Study of Multiple Stenosed Artery with Periodic Body Acceleration in Presence of Magnetic Field

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Abstract

The purpose of this paper is to study the characteristics of blood flow through the multi stenosed artery under the influence of periodic body acceleration in the presence of magnetic field. A mathematical model has been developed by considering blood as an incompressible Casson fluid and the small blood vessel as a circular tube. The non-linear equations governing the flow are solved and the effects of magnetic field, the body acceleration of blood on velocity, the volumetric flow rate in stenotic region and the wall shear stress on the surface of stenosis are obtained by applying perturbation method. The results are discussed in detail with the help of graphs for the variation of different flow parameters. Some obtained results of blood flow through a multi stenosed artery are of appreciable importance in Medical Science.

Keywords: Multi-stenosed artery, Casson fluid, Magnetic field, Body acceleration, Perturbation method.

Introduction

Stenosis is one of the serious cardiovascular diseases which causes abnormal blood flow in coronary arteries. Accumulation of fat, cholesterol and cellular waste products on an inner side of the blood vessels causes the path very narrow which makes blood flow through the blood vessels very difficult. This is a general reason for cardiovascular diseases especially heart attack, stroke,etc.

A good number of studies have been performed by Lee and Fung[8], Mehrotra et al.[10], Misra and Chakravarty[11], Young[23] by considering blood as a Newtonian fluid. Various mathematical models [21] have been developed to possess blood flow in stenosed artery by considering blood as both Newtonian and non-Newtonian fluid. A mathematical model on pulsatile flow of Herchel-Bulkey fluid

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analyzed by Shankar and Hemalatha[17]. Siddiqui et al.[19] conducted the experiments and suggested that the behaviour of blood at low shear rate can be described by Casson model. To explain the effects of stenosis on blood flow in an artery so many investigations have been made by Misra and Verma[12], Shukla et al.[18]. Pulsatile flow of Casson fluid flow discussed by Chaturani and Samy[3], by treating the blood as a non-Newtonian fluid. Several researchers have considered blood as a Newtonian fluid but some have taken it as non-Newtonian, since blood is a suspension of red cells in non magnetic colourless liquid part Plasma. Halder and Ghosh[7] have investigated Newtonian blood flow through indented artery under the effect of magnetic field in the presence of erythrocytes. Shah [16] presented a numerical model to determine the effect of magnetic field on blood flow in an axially non-symmetric but radially symmetric stenosed artery. Tanwar et al.[20] analyzed the magnetic field effect on oscillatory arterial blood flow in mild stenosis.

According to physiological importance of body acceleration many investigations have been developed for blood flow under the influence of body acceleration with and without stenosis. Belardinelli et al.[1] proposed mathematical models for the various forms of body acceleration. Chaturani and Palanisamy[2] analyzed the pulsatile blood flow under periodic body acceleration by assuming blood as a Casson fluid. Elshehawey et al.[6] studied the effect of body acceleration pulsatile flow through porous medium by treating blood a Newtonian fluid by applying transform method. Then El-Shahed[5] developed this research to a stenosed porous medium. Nagarani and Sarojamma[13] analyzed the effect of body acceleration on pulsatile casson fluid flow through a mild stenosed artery. Mandal et al.[9] developed a two dimensional mathematical model to explain the effect of extremely imposed periodic body acceleration on non-Newtonian blood flow through an elastic stenosed artery by assuming blood characterized power law model. The effect of magnetic field and body acceleration on axial velocity of pulsatile blood flow in inclined circular tube have been discussed by Sanyal et al.[15]. Das and Saha[4] analyzed about arterial MHD pulsatile blood flow under periodic body acceleration. Pulsatile flow of couple stress fluid through porous medium with periodic body acceleration and magnetic field has discussed by Rathod and Tanveer[14]. Varshney and Agarwal[22] worked on MHD pulsatile couple stress fluid flow through inclined circular tube with periodic body acceleration.

In view of the above, a mathematical model is developed to study the characteristics of blood flow through a multi-stenosed artery under the influence of periodic body acceleration in the presence of magnetic field. The blood is modeled as a casson fluid and the small blood vessel as a circular tube. By applying perturbation method, the effect of magnetic field, the body acceleration of blood on velocity, the volumetric flow rate in stenotic region and the wall shear stress on the surface of stenosis are obtained and shown graphically for the variations of different flow parameters.

