the weakness they presented their definition of soft multiset which is free from the short comings of [17] and [18]. Other operations and basic terms were defined to reflect the concept of soft set and multiset.

The organization of the paper is as follows: in section 2 basic notions about soft set, multiset and soft multiset are reviewed. Section 3 presented the application of soft multiset in information system. Lastly, section 4 summarizes the entire work

2. Preliminaries and basic definitions

In this section, we recollect the basic definitions of soft set, multiset and soft multiset with relevant examples. Some important results on soft multiset is also presented.

Soft set is defined in the following way. Let U be an initial Universe set and E be a set of parameters. Let P (U) denotes the power set of $U$and $A \subset E$.

A pair (F, A) is called a softset over U where F is a mapping given by $F: A \longrightarrow P(U)$.

In otherwords, a Softset over U is a parametrized family of subsets of the universe U.

For $e \in A,\ F(e)$ may be considered as the set of $e-$ approximate elements of the Softset(F, A). Obviously, a softset is not a set (Molodtsov, 1999)

Example 1:

Suppose the following:

U is the set of routes from town A to town B

E is the set of parameters. Each parameter is a word or a sentence.

E= {very good; good; bad; short}.

In this case, to define a soft set means to point out very good routes, good routes, bad routes, or short routes. The soft set (F, E) describes the nature of the routes from town A to town B.

We consider below the same example in more details for our next discussion.

Suppose that, there are six routes in the Universe U given by

U= $\{R_1, R_2, R_3, R_4, R_5, R_6\}$

E= $\{e_1, e_2, e_3, e_4,\}$ where

e_1 Stands for a parameter 'very good'

e_2 Stands for a parameter 'good'

e_3 Stands for a parameter 'bad'

e_4 Stands for a parameter 'short'

Suppose that

$F(e_1)$=$\{R_2, R_4\}$

$F(e_2)$ = $\{R_3, R_5\}$

$F(e_3)$ = $\{R_1, R_6\}$

$F(e_4)$ = $\{R_3, R_4, R_6\}$

The softset (F, E) is a parameterized family $\{F(e_i), i = 1,2, \ldots, 4\}$of subsets of the set U and gives us a collection of approximate description of the nature of the routes.

Therefore, we can view the softset (F, E) as a collection of approximations as below

(F, E)=$\{$very good routes $=$ $R2,R4,$, good routes $=R3,R5,$, bad routes $=$ $R1,R6,$, short routes $=R3,R4,R6,$

Where each approximation has two parts:

i. A predicate P; and

ii. An approximate value set v(or simply to be called value set).

For example, for the approximation "very good routes = $\{R_{2,} R_{4,}\}$, we have the following:

i. The predicate name is very good routes; and

ii. The approximate value set or value set is $\{R_{2,} R_{4,}\}$

Tabular representation of a soft set

"U"	"Very good"	"good"	"bad"	"Short"

R_1	0	0	1	0
R_2	1	0	0	0
R_3	0	1	0	1
R_4	1	0	0	1
R_5	0	1	0	0
R_6	0	0	1	1

Table 2.1

Thus, a softset (F, E) can be viewed as a collection of approximation below: (F, E)= $\{P_1 = V_1,\ P_2 = V_2, \dots, P_n = V_n\}$.

For the purpose of storing a soft set in a computer, we could represent a soft set in the form of table 3.1 above, (corresponding to the soft set in the example above).

Example 2

(i) Let (X, τ) be a topological space, that is, X is a set and τ is a toplogy (a family of subsets of X called the open sets of X). Then, the family of open neighbourhoods $T(x)$ of point x, where $T(x) = \{V \in \tau \mid x \in V\}$, may be considered as the soft set $(T(x), \tau)$.

(ii) Let A be a fuzzy set and μ_A be the membership function of the fuzzy set A, that is, μ_A is a mapping of U into $[0, 1]$, let $F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}, \alpha \in [0, 1]$ be a family of $\alpha -$ level sets for function μ_A. If the family F is known, $\mu_A(x)$ can be found by means of the definition: $\mu_A(x) = Sup_{\substack{\alpha \in [0,1], \\ x \in F(\alpha)}} \alpha$. Hence every fuzzy set A may be considered as the soft set $(F, [0, 1])$.

