

Analysis And Instability Of A Control System

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ABSTRACT

Control systems in an interdisciplinary field covering many areas of engineering and sciences. Control systems exist in many systems of engineering, sciences, and in human body. Some type of control systems affects most aspects of our day-to-day activities.

The study and design of a control system includes various steps such as modeling, simulation, analysis and optimization. Physical laws namely Kirchhoff's laws, Newton's laws, laws of thermodynamics. The Control System Toolbox has large number of built in functions which can be used for analysis and design of control systems.

Keywords: *laplace transform, Open Loop Control System, Closed Loop Control System, Geostationary Orbit*

INTRODUCTION

Control means to regulate, direct, command, or govern. A system is a collection, set, or arrangement of elements (subsystems). A control system is an interconnection of components forming a system configuration that will provide a desired system response. Hence, a control system is an arrangement of physical components connected or related in such a manner as to command, regulate, direct, or govern itself or another system

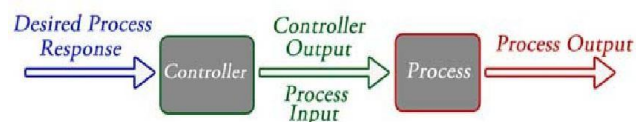
Types Of Control Systems

There are two main **types of control system**.

They are as follows:

1. Open loop control system

Open loop control systems are those that do not provide feedback of the actual state of the system in order to adjust the system. Figure below shows the block diagram of open control system.



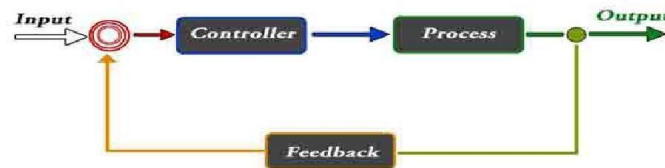
Practical Examples Of Open Loop Control System

- Electric Hand Drier – Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.

- Automatic Washing Machine – This machine runs according to the pre-set time irrespective of washing is completed or not.
- Bread Toaster – This machine runs as per adjusted time irrespective of toasting is completed or not.
- Automatic Tea/Coffee Maker – These machines also function for pre adjusted time only.
- Timer Based Clothes Drier – This machine dries wet clothes for pre – adjusted time, it does not matter how much the clothes are dried.
- Light Switch – lamps glow whenever light switch is on irrespective of light is required or not.
- Volume on Stereo System – Volume is adjusted manually irrespective of output volume level

2.Closed Loop Control System

Control system in which the output has an effect on the input quantity in such a manner that the input quantity will adjust itself based on the output generated is called **closed loop control system**. Open loop control system can be converted in to closed loop control system by providing a feedback. This feedback automatically makes the suitable changes in the output due to external disturbance. In this way closed loop control system is called automatic control system. Figure below shows the block diagram of closed loop control system in which feedback is taken from output and fed in to input.



Practical Examples Of Closed Loop Control System

- Automatic Electric Iron – Heating elements are controlled by output temperature of the iron.
- Servo Voltage Stabilizer – Voltage controller operates depending upon output voltage of the system.
- Water Level Controller– Input water is controlled by water level of the reservoir.
- Missile Launched & Auto Tracked by Radar – The direction of missile is controlled by comparing the target and position of the missile.
- An Air Conditioner – An air conditioner functions depending upon the temperature of the room.
- Cooling System in Car – It operates depending upon the temperature which it controls.

LAPLACE TRANSFORM

Laplace Transform helps to convert differential equations describing the behaviour of dynamic system into algebraic equation of a complex variable.

The Laplace transform of a function of time $f(t)$ is given by the following integral:

$$L[f(t)]=F(s)= \int e^{-st}f(t)dt.$$

Finding Laplace Tranform of the function $f(t)=e^{-t}(1-\cos(t))$

The following MATLAB command is used:

```
>> syms t
>> ft=exp(-t)*(1-cos(t));
>> fs=laplace(ft)
```

fs =

$1/(s + 1) - (s + 1)/((s + 1)^2 + 1)$

PARTIAL FRACTION EXPANSION

For problem in control systems, $F(s)$, Laplace Transform of $f(t)$ occurs as a ratio of two polynomials in

's' as $F(s) = \frac{B(s)}{A(s)}$.

