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On Relative Soft Set

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Abstract

Molodtsov in 1999 introduced the concept of soft set theory which can be used as a general mathematical tool for dealing with uncertainties about vague concepts. In this paper, we recall the definition of soft set, its properties and operations. We introduce the definition of relative soft set, its basic properties and operations such as union, intersection and complement. We give relevant examples for these concepts. Basic properties of the operations are also given.

Keywords: soft set, relative soft set, operations, parameters.

1. Introduction

Most of the challenges we come across in engineering, medical sciences, economics and social sciences have various uncertain features. Although a number of mathematical tools like probability theory, fuzzy sets [1], rough sets [2] and interval mathematics [3] are well known and effective models for dealing with uncertainties. However, each of them has distinguished advantages as well as certain limitations. One major weakness shared by these theories is possibly the inadequacy of parameterization tools as pointed out by Molodtsov [4].

The origin of soft set theory could be traced to the work of Pawlak [5] in 1993 titled Hard and Soft set in proceeding the international EWorkshop on rough sets and knowledge discovery at Banff. His notion of soft sets is a unified view of standard rough and fuzzy sets. This might have motivated D. Molodtsov's work in 1999 titled Soft set theory: first result. There in, the basic notions of the theory of soft sets and some of its possible applications were presented. This theory to some extent is free from the inadequacy of the parameterization tools of other classical set theory.

Recently, soft set theory has been developed rapidly and focused by some scholars in theory and practice. Based on the work of Molodtsov, Maji et al. [6] defined equality of two soft sets, subsets and super set of soft set, complement of a soft set, null soft set and absolute soft set with examples. They also defined binary operations such as AND, OR and the operation of union, intersection and De Morgan's law. Aktas and Cagman [7] introduced the basic properties of soft sets to the related concepts of fuzzy sets as well as rough sets and they gave a definition of soft group and derived the basic properties by using Molodtsov's definition of the soft sets. Liu and Yan [8] discussed the algebraic structure of fuzzy soft sets and gave the definition of fuzzy soft group. In their paper, they defined operations on fuzzy soft group and prove some results on them as well; they also presented fuzzy normal soft subgroup and fuzzy soft homomorphism and discussed their properties.

Feng et al. [9] have investigated the problem of combining soft sets with fuzzy sets and rough sets. M. I. Ali [10] discussed the concept of an approximation space associated with the soft set is defined.

Tutut Herawan [11] presented the notion of multi soft sets representing a multi valued information system in to binary valued information systems. The concept of topology on soft set is studied by researchers [12, 13, 14]. Y. Jiang et al. [15] present an adjustable approach to intuitionistic fuzzy soft set. Using rough set theory, Z. Zhang [16] proposes a new approach to intuitionistic fuzzy soft set..

Alkhazaleh et al. [17] extended the theory of soft set to soft multiset. They defined approximate value set, equal soft multiset, not set of soft multiset, complement of soft multiset, semi null soft multiset and absolute soft multiset. Union and intersection of soft multiset were also defined.

The organization of the paper is as follows: In section 2 basic notions about soft set is reviewed. Section 3 focuses on relative soft set with relevant examples. We also presented the basic properties of the operations. In section 4 we draw conclusion to the work done.

2. Preliminaries and Basic definitions

In this section, we recollect the basic definitions of soft set with relevant examples.

Soft set is defined in the following way. Let U be an initial Universe set and E be a set of parameters. Let P (U) denotes the power set of $U$and $A \subset E$.

Definition 2.1 A pair (F, A) is called a softset over U where F is a mapping given by $F: A \longrightarrow P(U)$.

In other words, a Soft set over U is a parametrized family of subsets of the universe U.

For $e \in A,\ F(e)$ may be considered as the set of $e-$ approximate elements of the Softset(F, A). Obviously, a softset is not a set (Molodtsov, 1999)

Example2. 1:

Suppose the following:

U is the set of routes from town A to town B

E is the set of parameters. Each parameter is a word or a sentence.

E= {very good; good; bad; short}.

In this case, to define a soft set means to point out very good routes, good routes, bad routes, or short routes. The soft set (F, E) describes the nature of the routes from town A to town B.

We consider below the same example in more details for our next discussion.