(iii) Let $U = \{C_1,\ C_2,\ C_3,\ C_4,\ C_5,\ C_6,\ C_7,\ C_8,\ C_9,\ C_{10}\}$ be a set of cars under consideration, E be a set of parameters.

$E =$ $\{e_1 =$ Expensive, $e_2 =$ beautiful, $e_3 =$ manual gear, $e_4 =$ cheap, $e_5 =$ automatic gear, $e_6 =$ in good repair, $e_7 =$ in bad repair$\}$.Then the soft set (F, E) describes the attractiveness of the cars.

2.1 Definition of Terms

Definition 2.1.1:value – class.

The class of all value sets of a softset (F, E) is called the value – class of the softset and is denoted by$C_{(F,E)}$. For the example above, $C_{(F,E)}$= $\{v_1, v_2, \dots, v_n\}$Clearly, $C_{(F,E)} \widetilde{\subset} P(U)$.

Definition 2.1.2:soft subset.

Let (F, A) and (G, B) be two softset over a commonuniverse U, we say that (F, A) is a softsubset of (G, B) if

i. $A \subset B$

ii. $\forall e \in A, F(e)$and $G(e)$ are identical approximations. We write$(F, A) \subseteq (G, B)$.

$(F,A)$is said to be a soft superset of (G,B) if $(G,B)$is a soft subset of (F,A). We denote it by$(F,A) \supseteq (\mathrm{G},\mathrm{B})$.

Definition 2.1.3: Equality of two softsets

Two softsets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a softsubset of (G,B) and $(G,B)$is a softsubset of (F,A).

Definition 2.1.4: Not set of a set of parameters.

Let $E = \{e_1,\ e_2, \dots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\urcorner E$ is defined by $\urcorner E = \{\urcorner e_1, \urcorner e_2, \dots, \urcorner e_n\}$where$\urcorner e_i =$ $not\ e_i, for\ all\ \ i.$

The following results are obvious

Proposition 2.1

1. $\urcorner(\urcorner A) = A$
2. $\urcorner(A \cup B) = (\urcorner A \cup \urcorner B)$
3. $\urcorner(A \cap B) = (\urcorner A \cap \urcorner B)$

Consider the example above:

Here, $\urcorner E$ = {not very good; not good; not bad; not short}.

Definition 2.1.5: Complement of a softset.

The complement of a softset (F,A) is denoted by $(F,A)^c$and is definedby$(F,A)^c = (F^c, \urcorner A)$, where$F^c(\alpha) = U - F(\urcorner\alpha), \forall\alpha \in \urcorner A$.

Let us call F^c to be the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.

Definition 2.1.6: Null softset.

A softset (F,A) over U is said to be a Null softset denoted by Φ, if $\forall\ \epsilon \in$ A, $F(\epsilon) = \emptyset$, (null-set).

Definition 2.1.7: Absolute softset.

A softset (F,A) over U is said to be absolute softset denoted by

Ã, if $e \in A, F(e) = U$

Clearly, $\tilde{A}^c = \Phi$ and $\Phi^c = \tilde{A}$

Definition 2.1.8: AND operation on two softsets

If (F,A) and (G,B) are two softsets then “(F,A) and (G,B)" denoted by $(F,A)\wedge(G,B)$ is defined by $(F,A)\wedge(G,B) = (H, A \times B)$, where

$\mathrm{H}(\alpha,\beta) = F(\alpha) \cap F(\beta), \forall(\alpha,\beta) \in \mathrm{A} \times \mathrm{B}$

Definition 2.1.9: OR operation on two soft sets

If (F,A) and (G,B) are two soft sets then

“(F,A) OR (G, B)” denoted by (F,A) V (G,B) is defined by (F,A) V (G,B) = (O, A×B), where $O(\alpha,\beta) = F(\alpha) \cup G(\beta), \forall(\alpha,\beta)\epsilon A \times B$.

