The Unification of the Fundamental Interactions

Dr. Andreas Gimsa

Stirling Technologie Institut Potsdam gemeinnützige GmbH, D-14478 Potsdam, Germany

Abstract
The known fundamental interactions of nature are to be examined for mathematical connections existing among themselves. This would be a further indication that nature has a mathematical basic framework. The investigation shall show whether a unification of the fundamental forces is reasonable and possible.

Keywords : Fundamental interactions, Fundamental forces, Strong force, Weak force, Electro-magnetic force, Gravitational force

Introduction

The search for unification of the fundamental interactions of nature is sometimes referred to as the search for the Holy Grail of physics. In general, four fundamental interactions are considered. The interactions are described in numerous publications in connection with their respective exchange particles and their ranges. This shall not be further investigated here, but it shall be mainly about the relations between the interactions. As an expression of the coupling strength of the interaction, a dimensionless coupling constant α is used. With this uniform measure the interactions can be compared with each other. The electromagnetic and the weak interaction could be united at high energies $(200 \, GeV)$ to the electroweak interaction [1.]. Before examining the coupling constants of the remaining three interactions, reference is made to the underlying forces.

1. Fundamental Force Effects

1.1 The known gravitational force between electron and proton

$$F_{grav} = \gamma \frac{m_e m_p}{r_{ep}^2} \qquad \text{with} \qquad \boxed{r_{ep} = \frac{m_e + m_p}{m_e m_p} \frac{\hbar}{\alpha_{em} \, c}} \quad \text{[2.]} \qquad (01)$$

In equation (01) $\hbar$ is the reduced quantum of action and α_{em} is the known (electro-magnetic) fine structure constant. Since the masses of electron m_e and proton m_p are very small, also the force F_{grav} is very small. The center distance r_{ep} between both masses is $5.296 \, 10^{-11} m$. This is only approximately the Bohr radius, because r_{ep} includes both masses in the calculation. A quantum mechanical consideration with the known radial wave function from the Schrödinger equation in spherical coordinates can show for the somewhat larger radius r_{ep} can show the following:

Not at the Bohr radius, but at the radius r_{ep} the probability density to measure the electron in the ground state of hydrogen becomes maximum. The gravitational force according to (01) with the radius r_{ep} is : $F_{grav} = 3.626 \, 10^{-47} N$.

1.2 The force between elementary charges

1.2.1 The known electric force between two elementary electric charges

$$F_e = \frac{e^2}{4\pi\varepsilon_0 r_{ep}^2} \qquad (02a)$$

The electric force between two electrons according to equation (02a) at the same distance as in equation (01) with $r_{ep} = 5.296 \ 10^{-11} m$ is $8.227 \ 10^{-08} N$.

1.2.2 The magnetic force between two magnetic elementary charges

$$F_m = \frac{p^2}{4\pi\mu_0 r_{ep}^2} \qquad (02b)$$

It could be proved that the force between two electric elementary charges must be as large as the force between two magnetic elementary charges. $p = 6.035 \ 10^{-17} Vs$ [3.]. The force is $8.227 \ 10^{-08} N$. However, magnetic monopoles have not been found so far.

1.2.3 The electromagnetic force between an electric and a magnetic elementary charge:

$$F_{em} = \frac{ep}{4\pi\sqrt{\varepsilon_0}\sqrt{\mu_0} \ r_{ep}^2} = \frac{epc}{4\pi \ r_{ep}^2} \qquad (02c)$$

From equations (02a) and (02b) an electromagnetic force between an electric and a magnetic elementary charge can be derived. This is given in equation (02c). It is also $8.227 \ 10^{-08} N$.

1.3 The strong force between two quarks

$$F_{strong} = \frac{\alpha_{strong} \ \hbar c}{r_{ep}^2} = \frac{\hbar c}{2 \ r_{ep}^2} \qquad (03)$$

For the strong force F_{strong} the formula (03) was found. At the distance $r_{ep} = 50 \ pm$ the coupling constant of the strong interaction has its maximum value. It is to be anticipated here that the maximum coupling constant of the strong interaction is $\alpha_{strong}(max) = 1/2$ is (see 2.3). The strong force between two quarks is according to equation (03) $5.637 \ 10^{-06} N$.

The ratio of each of the three fundamental forces to their respective coupling constant is the same, but it becomes smaller with space age (see also 5.):

$$\boxed{\frac{F_{grav}}{\alpha_{grav}} = \frac{F_{em}}{\alpha_{em}} = \frac{F_{strong}}{\alpha_{strong}} = \frac{\hbar c}{r_{ep}^2} = \frac{F_{super}}{\alpha_{super}}} \qquad (04)$$

The expression $\frac{F_{super}}{\alpha_{super}}$ of a superstrong interaction is explained under point 3. With the superstrong force F_{super} the possibility is created to summarize the three interactions of the three fundamental forces.