Suppose that, there are six routes in the Universe U given by

U= $\{R_1, R_2, R_3, R_4, R_5, R_6\}$

E= $\{e_1, e_2, e_3, e_4,\}$ where

e_1 Stands for a parameter ‘very good’

e_2 Stands for a parameter ‘good’

e_3 Stands for a parameter ‘bad’

e_4 Stands for a parameter ‘short’

Suppose that

$F(e_1) = \{R_2, R_4\}$

$F(e_2) = \{R_3, R_5\}$

$F(e_3) = \{R_1, R_6\}$

$F(e_4) = \{R_3, R_4, R_6\}$

The soft set (F, E) is a parameterized family $\{F(e_i), i = 1,2, \ldots, 4\}$of subsets of the set U and gives us a collection of approximate description of the nature of the routes.

Therefore, we can view the soft set (F, E) as a collection of approximations as below

(F, E)=$\{$very good routes $=$ $R2,R4,$, good routes $=R3,R5,$, bad routes $=$ $R1,R6,$, short routes $=R3,R4,R6,$

Where each approximation has two parts:

i. A predicate P; and

ii. An approximate value set v(or simply to be called value set).

For example, for the approximation “very good routes = $\{R_{2,} R_{4,}\}$, we have the following:

i. The predicate name is very good routes; and

ii. The approximate value set or value set is $\{R_{2,} R_{4,}\}$

Tabular representation of a soft set

“U”	“Very good”	“good”	“bad”	“Short”
R_1	0	0	1	0
R_2	1	0	0	0
R_3	0	1	0	1
R_4	1	0	0	1
R_5	0	1	0	0
R_6	0	0	1	1

Table 2.1

Thus, a soft set (F,E) can be viewed as a collection of approximation below: $(F,E) = \{P_1 = V_1,\ P_2 = V_2, \dots, P_n = V_n\}$.

For the purpose of storing a soft set in a computer, we could represent a soft set in the form of table 3.1 above, (corresponding to the soft set in the example above).

Definition 2.3: Value – class.

The class of all value sets of a softset (F,E) is called the value – class of the softset and is denoted by $C_{(F,E)}$. For the example above, $C_{(F,E)} = \{v_1, v_2, \dots, v_n\}$Clearly, $C_{(F,E)} \widetilde{\subseteq} P(U)$.

Definition 2.4: Soft subset.

Let (F,A) and (G,B) be two softset over a commonuniverse U, we say that (F,A) is a softsubset of (G,B) if

i. $A \subset B$

ii. $\forall e \in A, F(e)$and $G(e)$ are identical approximations. We write$(F,A) \subseteq (G,B)$.

$(F,A)$is said to be a soft superset of (G,B) if $(G,B)$is a soft subset of (F,A). We denote it by$(F,A) \supseteq (G,B)$.

Definition 2.5: Equality of two softsets

Two softsets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a softsubset of (G,B) and $(G,B)$is a softsubset of (F,A).

Definition 2.6: Not set of a set of parameters.

Let $E = \{e_1,\ e_2, \dots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \dots, \rceil e_n\}$where$\rceil e_i =$ $not\ e_i, for\ all\ \ i.$

The following results are obvious

Proposition 2.7

1. $\rceil(\rceil A) = A$
2. $\rceil(A \cup B) = (\rceil A \cup \rceil B)$
3. $\rceil(A \cap B) = (\rceil A \cap \rceil B)$

Definition 2.8: Complement of a softset.

The complement of a softset (F,A) is denoted by $(F,A)^c$and is definedby$(F,A)^c = (F^c, \rceil A)$, where$F^c(\alpha) = U - F(\rceil\alpha), \forall\alpha \in \rceil A$.

Let us call F^c to be the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.

Definition 2.9: Null softset.

A softset (F,A) over U is said to be a Null softset denoted by Φ, if $\forall\ \epsilon \in$ A, $F(\epsilon) = \emptyset$, (null-set).

Definition 2.10: Absolute softset.