Mathematical Formulation

Let us consider the motion of blood in a multi-stenosed artery under the influence of periodic body acceleration in the presence of external magnetic field. The geometry of stenosis in dimensionless form is as shown in Fig(1), which is given by

$$\begin{aligned} R(z) &= 1 - c[L_0^{s-1}\{(z-d) - (z-d)^s\}];\ d \leq z \leq d + L_0 \\ &= 1 \quad \text{otherwise}; \end{aligned} \tag{1}$$

where, L_0 is the length of stenosis, d denotes the position of stenosis, and c is a parameter which is given by

$$c = \frac{\delta}{R_0(L_0)^s} \tag{2}$$

where, s is a parameter determining the shape of stenosis, R_0 is the radius of normal artery and δ be the maximum height of the stenosis at

$$z = d + \frac{L_0}{s^{\frac{1}{s-1}}} \tag{3}$$

such that $\frac{\delta}{R_0} << 1$. To develope this mathematical model, let us assume that the blood flow is axially symmetric and fully developed flow which is flowing along z-direction only in a circular tube of uniform cross section. Since the red blood cell is a major biomagnetic substance, considering the blood as a magnetic fluid and hence the magnetization will form a rotational motion related to magnetic field with magnetic fluid particles.

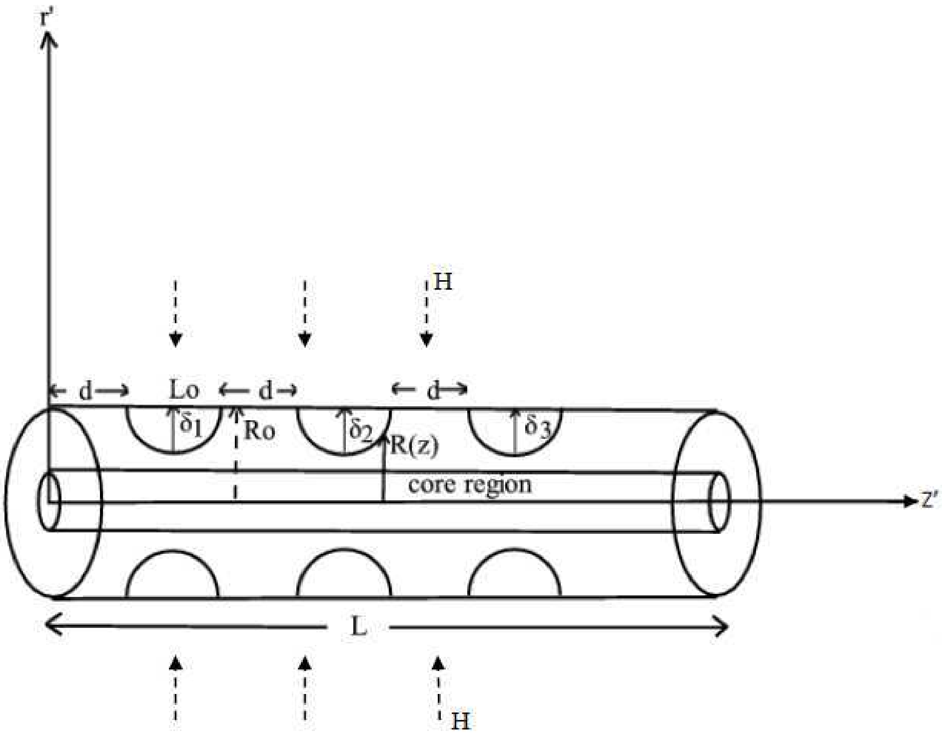

Figure 1: Geometry of Artery with multiple stenoses

The periodic body acceleration and the pressure gradient in axial direction are respectively given by

$$F(t') = a_0 \cos(\omega_b.t') \tag{4}$$

and

$$-\frac{\partial p'}{\partial z'} = A_0 + A_1 \cos(\omega_p.t') + B_1 \sin(\omega_p.t') \tag{5}$$

where, A_0 and A_1 are the steady component of pressure gradient and amplitude of oscillatory part respectively. Both A_0 and A_1 are the functions of z. $\omega_p = 2\ \pi\ f_p$; $\omega_b = 2\ \pi\ f_b$, where f_p and f_b represents frequency of pulse and body acceleration respectively. Also, a_0 is it's amplitude.