2. The Interaction Constants

2.1 The known coupling constant of the gravitational interaction α_{grav}

$$\alpha_{grav} = \frac{\gamma \ m_e \ m_p}{\hbar c} = 3.216 \ 10^{-42} \qquad (05)$$

Gravity is the weakest of the three interactions. The coupling constant according to (05), which results here with inclusion of the masses of electron and proton, has a very low value.

Although with the space age the proton mass and the electron mass become smaller, the coupling constant of the gravitational interaction remains constant. Because with the space age also the Newtonian gravitational value increases γ increases quadratically [4.]

2.2 The coupling constant of the elementary charge interactions

2.2.1 The known coupling constant α_e of the electric interaction

$$\alpha_e = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = 7.297 \; 10^{-03} \tag{06a}$$

This coupling constant is the best researched and has been known for a long time. It is sometimes also given as the ratio of the electron velocity in the hydrogen atom to the speed of light. This is approximately correct, but not exactly. Rather α_e the ratio of the total velocity v_g of electron and proton to the velocity of light $\alpha_e = 2\pi r_{ep}/(Tc) = v_g/c$ [5.].

2.2.2 The coupling constant α_m of the magnetic interaction [3.]

$$\alpha_m = \frac{p^2}{4\pi\mu_0 \hbar c} = 7.297 \; 10^{-03} \tag{06b}$$

Equivalent to equation (02b), which has the form of (02a), equation (06b) can be stated in the same form as (06a).

2.2.3 The coupling constant α_{em} of the electromagnetic interaction

$$\alpha_{em} = \frac{ep}{4\pi\sqrt{\varepsilon_0}\sqrt{\mu_0} \; \hbar c} = \frac{ep}{4\pi \; \hbar} = 7.297 \; 10^{-03} \tag{06c}$$

From equations (06a) and (06b) follows equation (06c) for the coupling constant α_{em} of the electromagnetic interaction. The rearrangement shows a formula for the reduced Planck's quantum of action, which is a novelty and lets recognize that a physical effect can occur in the interaction between electric and magnetic elementary charge. This equation was already derived by me in another way [6.].

$$\boxed{\hbar = \frac{ep}{4\pi\alpha_{em}}} \tag{06d}$$

2.3 The coupling constant of the strong interaction α_{strong}

$$\alpha_{strong} = \sqrt{\frac{\hbar}{m_p \, r_p \, c}} = 5.00 \; 10^{-01} = 1/2 \tag{07}$$

Equation (07), which follows from (03), is published here for the first time. The strong force leads at small energies and a distance of the quarks of about 0.2 femtometers (the proton radius itself is approx. 0.8 femtometers) leads to a strong coupling of the quarks. This coupling constant takes the value of $\alpha_{strong}(max) = 0.5$ which is a limit value. At larger distances the coupling constant does not grow any more. At higher energies and smaller distances the coupling constant becomes smaller, the quarks behave then like free particles [7.].

Equation (07) shows, as expected, that the mass and radius of the proton exert an influence on the coupling constant.

With space age, the proton mass becomes smaller and the proton radius larger. However, the product of mass and radius remains constant. $m_p r_p = const.$ [8.]. Thus also the coupling constant of the strong interaction remains constant.

3. The Relationships between the Interactions

Already with equation (04) the important connection according to (08) was shown, which was written there instead of the magnetic with the electromagnetic force F_{em} was written. Thus equation (08) reveals the direct connection of the magnetic attraction, as it follows from the complete Maxwell equations, and the gravity, by which the masses attract each other magnetically.

$$\boxed{\alpha_m F_{grav} = \alpha_{em} F_{grav} = \alpha_{grav} F_m} \tag{08}$$

In source [9.] the following connection with the masses of the universe has already been established m_{uni} of the proton m_p of the electron m_e and the Newtonian gravitational value γ was published:

$$\frac{m_{uni}}{m_e} = 2\left(\frac{c\,\hbar}{\gamma\,m_e\,m_p}\right)^2 \qquad \text{with} \quad m_{uni} = \frac{c^3 t_{uni}}{\gamma} = \frac{c^2 r_{uni}}{\gamma} \tag{09}$$

Thereby t_{uni} is the space age. With the equations (05) and (07) can be written now without problems:

$$\frac{m_{uni}}{m_e} = \frac{1}{\alpha_{strong}}\left(\frac{1}{\alpha_{grav}}\right)^2 \qquad \text{or} \qquad \boxed{\alpha_{strong}\alpha_{grav}{}^2\, m_{uni} = m_e} \tag{10}$$

This is an impressive correlation between the space mass m_{uni} and the electron mass m_e which is only the dimensionless coupling constants α_{strong} and α_{grav} are included.

Even more interesting is a connection between the radius of the universe r_{uni} and the proton radius r_p according to equation (12). For this it must be accepted that r_{uni} is exactly equal to the product of space age t_{uni} and the speed of light is equal to c corresponds.