A softset (F,A) over U is said to be absolute softset denoted by

$\tilde{A}$, if $e \in A, F(e) = U$

Clearly, $\tilde{A}^c = \Phi$ and $\Phi^c = \tilde{A}$

Definition 2.11: AND operation on two softsets

If (F,A) and (G,B) are two softsets then "(F,A) and (G,B)" denoted by $(F,A)\wedge(G,B)$ is defined by $(F,A)\wedge(G,B)=(H,A\times B)$, where

$H(\alpha,\beta)=F(\alpha)\cap F(\beta), \forall(\alpha,\beta)\in A\times B$

Definition 2.12: OR operation on two soft sets

If (F,A) and (G,B) are two soft sets then

"(F, A) OR (G, B)" denoted by (F,A) V (G,B) is defined by (F,A) V (G,B) = (O, A×B), where

$O(\alpha,\beta)=F(\alpha)\cup G(\beta), \forall(\alpha,\beta)\epsilon A\times B.$

Definition 2.13: Disjoint Softset.

Let $(F,A) and\ (G,B)$ be two softsets over a common universe$U$. Then $(F,A)and(G,B)$ are said to be disjoint if

$(F,A)\Cap(G,B)=(H,C)$. Where C $=A\cap B=\emptyset$ and for every$\varepsilon\epsilon C$, H(ε) = F(ε) ∩ G(ε) = ∅

3. Relative Soft Set

In this section, we introduce the definition of a relative soft set and its basic operations such as union, intersection and complement. We give relevant examples for these concepts. Essential properties of the operation are also given.

Let $\{U_i: i\in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i=\emptyset$ and let $\{E_{Ui}: i\in I\}$ be a collection of set of parameters. $U=P(U_i)$ denotes the power set of U_i, $E=E_{Ui}$ and $A\subseteq E$.

Definition 3.1. A pair (F,A) is called a relative soft set over U, where F is a mapping given by $F: A\to U$. In other words, a relative soft set over U is a parameterized family of subsets of the universe U. For $e\in A$, $F(e)$ may be considered as the set of e-approximate elements of the relative soft set (F,A). Based on the above definition, any change in the ordering of the universes will produce a different relative soft set.

As an illustration, suppose that there are two universes U_1 and U_2, let us consider a relative soft set (F,A) which describes the "attractiveness of cloths", and "shoes" that Mr. K is considering putting on for a job interview. Let $U_1=\{C_1,C_2,C_3,C_4,C_5\}$ be the set of cloths and $U_2=\{S_1,S_2,S_3,S_4\}$ be the set of shoes. Let $E_U=\{E_{U1},E_{U2}\}$ be the collection of sets of decision parameters, where

$E_{U1}=\{e_{U1},1=expensive, e_{U1},2=cheap, e_{U1},3=beautiful\}$

$$E_{U2}=\{e_{U2},1=expensive, e_{U2},2=made\ in\ italy, e_{U2},3=black\}$$

Let $U=P(U_i), E=E_{Ui}$ and $A\subseteq E$ such that $i=1,2.$ $A=\{a_1=(e_{U1},1,e_{U2},1), a_2=(e_{U1},1,e_{U2},2), a_3=(e_{U1},2,e_{U2},3), a_4=(e_{U1},3,e_{U2},2)\}$

Suppose that

$F(a_1)=(\{C_2,C_3\},\{S_1,S_4\}),$

$F(a_2)=(\{C_1,C_3\},\{S_2,S_3\}),$

$F(a_3)=(\{C_1,C_4,C_5\},\{\ \}),$

$F(a_4)=(\{C_2,C_5\},\{S_2,S_3\}).$

Then we can see the relative soft set (F,A) as consisting of the following approximations.

$(F, A) = \{(a_1, (\{C_2, C_3\}, \{S_1, S_4\})), (a_2, (\{C_1, C_3\}, \{S_2, S_3\})),$

$(a_3, (\{C_1, C_4, C_5\}, \{\ \})), (a_4, (\{C_2, C_5\}, \{S_2, S_3\}))\}$

. We can see that each approximation has two parts viz ; a predicate and an approximate value set. The illustration can logically be explained as follows: For $F(a_1) = (\{C_2, C_3\}, \{S_1, S_4\})$, IF $\{C_2, C_3\}$ is the set of expensive cloths to Mr. K . Then the set of relatively expensive shoes to him is $\{S_1, S_4\}$. It has been shown that relative soft set is a conditional relation.

Example 3.2. Suppose that there are three universes U_1, U_2 and U_3. Let us consider the relative soft set (F, A), which describes the condition of some states in a country Mr. X with enough capital is considering for the location of his manufacturing industries.

