Recognition Criteria and Application of Lake Eutrophication Evaluation Based On Set Pair Analysis

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Abstract
The set pair analysis evaluation method has been applied to evaluate water quality. However, in real-life applications, because traditional assessing methods usually yield extreme results and are of poor resolution, a hybrid recognition criterion, entropy weight and set pair analysis model is established. The concepts and calculated methods of assessment grade fuzzy characteristic and confidence criterion were proposed in order to solve the problem of determination of the magnitude of lake eutrophication. And the weight value was calculated by entropy weight method and finally got the evaluation grade as a confidence interval based on confidence level. Compared with the set pair analysis based on triangular fuzzy number method (SPA(TFN)) and the improved set pair analysis method (ISPA), the evaluating results of this model are more reasonable and its resolving power is higher. This study offer new insights and possibility to carry out an efficient way for lake eutrophication status evaluation. The study also provides scientific reference in lake risk management for local and national governmental agencies.

Keywords: Recognition criterion; Set pair analysis; Lake eutrophication evaluation; Confidence criterion.

1. Introduction

Lakes are extremely precious water resources on the surface of the earth, especially fresh water lakes. Because they can provide stable and clean water resources and aquatic products for the industrial and agricultural development and people's life along the lakes, they often become hot spots of human activities and development [1]. However, in recent ten years, with the rapid increase of population and the rapid development of industry and agriculture, most lakes in China have been or are facing the problem of eutrophication. Lake eutrophication will worsen water quality, increase the cost and difficulty of water treatment, destroy aquatic ecological environment and endanger human health, resulting in very negative influence on sustainable development of the local society and the economy. Concerns over water conflicts between human beings and ecosystems are increasing.

How to control and manage the trophic status of a water body timely and accurately is an important mission that scientists should carry out. Accurate comprehensive evaluation, diagnosis and assessment of lake eutrophication status and understanding of lake water quality classification and ranking are important means for effective management of water resources, which can provide scientific basis for decision-making and management of lake eutrophication prevention and control and is of great significance to promote sustainable development of lakes [2].

Over the past half century, many scholars have proposed a series of methods for evaluating lake eutrophication, of which the set pair analysis (SPA) is a more commonly used method at present and have achieved some achievements. However, in the existing lake eutrophication evaluation research based on set pair analysis, the evaluation results mostly adopt the judgment criteria similar to fuzzy evaluation-the maximum membership criterion. Generally, it can be used properly to obtain reasonable and effective judgment results, but sometimes there may be unreasonable decision results, because of being not good at deal with the problem of recognition criteria, leading to the distortion of evaluation results [3]. For example, according to the maximum membership criterion, the evaluation result of $\mu = 0.34 + 0.21i_1 + 0.29i_2 + 0.16j$

is "identity" and the effect is significant, so the decision result should be characterized by "identity". However, careful analysis shows that the recognition result lacks rationality, after all, 66% of the factors do not have "identity" characteristics.

In order to solve the problem, a lot of research work has been carried out. Among them, the confidence criterion for determining the evaluation grade proposed by Mr. Cheng Qiansheng [4,5] in attribute recognition evaluation also has the problem of distorting the evaluation results, while Chen Shouyu's research results[6] in engineering fuzzy set theory are only an average value, rather than a fuzzy set, and the interval size of the average value is rarely studied. In view of the above-mentioned common problems, in this study, the "confidence interval" was introduced into the SPA model of lake eutrophication evaluation, and a case study was carried out and used the confidence criterion to explore the identification criteria of the SPA method with multi-factor connection number, in order to provide more objective and reasonable results for lake eutrophication evaluation.

According to the characteristics of the China lakes, a hybrid recognition criterion, entropy weight and set pair analysis model is were established, The model built up by this method was applied to evaluate the eutrophication of ten lakes and the results were analyzed show that the algorithms are effective.