The specified momentum equation for the flow in cylindrical polar co-ordinate system is given by

$$\rho' \cdot \frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \frac{1}{r'} \cdot \frac{\partial}{\partial r'}(r'\tau_c') + F(t') + \mu_0 M \frac{\partial H'}{\partial z'} \tag{6}$$

$$\frac{\partial p'}{\partial r'} = 0 \tag{7}$$

where, r' and z' denote the radial and axial co-ordinates respectively. ρ' denote density, p' pressure and τ_c' is the shear stress. μ magnetic permeability, M magnetization, H' magnetic field intensity, u' axial velocity of blood, and t' is the time. For Casson fluid the relation between shear stress and shear rate is given by

$$\tau_c'^{\frac{1}{2}} \quad = \quad \tau_0'^{\frac{1}{2}} + \left[\mu\ (-\frac{\partial u'}{\partial r'})\right]^{\frac{1}{2}} \qquad if \ \ \tau_c' \geq \tau_0' \tag{8}$$

$$\frac{\partial u'}{\partial r'} \quad = \quad 0 \quad if \ \ \tau_c' \leq \tau_0' \tag{9}$$

where, τ_0' represents the yield stress and μ is the viscosity of blood.

The boundary conditions are

$$u' = 0 \quad \text{at} \quad r' = R'(z) \tag{10}$$

$$\tau_c' \quad \text{is} \quad \text{finite} \quad \text{at} \quad r' = 0 \tag{11}$$

$$\text{In core region,} \quad u' = u_c' \quad \text{at} \quad r' = R_c' \tag{12}$$

Introducing the dimensionless quantities as follows:

$$u' = \frac{A_0 R_0^2 u}{4\mu}; \quad z' = zR_0; \quad r' = rR_0; \quad R' = RR_0; \quad \delta' = \delta R_0; \quad t' = \frac{t}{\omega_p}; \quad \omega = \frac{\omega_b}{\omega_p}; \tag{13}$$

$$B = \frac{a_0}{A_0}; \quad e_1 = \frac{A_1}{A_0}; \quad e_2 = \frac{B_1}{A_0}; \quad H' = HH_0; \quad \tau_c' = \frac{A_0 R_0 \tau_c}{2}; \quad and \quad \alpha^2 = \frac{\rho R_0^2 \omega_p}{\mu}.$$

Here, α is called Womersley parameter.

The dimensionless momentum equation (6) becomes

$$\alpha^2 \frac{\partial u}{\partial t} = 4[1 + e_1 cost + e_2 sint + Bcos(\omega t)] + \frac{2}{r}\frac{\partial}{\partial r}(r\tau_c) + \frac{4\mu_0 M H_0}{A_0 R_0} \cdot \frac{\partial H}{\partial z} \tag{14}$$

Equation (8) and (9) can be written as

$$\tau_c^{\frac{1}{2}} = \tau_0^{\frac{1}{2}} + \frac{1}{\sqrt{2}}(-\frac{\partial u}{\partial r})^{\frac{1}{2}} \quad if \quad \tau_c \geq \tau_0 \tag{15}$$

$$\frac{\partial u}{\partial r} = 0 \quad if \quad \tau_c \leq \tau_0 \tag{16}$$

The boundary conditions (10),(11) and (12) reduce to

$$u = 0 \quad \text{at} \quad r = R(z) \tag{17}$$

$$\tau_c \quad \text{is} \quad \text{finite} \quad \text{at} \quad r = 0 \tag{18}$$

$$\text{In core region,} \quad u = u_c \quad \text{at} \quad r = R_c \tag{19}$$

Method of Solution

Applying perturbation method, the velocity u, and shear stress τ are expanded as follows:

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) \tag{20}$$

$$\tau_c(z,r,t) = \tau_{c_0}(z,r,t) + \alpha^2 \tau_{c_1}(z,r,t) \tag{21}$$

Substituting the equations (20) and (21) in equation (14) and equating the constant terms and α^2 terms, we get

$$\frac{\partial}{\partial r}(r \cdot \tau_{c_0}) = -2f(t) \cdot r - \frac{2\mu_0 M H_0}{A_0 R_0} \cdot \frac{\partial H}{\partial z} \cdot r \tag{22}$$

$$\frac{\partial u_0}{\partial t} = \frac{2}{r}\frac{\partial}{\partial r}(r \cdot \tau_{c_1}) \tag{23}$$

where, $f(t) = 1 + e_1 cost + e_2 sint + Bcos(\omega t)$.