The command RESIDUE can be used to perform partial fraction expansion directly (RESIDUE itself makes use of both ROOTS and DECONV behind the scenes). Providing two input vectors, which again represent the numerator and denominator polynomials, just like the DECONV command. MATLAB returns *three* outputs: the first is a vector of *residues*, the second a vector of *poles*, and the third a vector of coefficients for any *remainder* polynomial that exists.

Here's an example to show you how it works: if $F(s) = \frac{B(s)}{A(s)} = \frac{3s+1}{s^3 + 3s^2 + 2s}$ then you'd type

```
>>A=[1 3 2 0];
>>B=[3 1];
>>[R,P,K]=residue(B,A)
```

and MATLAB responds with

```
R =      -2.5000
         2.0000
         0.5000
```

```
P =      -2
        -1
         0
```

```
K =      []
```

The poles P are the roots of the denominator, the same as you'd get with roots([1 3 2 0]). The partial fraction expansion includes one term for each of them, for which the denominator is s minus the pole. The residues are placed in corresponding order as the numerators of each of these terms, like this:

$$F(s) = \frac{-2.5}{s+2} + \frac{2}{s+1} + \frac{0.5}{s}$$

STATE – SPACE REPRESENTATION OF DYNAMIC SYSTEMS

The state space model represents a physical system as n first order differential equations. This form is better suited for computer simulation than an n th order input-output differential equation.

Vector matrix format generally is given by:

State Space form

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

- A* - System matrix- $n \times n$
- B* - Input matrix- $n \times m$
- C* - Output matrix- $p \times n$
- D* - Feed forward matrix - $p \times m$

where **Y** is the output equation, and **X** is the state vector.

PARTS OF A STATE SPACE REPRESENTATION

State Variables: a subset of system variables which if known at an initial time t_0 along with subsequent inputs are determined for all time $t > t_0$

State Equations: n linearly independent first order differential equations relating the first derivatives of the state variables to functions of the state variables and the inputs.

Output equations: algebraic equations relating the state variables to the system outputs

Example: State Space to Transfer Function

Finding the transfer function of the system with state space representation

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}u = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -4 & -2 \end{bmatrix} \mathbf{q} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \mathbf{C}\mathbf{q} + \mathbf{D}u = [5 \ 1 \ 0] + 0 \cdot u$$

First find $(s\mathbf{I} - \mathbf{A})$ and the $\Phi = (s\mathbf{I} - \mathbf{A})^{-1}$. Rules for inverting a 3x3 matrix are

$$s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 4 & s+2 \end{bmatrix}$$

$$\Phi = (s\mathbf{I} - \mathbf{A})^{-1} = \frac{\begin{bmatrix} s^2 + 2s + 4 & 2 + s & 1 \\ -3 & s(2 + s) & s \\ -3s & -3 - 4s & s^2 \end{bmatrix}}{s^3 + 2s^2 + 4s + 3}$$

Now I can find the transfer function

To make this task easier, MatLab has a command (ss2tf) for converting from state space to transfer function.

```
>> % First define state space system
>> A=[0 1 0; 0 0 1; -3 -4 -2];
>> B=[0; 0; 1];
>> C=[5 1 0];
>> D=[0];
```

>> [n,d]=ss2tf(A,B,C,D)

n =
 0 0 1.0000 5.0000
 d =
 1.0000 2.0000 4.0000 3.0000

Transfer function is:

$$\frac{s + 5}{s^3 + 2s^2 + 4s + 3}$$

Instability of the Geostationary Orbit

The principle of such a satellite is that it serves as a mirror for electromagnetic signals. In order not to be forced to continuously aim the transmitters and receiving antennas at the satellite, it is desired that the satellite has a fixed position with respect to these devices. This also has the advantage that the satellite does not go down and rise, so that it can be used 24 hours a day. In order to simplify the example, Consider the motion of the satellite in the equator plane. By taking the origin at the center of the earth, the position of the satellite is given by its polar coordinates(r,θ). Introduce the following constants:

- M_E : =mass of the earth,
- G : =earth's gravitational constant,
- Ω : =earth's angular velocity,
- M_S : =mass of the satellite.