2. Methodology

2.1 Fundamental of Set Pair Analysis

Set pair analysis(SPA), proposed by Zhao in 1989[7], is a systematic analysis method to describe and process system uncertainty[8]. A set pair is formed by putting together the certainty and uncertainty sets. It is depicting the relationship between them from three aspects as identity, discrepant and contrary, which can be used to analyze the internal relationship between the whole and the part of a given system. In this system, certainty and uncertainty can influence each other, restrict each other and transform each other under certain conditions. The basic expression of the connection number is as follows:

$$\mu = a + bi + cj \qquad (1)$$

Equation (1) is called the three-element connection number [9]. Where j is the coefficient of the contrary degree, and is specified as -1; i is the coefficient of the discrepancy degree, and is an uncertain value between -1 and 1 in terms of various circumstances. As far as the current application based on SPA is concerned, there is no reasonable, systematic and acceptable framework, and the effectiveness of data information mining in practice may not be fully demonstrated. However, in practice, this depiction has not been able to meet the requirements of accuracy for many studies; two sets may have more than one kind of discrepancy degree.

Therefore, according to the developability principle of set pair analysis, it is necessary to extend the basic expression of connection degree at multiple levels in order to form a kind of multivariate connection number, which is as follows[9~11]:

$$\mu = a + b_1 i_1 + b_2 i_2 + \cdots + b_n i_n + c_1 j_1 + \cdots + c_n j_n \qquad (2)$$

At present, the five-element connection number [12] is widely used, and the expression of the five-element connection number is as follows:

$$\mu = a + b_1 i_1 + b_2 i_2 + b_3 i_3 + cj \qquad (3)$$

Where, $a, b_1, b_2, b_3, c \in [0,1]$且$a + b_1 + b_2 + b_3 + c = 1$，$a$ represents the identical degree of the set pair; b_1，b_2，b_3 represent discrepancy degree of the set pair, which are also explained as the different grades of the discrepancy degree, such as mild discrepancy, moderate discrepancy and severe discrepancy; c represents contrary degree of the set pair ; i_1、i_2、i_3 are the coefficients of the discrepancy degree, and are some uncertain values between -1 and 1, i.e. $i \in [-1,1]$. The"j" is the

coefficient of the contrary degree, and is specified as -1; i_1、i_2、i_3 and j can be regarded as the markers of the discrepancy degree and the contrary degree respectively.

2.2 Construction of five-element connection number evaluation model

There are k assessment lakes, with the property description lake of the m-th indicators, x_{mk} is the m-th indicator of the k-th lake (m = 1,2,…,M; k = 1,2,…K). The lake evaluation grade is divided into n levels (n = 1, 2,…,N) in this article to make N = 5. For lake eutrophication assessment, the lake evaluation grade is made into a set ($A_{mn} = [S_{m1},…,S_{m3},…,S_{m5}]$) and made the indicator values of 10 lakes factors into a set ($B_{mk} = [x_{m1},…,x_{m5},…,x_{m10}]$). Then, putting together set A_{mn} and set B_{mk} to form a set pair H (A_{mn}, B_{mk}). Finally, it can calculate the connection number between the evaluation unit and grade, respectively.

The eutrophic state of lakes was divided into five grades. In the five-level evaluation with grade boundary value as the grading standard, it can be seen from the multivariate connection number theory in SPA theory that the five-element connection number can represent the identity-discrepancy-contrary structure of lake eutrophication evaluation. The sample to be evaluated that meets Ⅰ level standard is defined as identity degree whose coefficient can be regarded as 1, higher than Ⅴ level standard is defined as contrary degree whose coefficient can be regarded as -1 and others are defined as discrepancy degree. The sample concentration meeting Ⅱ level standard was divided into the identity degree and the mild discrepancy degree to a certain proportion; the sample concentration meeting Ⅲ standard was divided into the mild discrepancy degree and the moderate discrepancy degree to a certain proportion; the sample concentration meeting Ⅳ standard was divided into the moderate discrepancy degree and the severe discrepancy degree to a certain proportion. It further refines the identity-discrepancy-contrary relation of set pair, so as to solve the problem that the granularity of measurement rules is too large [11~12].