Let $U_1 = \{S_1, S_2, S_3\}$ be a set of states with availability of land,

$U_2 = \{S_4, S_5, S_6\}$ be a set of states with availability of labour and

$U_3 = \{S_7, S_8, S_9\}$ be a set of state with availability of raw materials.

Let $E_U = \{E_{U1}, E_{U2}, E_{U3}\}$ be a collection of the set of parameters related to the above universes. Where

$$E_{U1} = \{e_{U1}, 1 = peaceful\ state, e_{U1}2 = commercial\ state, e_{U1}, 3 = armed\ robbery\ state, e_{U1}, 4 = good\ weather\ state, e_{U1}, 5 = densely\ populated\ state\}$$

$$E_{U2} = \{e_{U2}, 1 = powerstate, e_{U2}, 2 = harsh\ weather\ state, e_{U2}, 3 = violent\ state, e_{U2}, 4 = densely\ populated\ state$$

$$E_{U3} = \{e_{U3}, 1 = accesible\ state, e_{U3}2 = good\ weather\ state, e_{U3}, 3 = power\ state, e_{U3}, 4 = sparsely\ populated\ state\}$$

Let $U = P(U_i)$, E_{Ui} and $A \subseteq E$ such that $i = 1, 2, 3$.

$$A = \{a_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), a_2 = (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), a_3 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4), a_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4), a_5 = (e_{U1}, 5, e_{U2}, 1, e_{U3}, 1), a_6 = (e_{U1}, 1, e_{U2}, 4, e_{U3}, 4\}$$

Suppose that

$F(a_1) = (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})$,

$F(a_2) = (\{S_1, S_2\}, \{S_6\}, \{S_9\})$,

$F(a_3) = (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)$,

$F(a_4) = (\{S_2\}, \{S_6\}, \emptyset)$,

$F(a_5) = (\{S_3\}, \{S_5\}, \{S_8\})$,

$F(a_6) = (\{S_2, S_3\}, \{S_6\}, \emptyset)$,

Then we can view the relative soft set (F, A) as consisting of the following approximations:

$$(F, A) = \{(a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (a_2, (\{S_1, S_2\}, \{S_6\}, \{S_9\})), (a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (a_4, (\{S_2\}, \{S_6\}, \emptyset)), (a_5, (\{S_3\}, \{S_5\}, \{S_8\})), (a_6, (\{S_2, S_3\}, \{S_6\}, \emptyset))\}$$

Each approximation has two parts: a predicate name and an approximate value set.

We can logically explain the example1 as follows: For $F(a_1) = (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})$, IF $\{S_2, S_3\}$ is the set of peaceful states to Mr. X then the states he can relatively obtain regular electric power supply from is $\{S_5\}$ and IF $\{S_2, S_3\}$ is the set of peaceful states to Mr.

X and $\{S_5\}$ is the set of state he can obtain regular electric power supply THEN the set of relatively accessible state to him is $\{S_7, S_8\}$. It is obvious that the relative soft set is a conditional relation.

Definition 3.3. For any relative soft set (F, A), a pair $\left(\mathrm{e}_{\mathrm{Ui,j}}, F_{\mathrm{eUi,j}}\right)$ is called a $U_i -$ relative soft set part $\forall$ $\mathrm{e}_{\mathrm{Ui,j}} \in a_k$ and $F_{eUi,j} \subseteq F(A)$ is an approximate value set, where $a_k \in A, k = \{1,2,3, \dots, n\}, i \in \{1,2,3, \dots, m\}$ and $j \in \{1,2,3, \dots, r\}$.

Example 3.4

Consider Example 3.2. Then

$$\left(\mathrm{e}_{\mathrm{U1,j}}, F_{\mathrm{eU1,j}}\right) = \{(e_{U1}, 1, \{S_2, S_3\}),$$

$$(e_{U1}, 2, \{S_1, S_2\}),$$

$$(e_{U1}, 4, \{S_1, S_2, S_3\}), (e_{U1}, 3, \{S_4\}),$$

$$(e_{U1}, 1, \{S_2, S_3\})(e_{U1}, 5, \{ S_3\})\}$$

is a $U_1 - relative$ soft set part of (F, A)

Definition 3.5. For two relative soft sets (F, A) and (G, B) over U, (F, A) is said to be relative soft subset of (G, B) if

1. $A \subseteq B$ and
2. $\forall \ e_{Ui,j} \in a_k, \left(e_{Ui,j},\ F_{eUi,j}\right) \subseteq \left(e_{Ui,j},\ G_{eUi,j}\right)$

Where $a_k \in A, k = \{1,2,3, \dots, n\}, i \in \{1,2,3, \dots, m\}$ and $j \in \{1,2,3, \dots, r\}$.