Definition 2.1.10: Disjoint Soft set.

Let $(F,A) and\ (G,B)$ be two sof tsets over a common universe$U$. Then $(F,A) and (G,B)$ are said to be disjoint if

$(F,A) \Cap (G,B) = (H,C)$. Where $C = A \cap B = \emptyset$ and for every $\varepsilon \epsilon C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon) = \emptyset$

2.2 Concept of multiset

A multiset is an unordered collection of objects in which, unlike a standard (cantorian) set objects are allowed to repeat. In other words, a multiset A over a set D is a pair (D,F) such that $F: D \longrightarrow N$ is a function where N is the set of natural numbers, $\{0,1,2, \ldots, n, \ldots\}$. D is called the ground or domain set of all the multisets constructed from D. The number of times an element x occurs in A is called the multiplicity of that element denoted by $M_A(x)$ or $A(x)$ multiset is generally abbreviated as mset.

2.3 Some Basic Definitions in Multiset

a. The mset A for any ground set D is called empty, denoted by Ø or [ ], if $M_A(x) = 0$ for all $x \in D$.

b. For a given mset A, the set of all its objects is called its root set usually denoted by A^*, and the sum of the multiplicities of all its objects is called its cardinality usually denoted by $C(A)$ or $/A/$.

c. An mset is called regular or constant if all its objects occur with the same multiplicity. Also an mset is called simple if all its objects are the same, for example, $[x]_5$ is a simple mset containing x as its only object.

d. Two msets A and B are said to be cognate or similar if $\forall x (x \in A \Leftrightarrow x \in B)$, where x is an object. Thus, similar msets have equal root sets but need not be equal themselves.

e. Let A and B be two msets. A is an msubset or a submultiset of B, written as $A \sqsubseteq B$, or $B \sqsupseteq A$ if $m_A(x) \leq m_B(x)$ for all $x \in D$. Also, if $A \subseteq B$ and $A \neq B$, then A is called a proper submultiset of B.

g. Two msets A and B are equal or the same, written as $A = B$, iff for any object $x \in D$. $m_A(x) = m_B(x)$ or $A(x) = B(x)$. Equivalently, $A = B$ if every element of A is in B and conversely.

h. The power multiset of a given mset A, denoted by $P(A)$, is the multiset of all submultisets of A. However, in general, no combinatorial formula known to preserve cantor's power set theorem in multiset.

i. Let A and B be two msets over a given domain set D. The union of A and B denoted by $A \cup B$, is defined as $A \cup B = max\{m_A(x), m_B(x)\}$. For all $x \in D$.

j. Let A and B be two msets over a given domain set D. The intersection of two msets A and B denoted by $A \cap B$, is the mset defined by $A \cap B = min\{m_A(x), m_B(x)\}$ for all $x \in D$

k. Let A and B be two msets over a given domain set D. The direct sum or arithmetic addition of A and B denoted by $A + B$ or $A \uplus B$ is the mset defined by $A \uplus B = m_A(x) + m_B(x) = m_A(x) + m_B(x)$, for all $x \in D$

2.4 Soft multiset

Let $\{U_i\colon\ i \in I\}$ be a collection of universes such that there exists U_j,U_k and $U_j \cap U_k \neq \emptyset$. Suppose$U = \uplus_{i\in I} P(U_i)$, where $P(U_i)$ denotes the power set of U_i, and E be a set of parameters. A pair (F, A), where $A \subseteq E$, is called a soft multiset over U.F is a mapping given by $F\colon A \longrightarrow U$. That is, a soft multiset over U is a parametrized family of submultisets of U such that for $e \in A, F(e)$ is considered as the set of e–approximate element of the soft multiset (F, A).

Examples 2.4.1

Let $S_i\colon i \in N$ be a collection of states in a country and$U_i\colon i \in N$ be a collection of states with availability of land, labour and raw materials.