According to the characteristics of lake eutrophication evaluation index, all evaluation indexes can be divided into two categories. The cost index refers to the smaller measured valued as the better evaluation grade; but the benefit index is opposite, it refers to the smaller measured valued as the lower evaluation grade.

If the index is a cost-type index, the computational equations of u_{mk} are as follows:

$$\mu_{mk} = \begin{cases} 1 + 0i_1 + 0i_2 + 0i_3 + 0j, & x_{mk} \leq S_{m1} \\ \frac{S_{m1}+S_{m2}-2x_{mk}}{S_{m2}-S_{m1}} + \frac{2x_{mk}-2S_{m1}}{S_{m2}-S_{m1}}i_1 + 0i_2 + 0i_3 + 0j, & S_{m1} < x_{mk} \leq \frac{S_{m1}+S_{m2}}{2} \\ 0 + \frac{S_{m2}+S_{m3}-2x_{mk}}{S_{m3}-S_{m1}}i_1 + \frac{2x_{mk}-S_{m1}-S_{m2}}{S_{m3}-S_{m1}}i_2 + 0i_3 + 0j, & \frac{S_{m1}+S_{m2}}{2} < x_{mk} \leq \frac{S_{m2}+S_{m3}}{2} \\ 0 + 0i_1 + \frac{S_{m3}+S_{m4}-2x_{mk}}{S_{m4}-S_{m2}}i_2 + \frac{2x_{mk}-S_{m2}-S_{m3}}{S_{m4}-S_{m2}}i_3 + 0j, & \frac{S_{m2}+S_{m3}}{2} < x_{mk} \leq \frac{S_{m3}+S_{m4}}{2} \\ 0 + 0i_1 + 0i_2 + \frac{2S_{m4}-2x_{mk}}{S_{m4}-S_{m3}}i_3 + \frac{2x_{mk}-S_{m4}-S_{m3}}{S_{m4}-S_{m3}}j, & \frac{S_{m3}+S_{m4}}{2} < x_{mk} < S_{m4} \\ 0 + 0i_1 + 0i_2 + 0i_3 + j, & x_{mk} \geq S_{m4} \end{cases} \quad (4)$$

If the index is a benefit-type index, the five-element connection number of lake eutrophication evaluation is:

$$\mu_{mk} = \begin{cases} 1 + 0i_1 + 0i_2 + 0i_3 + 0j, & x_{mk} \geq S_{m1} \\ \frac{S_{m1}+S_{m2}-2x_{mk}}{S_{m2}-S_{m1}} + \frac{2x_{mk}-2S_{m1}}{S_{m2}-S_{m1}}i_1 + 0i_2 + 0i_3 + 0j, & \frac{S_{m1}+S_{m2}}{2} \leq x_{mk} < S_{m1} \\ 0 + \frac{S_{m2}+S_{m3}-2x_{mk}}{S_{m3}-S_{m1}}i_1 + \frac{2x_{mk}-S_{m1}-S_{m2}}{S_{m3}-S_{m1}}i_2 + 0i_3 + 0j, & \frac{S_{m2}+S_{m3}}{2} \leq x_{mk} < \frac{S_{m1}+S_{m2}}{2} \\ 0 + 0i_1 + \frac{S_{m3}+S_{m4}-2x_{mk}}{S_{m4}-S_{m2}}i_2 + \frac{2x_{mk}-S_{m2}-S_{m3}}{S_{m4}-S_{m2}}i_3 + 0j, & \frac{S_{m3}+S_{m4}}{2} \leq x_{mk} < \frac{S_{m2}+S_{m3}}{2} \\ 0 + 0i_1 + 0i_2 + \frac{2S_{m4}-2x_{mk}}{S_{m4}-S_{m3}}i_3 + \frac{2x_{mk}-S_{m4}-S_{m3}}{S_{m4}-S_{m3}}j, & S_{m4} < x_{mk} < \frac{S_{m3}+S_{m4}}{2} \\ 0 + 0i_1 + 0i_2 + 0i_3 + j, & x_{mk} \leq S_{m4} \end{cases} \quad (5)$$