This relationship is denoted by $(F, A) \widetilde{\subseteq} (G, B)$. In this case (G, B) is called a relative soft superset of (F, A).

Definition 3.6 Equal relative soft sets.

Two relative soft sets (F, A) and (G, B) over U are said to be equal if (F, A) is a relative soft subset of (G, B) and (G, B) is a relative soft subset of (F, A).

Example 3.7

Consider Example 3.2 . Let

$$A = \{a_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), a_2 = (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), a_3 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4), a_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4)\}$$

$$B = \{b_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), b_2 = (e_{U1}, 1, e_{U2}, 2, e_{U3}, 1),$$

$$b_3 = (e_{U1}, 2, e_{U2}, 3, e_{U3}, 1), b_4 = (e_{U1}, 5, e_{U2}, 4, e_{U3}, 2),$$

$$b_5 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 3), b_6 = (e_{U1}, 2, e_{U2}, 3, e_{U3}, 2)\}$$

Clearly $A \subseteq B$. Let (F, A) and (G, B) be two relative soft sets over the same U such that

$$(F, A) = (a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (a_2, (\{S_1, S_2\}, \{S_6\}, \{S_9\})), \left(a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)\right), \left(a_4, (\{S_2\}, \{S_6\}, \emptyset)\right)$$

$$(G, B) = \{(a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (a_2, (\{S_1, S_2\}, \{S_6\}, \{S_9\}))$$

$, (a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (a_4, (\{S_2\}, \{S_6\}, \emptyset)),$

$(a_5, (\{ S_3\}, \{S_5\}, \{ S_8\})), (a_6, (\{S_2, S_3\}, \{S_6\}, \emptyset)) \}$

Therefore, $(F, A) \tilde{\subseteq} (G, B)$.

Definition 3.8. NOT Set of a set of parameters.

Let $E = E_{Ui}, i = 1,2, \dots, m$ where E_{Ui} is a set of parameters. The NOT set of E denoted by $\urcorner E$ is defined by $\urcorner E = \urcorner E_{Ui}$

where $\urcorner E_{Ui} = \left\{\urcorner e_{Uij,}, = \text{not } e_{Uij,}, \forall i, j\right\}$.

Definition 3.9 Complement of a relative soft set.

The complement of a relative soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \urcorner A)$ *where* $F^c\colon \urcorner A \longrightarrow U$ is a mapping given by $F^c(\alpha) = U - F(\urcorner\alpha), \forall \alpha \in \urcorner A$.

Example 3.10

Consider Example 4.1 Here

$(F, A)^c$

$= \{(\urcorner a_1, (F(\urcorner a_1))), (\urcorner a_2, (F(\urcorner a_2))), (\urcorner a_3, (F(\urcorner a_3))),$

$(\urcorner a_4, (F\urcorner (\urcorner a_4))), (\urcorner a_5, (F(\urcorner a_5))), (\urcorner a_6, (F(\urcorner a_6))),$

$= \{(\urcorner a_1, (\{S_1\}, \{S_4, S_6\}, \{S_9\})),$

$(\urcorner a_2, (\{S_3\}, \{S_4, S_5\}, \{S_7, S_8\})),$

$(\urcorner a_3, (\emptyset, \{S_4, S_5\}, U_3)),$

$(\urcorner a_4, (\{S_1, S_3\}, \{S_4, S_5\}, U_3)),$

$(\urcorner a_5, (\{S_1, S_2\}, \{S_4, S_6\}, \{S_7, S_9\})),$

$(\urcorner a_6, (\{S_1\}, \{S_4, S_5\}, U_3)),$

Definition 3.11 Semi-null relative soft set.

A relative soft set (F, A) over U is called a semi-null relative soft set denoted by $(F, A)_{\approx\Phi i}$, if at least one of a relative soft set parts of (F, A) equals. ϕ.