Suppose

$U_1 = \{S_1, S_2, S_3\}$ be a set of states with availability of land,

$U_2 = \{S_2, S_4, , S_6,\}$ be a set of states with availability of labour

$U_3 = \{S_2, S_4, S_7,\}$be a set of state with availability of raw materials.

$U = P(U_1) \uplus P(U_2) \uplus P(U_3) =$

$$\left\{\begin{matrix} \{S_1\}, \{S_2\}, \{S_3\}, \{S_1, S_2\}, \{S_1, S_3\}, \{S_2, S_3\}, \{S_1, S_2, S_3\}, \\ \emptyset, \{S_2\}, \{S_4\}, \{S_6\}, \{S_2, S_4\}, \{S_2, S_6\}, \\ \begin{Bmatrix} S_4, \\ S_6 \end{Bmatrix}, \\ \{S_2, S_4, S_6\}, \emptyset, \{S_2\}, \{S_4\}, \{S_7\}, \{S_2, S_4\}, \{S_2, S_7\}, \\ \{S_4, S_7\}, \{S_2, S_4, S_7\}, \emptyset, \end{matrix}\right\}$$

$=$

$$\left\{\begin{matrix} \{S_1\}, 3\{S_2\}, \{S_3\}, 2\{S_4\}, 3\emptyset, \{S_6\}, \{S_7\}, \{S_1, S_2\}, \{S_1, S_3\}, \{S_2, S_3\}, 2\{S_2, S_4\}, \{S_1, S_2, S_3\}, \{S_2, S_4\}, \{S_4, S_6\}, \\ \{S_2, S_4, S_6\}, \{S_2, S_7\}, \{S_4, S_7\}, \{S_2, S_4, S_7\} \end{matrix}\right\}$$

Which is a multiset.

Let E be a set of decision parameters related to the above universes, where

E = {e_1 = peaceful, e_2 = kidnapping, e_3 = Army robbery, e_4= Accessibility, e_5 = market}.

Let A = {e_1 = peaceful, e_2 = kidnapping, e_3 = Army robbery, e_4 = Accessibility}.

The Soft multiset(F, A) is a parametrized family $\{F(e_i),\ i = 1, 2, \ldots, 4\}$of subsets of the set U and gives us a collection of approximate description of the conditions of some states in a country favourable to Mr. X for the location of his manufacturing industries.

Then we can view the soft multiset(F, A) as consisting of the following collection of approximations

Definition 2.4.1: Multi value-class. The class of all value set of a Soft multiset (F, A) is called the value class of the softmultiset and is denoted by $C^*(F, A) = \{V_1, V_2, \ldots, V_n\}$. Obviously $C^*_{(F, A)} \subseteq U$. Also, if there exists at least one i such that $V_i = V_j$,

$\forall\, i, j = 1,2, \ldots, n$, then the value-class of the soft multiset (F, A) is called Multivalue-class of the softmultiset (F, A) and is denoted by $Cm(F, A)$. Similarly $Cm(F, A) \subseteq U$.

Definition 2.4.2 Let $(F, A)$and (G, B) be two softmultiset over U, we say that (F, A) is a softmultisubset of (G, B) written as $(F, A) \subseteq (G, B)$, if $m_{(F,A)}(x) \leq m_{(G,B)}(x)$ for all $x \in U$, Also, if $A \subseteq B\ and\ A \neq B$, then (F, A) is called a proper Softmultisubset of (G, B) written as $(F, A) \subset (G, B)$.

Definition 2.4.3 Two Soft multisets (F, A) and (G, B) over U are said to be equal if and only if (F, A) is a softmultisubset of (G, B) and (G, B) is a softmultisubset of (F, A).

Definition 2.4.4 NOT Set of a set parameters.

Let E be a set of parameters The NOT set of E denoted by$\rceil E$ is defined by$\rceil E = \{\rceil e1, \rceil e2, \ldots, \rceil en\}$ where$\rceil ei =$ $not\ ei, \forall i$.