Where, m is the m-th evaluation index, $m=1,2,…, M$; k is the k-th evaluation sample, $k=1,2,…,K$; x_{mk} is the k-th evaluation sample value; S_{m1}, S_{m2}, S_{m3} and S_{m4} are the grade boundaries of each evaluation index.

2.3 Determination of evaluation index weight

The weight determination is important in SPA, because it has crucial effect on the assessment results. There are two main types of approach for obtaining constant weights, the subjective weights and the objective weights. In this study, the entropy weight method is introduced and applied to design the weights, based on the the principle of maximum entropy. As a measurement of the disorder degree of a system, information entropy can measure the amount of useful information with the provided data and has been prevalent in various weight-assigning problems. The calculation steps are as follows [13]:

1) If there are k lakes, each lake has m evaluation index, which constitute the judgment matrix R:

$$R = (r_{st})_{k \times m} \quad (s=1,2,3\ldots k, \quad t=1,2,3\ldots m) \tag{6}$$

Where, r_{st} is the measured value of the t-th index of the s-th lake.

2) Normalize the judgment matrix R, and the normalized matrix is $A = a_{st}$,

$$a_{st} = \frac{r_{st} - r_{min}}{r_{max} - r_{min}} \tag{7}$$

Where, r_{min} and r_{max} are respectively the minimum and maximum value of the measured values of each lake with the same index.

3) According to the definition of entropy, the entropy of each index is:

$$H_t = -\sum_{s=1}^{k} \frac{f_{st} \times \ln f_{st}}{\ln k} \quad (s=1,2,3\ldots k, \quad t=1,2,3\ldots m)$$

Where, $f_{st} = \frac{1 + a_{st}}{\sum_{s=1}^{k}(1 + a_{st})}$. $\tag{8}$

4) Calculate the entropy weight ω_t of each index:

$$\omega_t = \frac{1 - H_t}{m - \sum_{t=1}^{m} H_t} \tag{9}$$

Where, $\omega_t \in [0,1]$, and satisfy $\sum_{t=1}^{m} \omega_t = 1$。

2.4 Establishment of the total connection number formula

The total connection number μ of each lake is obtained by weighted sum of the connection degree of each lake, as shown in formula (10):

$$\mu = \sum_{t=1}^{m} \omega_t \mu_t = \sum_{t=1}^{m} \omega_t a_t + \sum_{t=1}^{m} \omega_t b_{t,1} i_1 + \sum_{t=1}^{m} \omega_t b_{t,2} i_2 + \sum_{t=1}^{m} \omega_t b_{t,3} i_3 + \sum_{t=1}^{m} \omega_t c_t j \tag{10}$$

Where, t is the t-th evaluation index; ω_t is the weight of index t; μ_t is the connection degree of each sub-item; a_t、$b_{t,1}$、$b_{t,2}$、$b_{t,3}$ and c_t are the identical degree, the mild discrepancy degree, the moderate discrepancy degree, the severe discrepancy degree and the contrary degree of each index respectively; i_1、i_2、i_3 and j are the coefficients of the mild discrepancy degree, the moderate discrepancy degree, the severe discrepancy degree and the contrary degree of each index respectively.

2.5 Fuzzy characteristic quantity of evaluation grade

Suppose that the evaluation grade is divided into v level. The domain of the evaluation grade is that $U = \{u_1, u_2, \cdots, u_v\}$, and define u_h, $h = 1,2,\ldots,v$. As h increases, the evaluation characteristics decrease and the evaluation level increases. The corresponding value field of evaluation grade Ω and the domain U is that $\Omega = \{f_{12}, f_{23}, \cdots, f_{hh+1}, \cdots, f_{vv+1}\} = \{f_1 - f_2, f_2 - f_3, \cdots, f_v - f_{v+1}\}$.