Example 3.12

Consider Example 3.2 again, with a relative soft set (F, A) which describes the conditions of some states in a country. Let

$$A = \{a_3 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4), a_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4), a_6 = (e_{U1}, 1, e_{U2}, 4, e_{U3}, 4\}$$

Then the relative soft set (F, A) is the collection of approximation as follows:

$(F, A)_{\approx\Phi 1} = \{a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset), (a_4, \{S_2\}, \{S_6\}, \emptyset), (a_6, \{S_2, S_3\}, \{S_6\}, \emptyset)\}$

Then $(F, A)_{\approx\Phi 1}$ is a semi-null relative soft set.

Definition 3.13 Null relative soft set.

A relative soft set (F, A) over U is called a null relative soft set denoted by $(F, A)_{\Phi}$, if all the relative soft set parts of (F, A) equals $\emptyset$.

Example 3.14

Consider Example 3.2 again, with a relative soft set (F,A) which describes the condition of some cars Mr. X is considering to purchase. Let

$A=\{a_1=(e_{U1},4,e_{U2},3,e_{U3},4),a_2=$

eU1,4,eU2,4,eU3,4. Then relative soft set *F,A* is the collection of approximations as below:

$(F,A)_{\Phi}=\{(a_1,(\emptyset,\emptyset,\emptyset)),(a_2,(\emptyset,\emptyset,\emptyset))\}$.

Then $(F,A)_{\approx\Phi 1}$ is a null relative soft set.

Definition 3.15 Semi-absolute relative soft multiset.

A relative soft set (F,A) over U is called a semi-absolute relative soft set denoted by $(F,A)_{\approx Ai}\ if\ (e_{Ui,j},\ F_{eUi,j})=U_i.$ for at least one $i,\ a_k\in A,k=\{1,2,3,\dots,n\},i\in \{1,2,3,\dots,m\}\ and\ j\in\ \{1,2,3,\dots,r\}$.

Example 3.16

Consider Example 3.2 again, with a relative soft set (F,A) which describes the conditions of some states with respect to the nearness of the state to the capital of the country, availability, labour and raw materials.

Let $A=\{(a_1=(e_{U1,}4,e_{U2,}1,e_{U3,}1),a_2=(e_{U1,}4,e_{U2,}3,e_{U3,}1),$

$a_3=(e_{U1,}4,e_{U2,}3,e_{U3,}3)\}$.

The relative soft set (F,A) is the collection of approximations as given below:

$(F,A)_{\approx Ai}$

$=\{(a_1(U_1,\{S_5\},\{S_7,S_8\})),(a_2(U_1,\{S_6\},\{S_9\})),$

$(a_3,(U_1,\{S_6\},\ \emptyset))\}$.

Then $(F,A)_{\approx\Phi 1}$ is a semi-absolute relative soft set.

Definition 3.17 Absolute relative soft set.

A relative soft set (F,A) over U is called an absolute relative soft set denoted by $(F,A)_A\ if\ (e_{Ui,j},\ F_{eUi,j})=U_i.\forall_i$.

Example 3.18

Consider Example 3.2 again, with a relative soft set (F,A) which describes the condition of some states with respect to the nearness of the state to the capital of the country , amount of rainfall the state receive in a year, nature of soil in the state. Let $A=\{a_1=(e_{U1,}4,e_{U2,}3,e_{U3,}4),a_2=$

eU1,4, eU2,4,eU3,4.

The relative soft set (F,A) is the collection of approximations as shown below:

$(F,A)_A=\{(a_1,(U_1,U_2,U_3)),(a_2,(U_1,U_2,U_3))\}$.

Then, $(F,A)_A$ is an absolute relative soft set.

Proposition 3.19

If (F,A) is a relative soft set over U, then

1. $((F,A)^c)^c=(F,A)$,
2. $(F,A)^c{}_{\approx\Phi i}=(F,A)_{\approx Ai}$,

3. $(F,A)^c{}_{\Phi} = (F,A)_A$,

4. $(F,A)^c{}_{\approx Ai} = (F,A)_{\approx \Phi i}$,

5. $(F,A)^c{}_A = (F,A)_{\Phi}$.

Proof:

The proof is straightforward.

Definition 3.20 The union of two relative soft sets.