Proposition 2.1

1. $\rceil(\rceil A) = A$
2. $\rceil(A \cup B) = (\rceil A \cup \rceil B)$
3. $\rceil(A \cap B) = (\rceil A \cap \rceil B)$

Considering the example 4.1above

Definition 2.4.5 Similar Soft multisets

Two Soft multisets(F, A) and (G, B) are said to be 'Cognate' or similar if $\forall x$ $(x \in (F, A) \Leftrightarrow x \in (G, B))$ where x is an object. Therefore, similar Softmultisets have equal root sets but need not be equal themselves.

Definition 2.4.6 Let (F, A) and (G, B) be two Softmultisets over U. (F, A)⋓ (G, B) is the Softmultiset defined by

$M_{(F,A)} \cup M_{(G,B)}(x) = M_{(F,A)}(x) \cup M_{(G,B)}(x) =$ $maximum\left(M_{(F,A)}(x), M_{(G,B)},(x)\right)$,

being the union of two numbers.

Definition 2.4.7 Let (F, A) and (G, B) be two soft multisets over U. Then, the intersection of (F, A) and (G, B) written as $(F, A) \Cap (G, B)$ is the Softmultiset defined by

$M_{(F,A)} \cap M_{(G,B)}(x) = M_{(F,A)}(x) \cap M_{(G,B)}(x) =$ $minimum\ (M_{(F,A)}(x), M_{(G,B)}(x)\ )$ being the intersection of two numbers. That is, an object x occurring a times in (F, A) and b times in (G, B), occurs minimum (a, b) times in $(F, A) \Cap (G, B)$, which always exists.

Difference

Definition 2.4.8 Let (F, A) and (G, B) be two softmultisets over U, and $(G, B) \subseteq (F, A)$, Then $M_{(F,A)} - M_{(G,B)}(x) =$ $M_{(F,A)}(x) - M_{(F,A)} \cap M_{(G,B)}(x)$ for all$x \in U$.

It is sometimes referred to as the arithmetic difference of (G, B) from (F, A). Note that, even if $(G, B) \subset (F, A)$, this definition still holds good

Definition 2.4.9 Let (F, A) and (G, B) be two Softmultisets defined by

$M_{(F,A)} \uplus M_{(G,B)}(x) = M_{(F,A)}(x) + M_{(G,B)}(x)$, for any $x \in U$, direct sum of two numbers. That is, an object x occurring a times in (F, A) and b times in (G, B), occurs $a + b$ times in $(F, A) \uplus (G, B)$.

Definition 2.4.10 The complement of a Soft multiset(F, A) is denoted by (F, A) and is defined by

$\overline{(F, A)} = (F, \bar{7}A), where\ F: 7A \rightarrow$ $U\ is\ a\ mapping\ given\ by$

$F\ (x) = U - F\ (7\ x), \forall\ x \in 7A.$

The Complement of a Soft multiset (F, A) is denoted by (F, A) and is defined by

$\overline{(F, A)} = U - (F, A) = Mu(x) -$ $M(F, A)\ (x)\ for\ all\ x \in U.$

Proposition 2.2

1. $\overline{\overline{(F,A)}} = (F,A)$,

Proposition 2.3 $If\ (F,A),(G,B)\ and\ (H,C)$are three Softmultisets over U, then

1. $(F,A) \Cup ((G,B) \Cup (H,C) = ((F,A) \Cup (G,B)) \Cup (H,C)$,
2. $(F,A) \Cup (F,A) = (F,A)$,
3. $(F,A) \Cap ((G,B) \Cap (H,C) = ((F,A) \Cap (G,B)) \Cap (H,C)$,
4. $(F,A) \Cap (F,A) = (F,A)$,