The evaluation result of multi-objective set pair analysis is the membership vector corresponding to the evaluation grade, which is as follows:

$$\mu_A = \{\mu_{A1}, \mu_{A2}, \cdots, \mu_{Av}\}, \text{ here it is assumed that } \mu_{A1} = a, \mu_{A2} = b_1, \mu_{A3} = b_2, \cdots, \mu_{Av} = c.$$

According to the definition of Xu Kaili [14], the characteristic quantity of the evaluation level is expressed by the following formula:

$$H_{\mu_A} = [H_{\mu_A}^-, H_{\mu_A}^+] = \sum_{h=1}^{v} \mu_{A_h} X(\mu_{A_h})$$

$$= \left[\sum_{h=1}^{v} \mu_{A_h} |f_h + \mu_{A_h}(f_{h+1} - f_h) / 2|, \sum_{h=1}^{v} \mu_{A_h} |f_{h+1} + \mu_{A_h}(f_{h+1} - f_h) / 2| \right] \quad (11)$$

In particular, when

$\Omega = \{f_1 - f_2, f_2 - f_3, \cdots, f_v - f_{v+1}\} = \{0.5\sim1.5, 1.5\sim2.5, \cdots, v\text{-}0.5\sim v +0.5\}$, the evaluation grade can be expressed by a hierarchical system of customary concepts, and the opposite can also be defined.

2.6 Determination of confidence criterion and evaluation grade

The following confidence criterion is defined. When $[H_{\mu_A}^-, H_{\mu_A}^+] \subseteq [f_h, f_{h+1}]$, the confidence degree of the evaluation grade judged as A_h is 100%; When $[H_{\mu_A}^-, H_{\mu_A}^+] \subseteq [f_h, f_{h+2}]$, the confidence degree of the evaluation grade judged as A_h is:

$$\pi_h = \frac{\left| \int_{H_{\mu_A}^-}^{f_{h+1}} \mu_{FA_h}(f)df \right|}{\left| \int_{H_{\mu_A}^-}^{f_{h+1}} \mu_{FA_h}(f)df \right| + \left| \int_{f_{h+1}}^{H_{\mu_A}^+} \mu_{FA_{h+1}}(f)df \right|} \quad (12)$$

Where, $\mu_{FA_h}(f)$ is the fuzzy number used to calculate the fuzzy characteristic quantity of evaluation grade, and the specific determination method is referred to the paper [15]. The confidence degree of the evaluation grade judged as A_{h+1} is $\pi_{h+1} = 1 - \pi_h$.

3. Case study

Based on the monitoring data and the Classification Standard of Lake Eutrophication in China [16] (Table 1), five indexes were applied to quantitatively describe the eutrophication state: chemical oxygen demand (COD_{Mn}), total nitrogen (TN), total phosphorus (TP), Secchi disk depth (SD) and biomass (BIO) . At the same time Table 2 shows the sample indexes values about ten lakes in China [17]. To examine the feasibility and effectiveness of the model, the suggested method has been applied to lake eutrophication assessment.