Integrating equation (22) and applying the boundary condition (17), we get the relationship

$$\tau_{c_0} = -g(t) \cdot r \tag{24}$$

$$\text{where,} \quad g(t) = 1 + e_1 cost + e_2 sint + Bcos(\omega t) + F \cdot \frac{\partial H}{\partial z} \tag{25}$$

Substituting the equations (20) and (21) in equation (15) and equating the constant terms and α^2 terms, we get

$$-\frac{\partial u_0}{\partial r} = 2\tau_{c_0} + 2\tau_0 - 4\sqrt{\tau_{c_0} \cdot \tau_0} \tag{26}$$

$$-\frac{\partial u_1}{\partial r} = 2\tau_{c_1}\left[1 - \sqrt{\frac{\tau_0}{\tau_{c_0}}}\right] \tag{27}$$

Integrating the equation (26), using the relation (24) and applying the boundary condition (17), we get

$$u_0 = g(t)R^2\left[1 - (\frac{r}{R})^2 - \frac{8k}{3\sqrt{R}}\left\{1 - (\frac{r}{R})\right\}^{\frac{3}{2}} + \frac{2k^2}{R}\left\{1 - (\frac{r}{R})\right\}\right] \tag{28}$$

$$\text{where,} \ \ k^2 = \frac{\tau_0}{g(t)} \quad \text{and R denotes R(z).} \tag{29}$$

The plug core velocity u_{0p} can be obtained from equation (28) as

$$u_{0p} = g(t)R^2 \left[1 - (\frac{R_{0p}}{R})^2 - \frac{8k}{3\sqrt{R}} \left\{1 - (\frac{R_{0p}}{R})\right\}^{\frac{3}{2}} + \frac{2k^2}{R} \left\{1 - (\frac{R_{0p}}{R})\right\}\right] \tag{30}$$

Similarly, the solution for u_1 becomes by integrating the equation (27) and applying the boundary condition(17) as follows:

$$u_1 = \frac{g'(t)R^4}{16} \left[(\frac{r}{R})^4 - 4(\frac{r}{R})^2 + 3 + \frac{k}{\sqrt{R}} \left\{\frac{16}{3}(\frac{r}{R})^2 - \frac{424}{147}(\frac{r}{R})^{\frac{7}{2}} + \frac{16}{3}(\frac{r}{R})^{\frac{3}{2}} - \frac{1144}{147}\right\} + \frac{k^2}{R} \left\{\frac{128}{63}(\frac{r}{R})^3 - \frac{64}{9}(\frac{r}{R})^{\frac{3}{2}} + \frac{320}{63}\right\}\right] \tag{31}$$

From the equation (20), the velocity expression becomes

$$u = g(t)R^2 \left[1 - (\frac{r}{R})^2 - \frac{8k}{3\sqrt{R}} \left\{1 - (\frac{r}{R})\right\}^{\frac{3}{2}} + \frac{2k^2}{R} \left\{1 - (\frac{r}{R})\right\} + \frac{\alpha^2 R^2 c}{16} \left((\frac{r}{R})^4 - 4(\frac{r}{R})^2 + 3 + \frac{k}{\sqrt{R}} \left\{\frac{16}{3}(\frac{r}{R})^2 - \frac{424}{147}(\frac{r}{R})^{\frac{7}{2}} + \frac{16}{3}(\frac{r}{R})^{\frac{3}{2}} - \frac{1144}{147}\right\} + \frac{k^2}{R} \left\{\frac{128}{63}(\frac{r}{R})^3 - \frac{64}{9}(\frac{r}{R})^{\frac{3}{2}} + \frac{320}{63}\right\}\right)\right] \tag{32}$$

where, $\mathrm{c} = \frac{g'(t)}{g(t)}$

On applying perturbation method in the equations (26) and (27) using the boundary condition (18), the wall shear stress can be written as

$$\tau_c = g(t)R \left\{1 + \frac{\alpha^2 R^2 c}{8}(1 - \frac{8k}{7\sqrt{R}})\right\} \tag{33}$$