Proof: The proof is straight forward

Some Properties holding for Soft multiset Operations

i. **Commutativity:**

$$(F,A) + (G,B) = (G,B) + (F,A)$$

$$(F,A) \Cup (G,B) = (G,B) \Cup (F,A)$$

ii. **Associativity:**

$$(F,A) \uplus ((G,B) \uplus (H,C)) = ((F,A) \uplus (G,B)) \uplus (H,C)$$

iii. **Idempotency:**

$$(F,A) \Cup (F,A) = (F,A),$$

$$(F,A) \Cap (F,A) = (F,A)\ but\ (F,A) \uplus (F,A) \neq (F,A)$$

iv. **Distributivity:**

$$(F,A) \uplus ((G,B) \Cup (H,C)) = ((F,A) \uplus (G,B)) \Cup (F,A) \uplus (H,C)),$$

$$(F,A) \uplus ((G,B) \Cap (H,C)) = ((F,A) \uplus (G,B)) \Cap ((F,A) \uplus (H,C)),$$

$$(F,A) \Cup ((G,B) \Cap (H,C)) = ((F,A) \Cup (G,B)) \Cap (F,A) \Cup (H,C)),$$

$$(F,A) \Cap ((G,B) \Cup (H,C)) = ((F,A) \Cap (G,B)) \Cup ((F,A) \Cap (H,C))$$

The proof of all these identities follows from the interpretation of $\Cup,\Cap$
and $\uplus$ of two natural numbers as maximum, minimum and (direct) Sum respectively.
It is easy to see that $\uplus$ is stronger than both
$\Cup\ and\ \Cap$ in the sense that neither $\Cup\ nor\ \Cap$
distributes over $\uplus$, where as $\uplus$ distributes over
both $\Cup\ and\ \Cap$.

Definition 2.2 (Knowledge representation system)

A knowledge representation system can be formulated as follows: Knowledge representation system is a pair $S = (U,A)$, where
U =a nonempty, finite set called the universe,
A = a nonempty, finite set of primitive attributes
Every primitive attribute a∈A is a total function
$a: U \rightarrow Va\ where\ Va$is the set of values of a, called the domain of a.

Definition 2.3

With every subset of attribute $B \subseteq A$, we associate a binary relation $IND\,(B)$, called an indiscernibility relation, defined by $IND\,(B) = \{(x,y) \in U2: for\ every\ a \in B, a\,(x) = a(y)\}$.
Obviously, $IND\,(B)$ is an equivalence relation and IND (B)= ∩ IND (a).
Every subset $B \subseteq A$ will be called an attribute. If B is a single element set, then B is called primitive, otherwise the attribute is said to be compound.

Attribute B may be considered as a name of the relation $IND\ (B)$, or in other words-as a name of knowledge represented by an equivalence relation $IND\ (B)$.

Definition 2.4

Let R be a family of equivalence relations and $A \in R$. we will say that A is dispensable in $R\ if\ IND(R) = IND\ (R - \{A\})$; otherwise A is indispensable in R. the family R is independent if each $A \in R$ is indispensable in R; otherwise R is dependent.

If R is independent and $P \subseteq R$, then P is also independent $A \subset P$ is a reduct of $P\ if\ Q$ is $k = (U, R)$independent and $IND(Q) = IND(P)$. Intuitively, a reduct of knowledge is its essential part, which suffices to define all basic concepts occurring in the considered knowledge, where as the core is in a certain sense its most important part. The set of all indispensable relations in P will be called the core of P, and will be denoted $CORE(P)$.Clearly $CORE(P) = \cap RED(P), where\ RED(P)$ is the family of all reducts of P.

The use of the concept of the Core is two fold. First it can be used as a basis for computation of all reducts, for the core is included in every reduct, and its computation is straight forward.

Secondly, the Core can be interpreted as the set of the most characteristics part of knowledge, which cannot be eliminated when reducing the knowledge.

Definition 2.5: dependency of knowledge the dependency can be defined as below; Let be a knowledge base and let $P, Q \subseteq R$.