The union of two relative soft sets (F,A), and (G,B), denoted by $(F,A) \cup (G,B)$is defined by $(F,A) \cup (G,B) = (H,C)$ such that for all $e \epsilon C$

$$\mathrm{H}(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in \mathrm{A} - \mathrm{B} \\ G(\varepsilon), & if\ \varepsilon \in \mathrm{B} - \mathrm{A} \\ F(\varepsilon) \cup G(\varepsilon), & if\ \varepsilon \in \mathrm{A} \cap \mathrm{B}. \end{cases}$$

Example 3.21

Consider Example 3.2. Let

$A = \{a_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), a_2 = (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), a_3 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4),$

$a_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4)$ and

$B = \{b_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), b_2 = (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), b_3 = (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4),$

$b_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4), b_5 = (e_{U1}, 5, e_{U2}, 1, e_{U3}, 1), b_6 = (e_{U1}, 1, e_{U2}, 4, e_{U3}, 4)\}$.

Suppose $(F,A)$and (G,B) are two relative soft set over the same U such that

$(F,A) = \{(a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (a_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\}$

$(a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (a_4, \{S_2\}, \{S_6\}, \emptyset,))\}$and

$(G,B) = \{(b_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (b_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\}$

$(b_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (b_4, \{S_2\}, \{S_6\}, \emptyset,))\}$

$(b_5, (\{S_3\}, \{S_5\}, \{S_8\})), (b_6, (\{S_2, S_3\}, \{S_6\}, \emptyset))\}$.

Therefore, $(F,A) \tilde{\cup} (G,B) = (H,C)$

$= \{(C_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (C_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\})),$

$(C_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (C_4, \{S_2\}, \{S_6\}, \emptyset,)), \}$

$(c_5, \{S_3\}, \{S_5\}, \{S_8\})), (c_6, (\{S_2, S_3\}, \{S_6\}, \emptyset))\}$

Proposition 3.22. If $(F,A), (G,B)\ and\ (H,C)$ are three relative soft sets over U, then

1. $(F,A) \tilde{\cup} ((G,B) \tilde{\cup} (H,C)) = ((F,A) \tilde{\cup} (G,B)) \tilde{\cup} (H,C)$,

2. $(F,A) \tilde{\cup} (F,A) = (F,A)$,

3. $(F,A) \tilde{\cup} (G,A)_{\approx \Phi i} = (R,A)$,

4. $(F,A) \tilde{\cup} (G,A)_{\Phi} = (F,A)$,

5. $(F,A) \tilde{\cup} (G,B)_{\approx \Phi i} = (R,D)$,

6. $(F,A) \tilde{\cup} (G,B)_{\Phi} = \begin{cases} (F,A) & if\ A = B, \\ (R,D) & otherwise \end{cases}, where\ D = A \cup B,$

7. $(F,A) \tilde{\cup} (G,A)_{\approx \mathrm{A}i} = (R,A)_{\approx \mathrm{A}i}$,

8. $(F,A)\,\tilde{\cup}\,(G,A)_{\mathrm{A}} = (G,A)_{\mathrm{A}}$,

9. $(F,A)\,\tilde{\cup}\,(G,B)_{\approx Ai} =$

$\begin{cases}(R,D)_{\approx Ai} & if\ A = B, \\ (R,D) & otherwise\end{cases}$, $where\ D = A \cup B$,

10. $(F,A)\,\tilde{\cup}\,(G,B)_{A} =$

$\begin{cases}(G,B)_{A} & if\ A \subseteq B, \\ (R,D) & otherwise\end{cases}$, $where\ D = A \cup B$.

Definition 3.23 Intersection of two relative soft sets.

The intersection of two relative soft sets (F,A) and (G,B) over U denoted by $(F,A)\,\tilde{\cap}\,(G,B)$ is the relative soft set $(H,C)\ where\ C = A \cup B, and\ \forall \varepsilon \subset C$,

$$\mathrm{H}(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in \mathrm{A} - \mathrm{B} \\ G(\varepsilon), & if\ \varepsilon \in \mathrm{B} - \mathrm{A} \\ F(\varepsilon) \cap G(\varepsilon), & if\ \varepsilon \in \mathrm{A} \cap \mathrm{B}. \end{cases}$$

Example 3.24 Consider Example 3.2. Let

$A = \{a_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), a_2$

$= (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), a_3$

$= (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4)$,

$a_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4)$ and

$B = \{b_1 = (e_{U1}, 1, e_{U2}, 1, e_{U3}, 1), b_2$

$= (e_{U1}, 2, e_{U2}, 4, e_{U3}, 2), b_3$

$= (e_{U1}, 4, e_{U2}, 3, e_{U3}, 4)$,

$b_4 = (e_{U1}, 3, e_{U2}, 4, e_{U3}, 4), b_5$

$= (e_{U1}, 5, e_{U2}, 1, e_{U3}, 1), b_6$

$= (e_{U1}, 1, e_{U2}, 4, e_{U3}, 4)\}$.