Table 1: Standard values of the eutrophication of the lakes in China

TP (mg.m^{-3})	COD$_{Mn}$ (mg.L^{-1})	SD (m)	TN (mg.m^{-3})	BIO ($10^4.L^{-1}$)	Trophic state
[0,1]	[0,0.09]	[12,37]	[0,0.02]	[0,4]	Oligotrophic (Grade I)
[1,4]	[0.09,0.36]	[2.4,12]	[0.02,0.06]	[4,15]	Oligo-meso (Grade II)
[4,23]	[0.36,1.8]	[0.55,2.4]	[0.06,0.31]	[15,50]	Mesotrophic(Grade III)
[23,110]	[1.8,7.1]	[0.17,0.55]	[0.31,1.2]	[50,100]	Meso-eutro (Grade IV)
[110,660]	[7.1,27.1]	[0,0.17]	[1.2,4.6]	[100,1000]	Eutrophic (Grade V)

Table 2: Surveyed data of evaluation water quality in China lakes

Lakes	TP (mg.m^{-3})	COD$_{Mn}$ (mg.L^{-1})	SD (m)	TN (mg.m^{-3})	BIO ($10^4.L^{-1}$)
Qinghai lake	20	1.40	4.50	0.22	14.6
Fuxian lake	20	1.61	7.03	0.21	19.0
Tai lake	20	2.83	0.50	0.90	100
Hongze lake	100	5.50	0.30	0.46	11.50

Chao lake	30	8.26	0.25	1.67	25.30
Erhai lake	34	2.11	0.30	0.49	22.36
Dianchi lake	20	10.13	0.50	0.23	189.20
East lake in Wuhan	105	10.70	0.40	2.00	1913.70
Hulun lake	80	8.29	0.50	0.13	11.6
West lake in Hangzhou	130	10.30	0.35	2.76	6920

The specific data processing and calculation steps are as follows:

Step 1: Substitute the data in Table 1 and Table 2 into formulas (4) and (5) to calculate the sub-item connection degree μ_{mk} of each index of each lake. The results are shown in Table 3.

Table 3: Calculation results of the index connection numbers of ten lakes

Lakes	TP (mg·L^{-1})	COD$_{Mn}$ (mg·L^{-1})	SD (m)	TN (mg·L^{-1})	BIO (10^4·L^{-1})
Qinghai lake	$0.88i_2+0.12i_3$	$0.91i_2+0.09i_3$	$0.53i_2+0.47i_3$	$0.94i_2+0.06i_3$	$0.78i_1+0.22i_2$
Fuxian lake	$0.88i_2+0.12i_3$	$0.84i_2+0.16i_3$	$0.97i_2+0.03i_3$	$0.38i_1+0.62i_2$	$0.59i_1+0.41i_2$
Tai lake	$0.88i_2+0.12i_3$	$0.48i_2+0.52i_3$	J	$0.52i_2+0.48i_3$	j
Hongze lake	$0.23i_3+0.77j$	$0.60i_3+0.40j$	J	$0.96i_2+0.04i_3$	$0.91i_1+0.09i_2$
Chao lake	$0.69i_2+0.31i_3$	$0.32i_3+0.68j$	J	j	$0.31i_1+0.69i_2$
Erhai lake	$0.61i_2+0.39i_3$	$0.70i_2+0.30i_3$	J	$0.46i_2+0.54i_3$	$0.44i_1+0.56i_2$
Dianchi lake	$0.88i_2+0.12i_3$	j	J	$0.67i_3+0.33j$	j
East lake in Wuhan	$0.11i_3+0.89j$	j	J	j	j
Hulun lake	$0.69i_3+0.31j$	j	J	$0.46i_2+0.54i_3$	$0.91i_1+0.09i_2$
West lake in Hangzhou	j	j	J	j	j

Step 2: Calculate the weight of each evaluation index according to formulas (6)~(9), the calculation result is ω_t= (0.201, 0.143, 0.183, 0.212, 0.261).

Step 3: According to formula (10), each index weight and μ_{mk}, calculate the total connection degree μ of each lake.

Step 4: Calculate the fuzzy feature vector and the possibility of evaluation level according to formulas (11) ~ (12), the expression of the total connection degree μ of each lake. The calculation results are shown in Table 4.