1. Knowledge Q depends on knowledge $P\ iff\ IND(P) \subseteq IND(Q)$
2. Knowledge $P\ and\ Q$are equivalent, denoted as $P \equiv Q, iff\ P \Rightarrow Q\ and\ Q \Rightarrow P.$
3. Knowledge $P\ and\ Q$are independent, denoted as $P \not\equiv Q\ iff\ neither\ P \Rightarrow Q\ nor\ Q \Rightarrow P$ hold.

Clearly, $P \equiv Q, iff\ IND\ (P) = IND(Q)$

3. An Application of soft multiset theory in information system

Molodtsov [4] presented some applications of soft set theory in several directions, which includes: the study of smoothness of functions, game theory, operations research, Riemann-integration, perron integration, probability, theory of measurement, etc. It has been shown that there is compact connection between soft sets and information system. From the concept and the example of soft multisets given in the foregoing section, it can be seen that a soft multi set is a multi-valued information system.

Definition 3.1 A multi-valued information system is a quadruple $I = (X, A, f, V)$ where X is a non empty finite set of objects, A is a non empty finite set of attribute, $V = \cup_{a \in A} Va$ where V is the domain (value set) set of attribute a which has multi value $(IVa\ l \geq 3)$ and f: $U\ x\ A \Rightarrow V$ is a total function such that f $(U, a) \in V_a$ for every (U, a) $\in XxA$.

Proposition 3.1 If (F, A) is a softmultiset over universe $U(F, A)$then is a multi-value information system.

Proof: Let (F, A) be a softmultiset over U. We define a mapping f where

$f: UxA \Rrightarrow V \; as \; f(U, a) = CF(a) \; (U) \; where \; C$is the count of element U in the multiset F(a). Hence, $V = U_{a \in A} V_a \; where \; V_a$ is the set of all counts of U in $F(a)$. Then the multi-valued information system (U, A, f, V) represents the soft multiset (F, A)

Example Let U = $\left\{ {}^{C_1}/_{S_1}, {}^{C_2}/_{S_2}, {}^{C_3}/_{S_3}, {}^{C_4}/_{S_4}, {}^{C_5}/_{S_5}, {}^{C_6}/_{S_6} \right\}$ and

Let $A = \{Peaceful, Accessible, Market\}$

Then $F\ (Peaceful) = \left\{ {}^{C_1}/_{S_1}, {}^{C_3}/_{S_3}, {}^{C_6}/_{S_6} \right\}$

$$F\ (Accessible) = \left\{ {}^{C_1}/_{S_1}, {}^{C_5}/_{S_5} \right\}$$

$$F\ (Market) = \left\{ {}^{C_2}/_{S_2}, {}^{C_4}/_{S_4} \right\}$$

Then the soft multiset defined above describes the conditions of some states in a country. Then the quadruple I= (X, A, f, V) corresponding to the soft multi set given above is a multivalued information system.

Where $X = U$ and A is the set of parameters in the softmultiset and $V_{peaceful}$ = $\{C_1, C_3, C_6\}$, $V_{accessible}$= $\{C_1, C_5\}$ and V_{market} = $\{C_2 C_4\}$. For the pair (S_1, Peaceful) we have f(S_1, peaceful) = C_1, for (S_2, market), we have f(S_2,, market) = C_2,. Continuing in this way we obtain the values of other pairs. Therefore, according to the result above, it is seen that soft multisets are multi-valued information systems. Nevertheless, it is obvious that multi-valued information systems are not necessarily soft multisets.

We can construct an information table representing soft multiset (F, A) defined above as follows.

An Information Table

‘States’	*‘Peaceful’*	*‘Accessible’*	*‘Market’*
S_1	C_1	C_1	0
S_2	0	0	C_2
S_3	C_3	0	0
S_4	0	0	C_4
S_5	0	C_5	0
S_6	C_6	0	0

Table 3.1

4 Conclusion

In this paper, the basic concepts of soft sets, multiset and soft multiset were reviewed. Basic definitions of terms in soft set, multiset and soft multiset are presented. Some important proposition and results were stated and proved. Basic supporting tools in information system were also defined. Application of soft multiset in information system were presented and discussed.

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