The volumetric flow rate Q is given by the equation

$$Q = 4 \int r \cdot u(z, r, t) \ dr \tag{34}$$

Integrating the equation (34) and applying the boundary condition (18), the volumetric flow rate Q can be written as

$$Q = g(t)R^4 \left[\frac{1}{4} - \frac{4}{7}(\frac{k}{\sqrt{R}}) + \frac{1}{3}(\frac{k}{\sqrt{R}})^2 + \frac{\alpha^2 R^2 c}{16} \left\{\frac{2}{3} + \frac{120}{77}(\frac{k}{\sqrt{R}}) + \frac{32}{35}(\frac{k}{\sqrt{R}})^2\right\}\right] \tag{35}$$

Numerical Results and Discussion

The objective of this investigation is to study the blood flow through a multi-stenosed artery under the action of periodic body acceleration as well as an external magnetic field by modeling blood as a Casson fluid. The relevant computational work has been performed for some specific cases using available experimental data.

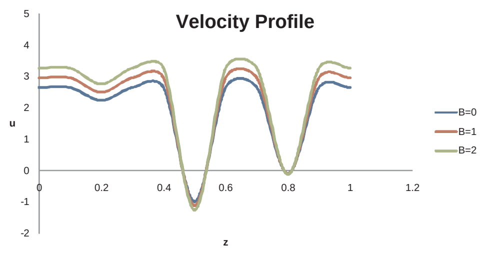

Figure 2: Variation of axial velocity along z-axis for different values of body acceleration B

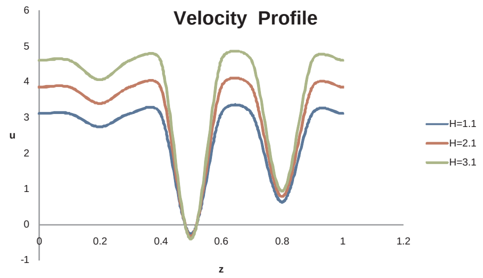

Figure 3: Variation of axial velocity along z-axis for different values of magnetic field gradient H

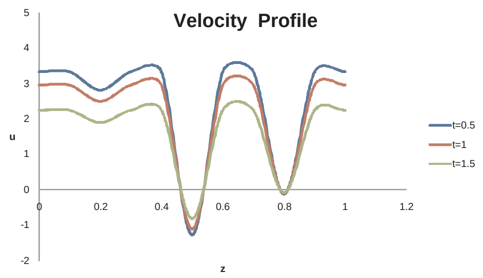

Figure 4: Variation of axial velocity along z-axis for different values of time t

The axial velocity profile is computed by using the velocity expression (32) for different values of periodic body acceleration B, the induced magnetic field gradient $\mathbf{H} = \frac{\mathbf{dH}}{\mathbf{dz}}$ and the variation of time t. Hence, we can control the process of flow. From Figures (2) and (3), we notice that when the body acceleration and the magnetic field gradient increases, the amplitude of axial velocity also increases. Figure (4) shows that by increasing time t, the amplitude of axial velocity decreases.

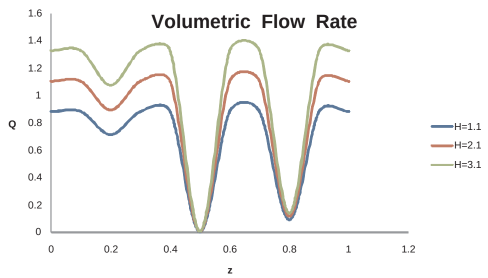

Figure 5: Variation of volumetric flow rate along z-axis for different values of magnetic field gradient H