Up to now in cosmology it is assumed that the r_{uni} is only a visible radius, but it is also the actual radius of the universe. Equation (09) shows the actual and not only the visible mass of the universe m_{uni}. In source [10.] was published the following relation for the proton radius r_p was published:

$$r_p = 2\, r_{uni}\, \frac{\gamma\, m_e m_p}{\hbar\, c} \tag{11}$$

Using equations (05) and (07), it is easy to write for equation (11):

$$\boxed{\alpha_{grav}\, r_{uni} = \alpha_{strong}\, r_p} \tag{12}$$

Equation (12) shows the perfect connection between the radius of the universe r_{uni} and the proton radius r_p which brings the strong and the gravitational interaction into an ingeniously simple relation. The equations (10) and (12) can now also be inserted into each other again, which, however, does not provide any new information with respect to the contents.

Equation (04) can be used to transform (12) into a simple relation which shows an interesting relationship between the strong force and the gravitational force:

$$\boxed{F_{grav}\, r_{uni} = F_{strong}\, r_p} \tag{13}$$

If in equation (05) instead of the electron mass m_e the mass of the universe m_{uni} a theoretically superstrong interaction arises. With the already presented connections this superstrong interaction can be traced back to the already described interactions:

$$\alpha_{super} = \frac{\gamma \, m_{uni} \, m_p}{\hbar \, c} = \frac{1}{\alpha_{grav} \, \alpha_{strong}} = \sqrt{\frac{1}{\alpha_{strong}} \frac{m_{uni}}{m_e}} = 6.21 \; 10^{41} \qquad (14)$$

The superstrong force derived from equation (04) is:

$$F_{super} = \alpha_{super} \frac{\hbar \, c}{r_{ep}{}^2} = 7.00 \; 10^{36} N \qquad (15)$$

If one puts the force F_{super} in relation to the gravitational force F_{grav} according to equation (01), we get the ratio of the space mass m_{uni} to the mass of the electron m_e comes out:

$$\frac{F_{super}}{F_{grav}} = \frac{m_{uni}}{m_e} = 1.93 \; 10^{83} \qquad (16)$$

This can be understood with equations (01), (04), (05), (07) and (14) compared with equation (10).

Another interesting relation for the superstrong force can be obtained from equation (16) with (13):

$$F_{super} = \frac{r_p}{r_{uni}} \frac{m_{uni}}{m_e} F_{strong} \qquad (17)$$

Equations (04), (10), and (17) yield a notation for the superstrong force that includes all three interactions:

$$F_{super} = \frac{F_{em}}{\alpha_{grav} \, \alpha_{em} \, \alpha_{strong}} \qquad (18)$$

Equation (18) represents a mathematical union of the three interactions. Whether an actual physical meaning can be ascribed to this superstrong force remains to be seen.

From equations (17) and (18) the following new and interesting relation between the strong force F_{strong} and the electromagnetic force F_{em} is obtained:

$$F_{strong} = \frac{F_{em}}{\alpha_{grav} \, \alpha_{em} \, \alpha_{strong}} \frac{m_e}{m_{uni}} \frac{r_{uni}}{r_p} \qquad (19)$$

4. Discussion

This publication shows simple and comprehensible mathematical relationships between the three fundamental interactions. This is especially amazing because the interactions are phenomena from both the microscopic and macroscopic realms of the universe. That means also the quantum physics with energetic and geometrical sizes of the particles with particle and wave properties, as for example the mass and the radius of the proton, arranges itself here finally with. An important condition for the presented calculability of the interactions, is the mass calculation of the universe from its age t_{uni} or its radius r_{uni} (see equation (09)). Since the age and the radius become larger, the mass of the universe must fall with quadratic growth of the gravitational value γ fall [4.]

5. Summary

An emergence of the investigated three basic forces from one single force is not possible even at very high energy. This follows from the fact that with invariable coupling constants the ratios between the forces must always remain constant. This is dictated by equation (04). The forces are indeed variable with the age of the space, but the ratios of the forces among each other remain absolutely constant. Each force by itself falls quadratically with the age of the universe. Thereby applies for each of the forces F_x: $(F_{grav}, F_e, F_m, F_{em}, F_{strong}, F_{super})$ after a certain period of time Δt and starting from the actual age of the universe t_{uni} and the original force F_{x0}:

$$F_x = \frac{F_{x0}}{\left(1+\frac{\Delta t}{t_{uni}}\right)^2} \tag{20}$$

With equation (18) a unification of the three fundamental interactions with eternally constant ratios of the fundamental forces has been achieved.

Valuable physical connections between the microcosm and the macrocosm of the universe could be found. This is proved above all by the equations (12), (13), (17) and (19). Thus an age-old wish of the physicists is fulfilled: Useful relations in the structure and the interaction of the matter between the microscopic and the macroscopic area were found.

6. References

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