Assume that the satellite has on-board jets which make it possible to exert forces $F_r(t)$ and $F_\theta(t)$ to the satellite in the direction of r and θ respectively. Using Newton's law it can be verified that the equations of motion of the satellite are given by

$$\begin{aligned} \ddot{r}(t) &= r(t)\dot{\theta}(t)^2 - \frac{GM_E}{r(t)^2} + \frac{F_r(t)}{M_S}, \\ \ddot{\theta}(t) &= -2\frac{\dot{r}(t)\dot{\theta}(t)}{r(t)} + \frac{F_\theta(t)}{M_S r(t)}. \end{aligned}$$

The desired geostationary orbit is given by

$$\begin{aligned} \theta(t) &= \theta_0 + \Omega t, \\ r(t) &= R_0, \\ F_r(t) &= 0, \\ F_\theta(t) &= 0, \end{aligned}$$

Where R_0 still has to be determined. First check that this indeed yields a solution to the equations of motion for suitable R_0 . By substitution in the differential equations we obtain

$$0 = R_0 \Omega^2 - \frac{GM_E}{R_0^2},$$

which yields

$$R_0 = \sqrt[3]{\frac{GM_E}{\Omega^2}}$$

By taking the appropriate values for the physical constants in this formula, we find that R_0 is approximately equal to 42000 km. Thus the geostationary orbit is a circular orbit in the equator plane, at an altitude of approximately 35620 km over the equator (the radius of the earth being approximately 6380 km). It is convenient to replace the equations of motions by an equivalent system of four first order differential equations by putting

$$\begin{aligned} x_1 &:= r(t) - R_0, \\ x_2 &:= \dot{r}(t), \\ x_3 &:= \theta(t) - (\theta_0 + \Omega t), \\ x_4 &:= \dot{\theta} - \Omega. \end{aligned}$$

Note that x_3 is the deviation from the desired angle. This value can at any time instant be measured by an observer on the equator by comparing the actual position of the satellite to the desired position. In terms of these new variables, the system is described by

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{pmatrix} = \begin{pmatrix} x_2(t) \\ (x_1(t) + R_0)(x_4(t) + \Omega)^2 - \frac{GM_E}{(x_1(t) + R_0)^2} + \frac{F_r(t)}{M_S} \\ x_4(t) \\ -\frac{2x_2(t)(x_4(t) + \Omega)}{x_1(t) + R_0} + \frac{F_\theta(t)}{M_S(x_1(t) + R_0)} \end{pmatrix} \quad (1)$$

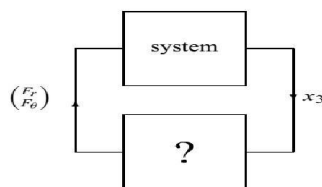
The equations (1) constitute a nonlinear state-space model with inputs and outputs. The control input is $u=(F_r, F_\theta)^T$, for the measured output one could take x_3 . The geostationary orbit corresponds to the equilibrium solution given by

$$(x_1, x_2, x_3, x_4) = (0, 0, 0, 0), F_r = 0, F_\theta = 0$$

By Kepler's law it is clear that if at time t_0 the equilibrium solution is perturbed to, say,

$$(x_1(t_0), x_2(t_0), x_3(t_0), x_4(t_0)) = (\xi_1, \xi_2, \xi_3, \xi_4)$$

then the resulting orbit will be an ellipsoid in the equator plane with the earth in one of its foci. The angular velocity of the satellite with respect to the center of earth will then no longer be constant, so to an observer on the equator the satellite will not be in a fixed position, but will actually go down and rise periodically. Mathematically this can be expressed by saying that the equilibrium solution is not locally asymptotically stable. There is the possibility to exert forces to the satellite in the r -direction and in the θ -direction. Also, the variable x_3 can be measured. The control synthesis problem can now be formulated as: find a feedback controller that generates a control input $u=(F_r, F_\theta)^T$ on the basis of the measured output x_3 , in such a way that the equilibrium solution corresponding to the geostationary orbit becomes locally asymptotically stable. Of course, it is not clear a priori whether such controller exists.



CONCLUSION

Topics covered in this paper emphasis on finding Laplace Transform, Partial fraction and some real life example of control system upto the limit of my knowlwdge.

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