Table 4: Eutrophication evaluation results of ten lakes

Lakes	Connection degree expression μ	Fuzzy characteristic	Confidence of grade	
Qinghai lake	$0.16i_1+0.70i_2+0.14i_3$	[2.7476,3.2124]	III 100%	IV 0
Fuxian lake	$0.12i_1+0.81i_2+0.07i_3$	[2.7877,3.1123]	III 100%	IV 0
Tai lake	$0.28i_2+0.26i_3+0.46j$	[3.8588,4.5012]	IV	V

Lakes	Expression	Interval	Grade	%	Grade	%
				99.73%		0.27%
Hongze lake	$0.19i_1+0.12i_2+0.27i_3+0.43j$	[3.61915,4.32085]	IV 100%		V 0	
Chao lake	$0.06i_1+0.28i_2+0.13i_3+0.53j$	[3.8199,4.4401]	IV 100%		V 0	
Erhai lake	$0.09i_1+0.47i_2+0.25i_3+0.20j$	[3.25075,3.92925]	III 39.29%		IV 60.71%	
Dianchi lake	$0.36i_2+0.04i_3+0.60j$	[3.9856,4.4944]	IV 100%		V 0	
East lake in Wuhan	$0.02i_3+0.98j$	[4.9604,4.9996]	IV 0		V 100%	
Hulun lake	$0.26i_1+0.14i_2+0.14i_3+0.46j$	[3.4592,4.1408]	III 8.41%		IV 91.59%	
West lake in Hangzhou	j	[5,5]	IV 0		V 100%	

In order to explain the validity of the evaluation method, Table 5 lists the comparison of the set pair analysis method based on recognition criteria (SPA(RC)) with the set pair analysis method based on triangular fuzzy number (SPA(TFN)) and the improved set pair analysis method (ISPA).

Table 5: Evaluation results of ten lakes and comparison of other methods

Lakes	Grades SPA(RC)	Grades ISPA[18]	Grades SPA(TFN)[19]
Qinghai lake	III	III	III
Fuxian lake	III	III	III
Tai lake	IV	IV	IV
Hongze lake	IV	IV	IV
Chao lake	IV	IV	IV
Erhai lake	IV	III	III
Dianchi lake	IV	IV	IV
East lake in Wuhan	V	V	IV
Hulun lake	IV	III	IV
West lake in Hangzhou	V	V	V

As seen from Table 5 above the set pair analysis based on recognition criterion SPA (RC) evaluation results are coherent to the results of the (ISPA) used in reference [18] and the results of the SPA(TFN) used in reference [19] which shows that the SPA (RC) is feasible and reliable in lake eutrophication evaluation. The SPA (RC) method doesn't only give a concrete grade of evaluation result but also gives the evaluation grade as a confidence interval, along with the confidence level, which has advantages of high resolution and information utilization. Besides, the SPA(RC) method will help to keep away from pointlessly rigid interpretation of lake eutrophication status evaluation results as we advance towards confirm based practice.

4. Conclusions

To assess the lake eutrophication status, a hybrid recognition criterion, entropy weight and set pair analysis model is established, which takes recognition criterion, entropy weight and set pair analysis as a theory basis. The model is used to assess the lake trophic status. The main conclusions are as follows.

1) The proposed model improve previous studies in (a) systematically reflecting lake in terms of multiple factors, and (b) consideration of uncertain parameters in the evaluation process. The traditional evaluation method can only get a rough assessment of the evaluation grade, and can't further distinguish between the same grade. The proposed model can not only give the specific evaluation grade, but also obtain a confidence interval.

2) Considering the dynamic characteristics of lake eutrophication, the proposed confidence criterion innovatively reflects the relative proportion of lake eutrophication status relative to each evaluation level, which provides an effective method for uncertain multi-objective decision-making problems.

3) The entropy weight method can simultaneously consider the relationship between multiple samples and the relative importance of different evaluation indicators, avoiding the subjectivity of weight assignment, and ultimately making the evaluation results more objective and reasonable.

4) The computational results demonstrate that the model presented in this study to assess lake eutrophication status is reasonable, reliable and applicable.

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