Suppose (F,A) and (G,B) are two relative soft set over the same U such that

$$(F,A) = \{(a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})),$$
$$(a_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\})),$$
$$(a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (a_4, \{S_2\}, \{S_6\}, \emptyset,))\}\text{and}$$
$$(G,B) = \{(b_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})),$$
$$(b_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\})),$$
$$(b_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (b_4, \{S_2\}, \{S_6\}, \emptyset,))\}$$

Therefore, $(F,A)\,\tilde{\cup}\,(G,B) = (H,C)$

$=$

$\{(C_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})), (C_2, (\{S_1, S_2\}, \{S_6\}, \{S_6\})),$

$(C_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)), (C_4, \{S_2\}, \{S_6\}, \emptyset,)), \quad \}$

$(c_5, \{S_3\}, \{S_5\}, \{S_8\})), (c_6, (\{S_2, S_3\}, \{S_6\}, \emptyset))\}$

Proposition 3.25

If $(F,A), (G,B)\ and\ (H,C)$ are three relative soft sets over U, then

1. $(F,A)\,\tilde{\cap}\,((G,B)\,\tilde{\cap}\,(H,C)) = ((F,A)\,\tilde{\cap}\,(G,B))\,\tilde{\cap}\,(H,C)$,
2. $(F,A)\,\tilde{\cap}\,(F,A) = (F,A)$,
3. $(F,A)\,\tilde{\cap}\,(G,A)_{\approx\Phi i} = (R,A)_{\approx\Phi i}$,
4. $(F,A)\,\tilde{\cap}\,(G,A)_{\Phi i} = (G,A)_{\Phi i}$,
5. $(F,A)\,\tilde{\cap}\,(G,B)_{\approx\Phi i} =$

   $\begin{cases}(R,D)_{\approx\Phi i} & if\ A \subseteq B, \\ (R,D) & otherwise\end{cases}$, $where\ D = A \cup B$,

6. $(F,A)\tilde{\cap}(G,B)_{\Phi} =$

$\begin{cases}(R,D)_{\Phi} & if\ A \subseteq B,\\ (R,D) & otherwise\end{cases}$, $where\ D =$ $A\cup B$,

7. $(F,A)\tilde{\cap}(G,A)_{\approx \mathrm{A}i} = (R,D)$

8. $(F,A)\tilde{\cap}(G,A)_{\mathrm{A}} = (F,A)$,

9. $(F,A)\tilde{\cap}(G,B)_{\approx \mathrm{A}i} = (R,D)$,

10. $(F,A)\tilde{\cap}(G,B)_{A} =$

$\begin{cases}(F,A) & if\ A \subseteq B,\\ (R,D) & otherwise\end{cases}$, $where\ D = A \cup$ B.

Proposition 3.26. If $(F,A),(G,B)$ and (H,C) are three relative soft sets over U, then

1. $(F,A)\tilde{\cup}\big((G,B)\tilde{\cap}(H,C)\big) = \big((F,A)\tilde{\cup}(G,B)\big)\tilde{\cap}\big((F,A)\tilde{\cup}(H,C)\big)$,

2. $(F,A)\tilde{\cap}\big((G,B)\tilde{\cup}(H,C)\big) = \big((F,A)\tilde{\cap}(G,B)\big)\tilde{\cup}\big((F,A)\tilde{\cap}(H,C)\big)$.

4. Conclusion

Soft set theory invented by Molodtsov offers a general mathematical frame work for dealing with uncertain or vague objects. In this paper basic concepts of soft set are reviewed. We introduced the concept of relative soft set as an extension of soft set. The basic properties of relative soft set are also presented and discussed. This extension does not only provides an important addition to existing theories for handling uncertainties but also leads to potential areas of further research and pertinent applications.

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