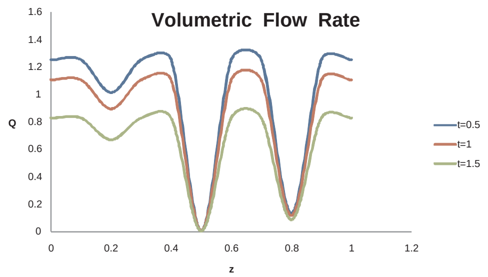

Figure 6: Variation of volumetric flow rate along z-axis for different values of time t

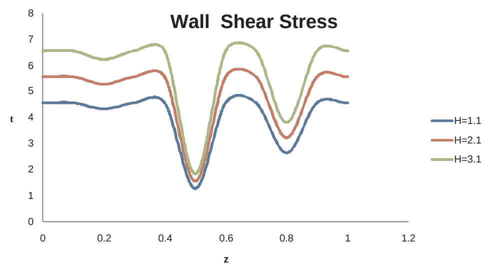

Figure 7: Variation of wall shear stress along z-axis for different values of magnetic field gradient H

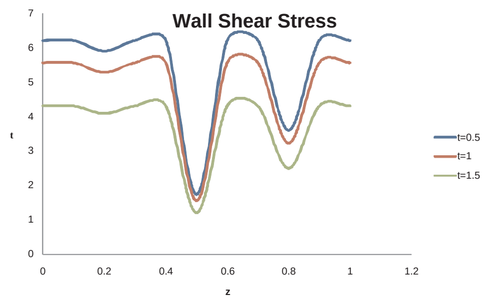

Figure 8: Variation of wall shear stress along z-axis for different values of time t

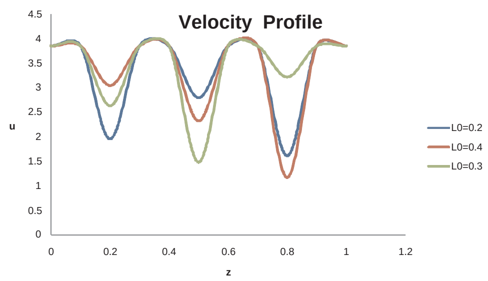

Figure 9: Variation of axial velocity along z-axis for different values of height of stenosis

The volumetric flow rate Q of blood has been computed by applying the equation (35) for different values of magnetic field gradient $\mathbf{H}$ and the variation of time t. From figures (5) and (6), it is observed that the volumetric flow rate along z-axis increases by increasing the magnetic field gradient $\mathbf{H}$ whereas it decreases with increasing of time t.

The wall shear stress τ_c has been computed by using the equation (33) for different values of magnetic field gradient $\mathbf{H}$ and the variation of time t. From figures (7) and (8), we notice that the wall shear stress τ_c along z-axis increases by increasing magnetic field gradient $\mathbf{H}$ whereas it decreases by increasing time t like the effect of volumetric flow rate.

Figure 9 illustrates that when the height of stenosis is greater than the length of the stenosis then flow of blood decreases and when $\delta = 0.4$ which is nearly half the radius of the tube, then there is a reverse flow of blood. From Fig.10, we notice that when the length of the stenosis is greater than the height of the stenosis then flow of blood increases and when $L_0 = 0.5$, there is a reverse flow of blood.

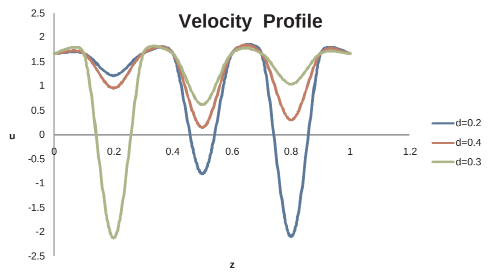

Figure 10: Variation of axial velocity along z-axis for different values of length of stenosis

Conclusion

The non-linear equations governing the flow are solved by applying perturbation method. We can conclude that

- The amplitude of axial velocity increases by increasing periodic body acceleration B.
- By increasing the induced magnetic field gradient $\mathbf{H}$, the amplitude of axial velocity also increases. But decreases by increasing the time t.
- The volumetric flow rate and wall shear stress of stenotic region increases by increasing the magnetic field gradient and shows a decrease by increasing time t.

. The results obtained in this paper can be applied to the pathological situations of blood flow in stenosed arteries when fatty plaques of cholestrol and artery blocking blood clots are formed in the lumen of the coronary artery.

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