International Journal of Scientific Research and Management (IJSRM)
||Volume||09||Issue||10||Pages||P-2021-12-15||2021||
Website: www.ijsrm.in ISSN (e): 2321-3418
DOI: 10.18535/ijsrm/v9i10.p01

Rotational Ground-State Bands of Doubly Even-Even ^{170,172}Hf Isotopes

[1]I. Hossain, [2]Hewa Y. Abdullah, [3]I.M. Ahmed, [4,5]Fadhil I. Sharrad

[1]Department of Physics, Rabigh College of Science and arts, King Abdulaziz University, Post Box 344, Rabigh 21911, Saudi Arabia
[2]Physics education department, Faculty of Education, Tishk International University, Erbil-Iraq
[3]Department of Physics, College of Education, Mosul University, 09334, Mosul, Iraq
[4]Department of Physics, College of Science, University of Kerbala, Iraq.
[5] College of Health and Medical Technology, Al-Ayen University, An Nasiriya, Thi Qar, Iraq.

Abstract
Using phenomenological fitting, Sood's semi-empirical formula have been investigated the rotational structure of even-even rare-earth ^{170,172}Hf nuclei. The rotational energy band of an axially symmetric rotator shows the properties J(J+1) requirement if we take adiabatic approximations for the motions of all the particles in the nucleus. The low-lying energy levels have follows spin-parity sequence $J^{\pi} = 0^+, 2^+, 4^+, 6^+$ … for the even parity rotational ^{170,172}Hf nuclei. The calculated values of energy levels based on Sood's semi-empirical formula are presented with experimental values of low-lying rotational energy levels where $K^{\pi} = 0^+$ and J^{π} take even values from 0^+ up to 18^+. It showed that in ^{172}Hf and ^{170}Hf isotopes, calculated energies fits the experimental values to a remarkable degree of accuracy with deviation less than 3 percent at $J^{\pi} = 18^+$ state and Sood's semi-empirical formula predicted rotational energies. The maximum deviation is 2.96 % in ^{170}Hf at 16^+ levels and 0.68 % in ^{172}Hf at 18^+ levels.

Introduction

The even-even Hf isotopes are one of the most important in nuclear science. In the collective model proposed by Bohr and Mottelson (1953) a parameter called nuclear deformation is introduced, which the surface of the nucleus may undergo oscillations in a rotating nucleus. These results in the prediction of rotational energy levels being restricted to even values due to oscillating symmetry and it is easily observed with nuclei having number of nucleons far from closed shells. Experimental data have been studied by Wallace and Welch (1960) comes to a good agreement with these predictions, and applications such as probability for B(E2) transitions, magnetic moments, quadrupole moments and isomeric transitions are so far successful.

In the macroscopic interpretation, the common simplest initial fact of modelling the atomic nucleus is based on semi-classical liquid drop model by Feenberg (1947) and Green and Engler (1953) where the nucleus is assumed to behave akin to a drop of dense incompressible liquid where the spherical shape is the result of attractive forces between all the particles contained in the nucleus. This model is then prolonged to explain quantum mechanical collective motion such as rotation and vibrations, foremost to the Bohr and Mottelson collective model (1953). In the collective model, a new parameter called nuclear deformation is introduced, which the surface of the nucleus may undergo oscillations in a rotating nucleus. These results in the prediction of rotational energy levels being restricted to even values due to oscillating symmetry and it is easily observed with nuclei having number of nucleons far from closed shells.

In this study, Sood's semi-empirical formula (SSEF) are utilized to calculate the ground state bands of Hf nuclei with mass A=70, 72 and spin-parity up to 18^+. Hafnium isotope was selected since it lies in the transition area of well deformed rare-earth nuclei and the closed shell of spherical ^{208}Pb nucleus (Kumar and Gunye, 1980).

Materials and Methods

1. SOOD SEMI-EMPIRICAL FORMULA

The shape of atomic nuclei with higher number of nucleons outside the closed shells is axially symmetric and permanently deformed by Bohr and Mottelson (1953). The rotational energy band of an axially symmetric rotator shows the properties J(J+1) requirement if we take adiabatic approximations for the motions of all the particles in the nucleus. The low-lying energy levels follows spin-parity sequence $J^{\pi} = 0^+, 2^+, 4^+, 6^+$, and So on for the even parity rotational nuclei (Preston and Bhaduri; 1975).

$$E_J = AJ(J+1) \quad (1)$$

Where rotational parameter A is inversely proportional to the nuclear moment of inertia $\Im$, such that $A = \hbar^2/2\Im$. The moment of inertia is also assumed to be constant following an approximation where the energy ratio $E(4^+)/E(2^+) = 10/3$, $E(6^+)/E(2^+) = 7$, $E(8^+)/E(2^+) = 12$. Deviations by a few per-cent from these ratios are thought to be coupling of vibrational and rotational modes of the nuclei. This coupling depends on J^2 and constitutes changes in rotational energy from the first-order perturbation theory. However, for fitting the first three or four excited states, first-order correction term can be added to equation (1) where the rotational energy by Preston and Bhaduri (1975); Saethre, Hjorth, and Johnson *et al*. (1973).

$$E_J = AJ(J+1) - BJ^2(J+1)^2 \quad (2)$$

Even with addition of correction terms in equation (2), at higher rotational state where J > 10 the approximation will deviate significantly from experimental values. Stephens, Diamond and Perlman (1959) and Singh and Sahota (1985) added second-order $CJ^3(J+1)^3$ and third-order $DJ^4(J+1)^4$ correction terms respectively to account the deviations in energy bands for even- ass actinide nuclei. The rotational energy stated in terms of an infinite power series (Sood (1968):

$$E_J = AJ(J+1) + BJ^2(J+1)^2 + CJ^3(J+1)^3 + DJ^4(J+1)^4 \quad (3)$$

Assuming that the coefficient parameters are constant through each successive order of correction terms, where $CB^{-1} = DC^{-1} = N(BA^{-1})$. Sood (1968) derivative a semi-empirical formula for rotational energy by summing of infinite series in equation (3):

$$E_J = AJ(J+1)\left[\frac{1 + (N-1)\left(\frac{B}{A}\right)J(J+1)}{1 + N\left(\frac{B}{A}\right)J(J+1)}\right] \quad (4)$$

where ratio $BA^{-1} = (10\text{-}3R)/(200\text{-}18R)$ and $R = E(4^+)/E(2^+)$. The constant N was obtained by Sood (1968) via empirical method was:

$$N = 2.85 - 0.05J \quad (5)$$

Results and Discussion

The calculated values of energy levels based on Sood's semi-empirical formula are presented with experimental values of low-lying rotational energy levels where $K^{\pi} = 0^+$ and J^{π} take even values from 0^+ up to 18^+ are shown in Table 1. The experimental energy levels of ^{170}Hf in $2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+, 16^+, 18^+$ are 100.80(17), 321.99(20), 642.9(30), 1043.1, 1505.2, 2016.1, 2567.0, 3151.3 and 3766.5 keV respectively. The corresponding theoretical calculation of those yrast levels are 100.883, 324.677, 651.006, 1060.134, 1536.319, 2066.919, 2640.387, 3244.545, and 3865.336 keV respectively. The deviation of theoretical and experimental calculation of ^{170}Hf yrast level $2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+, 16^+, 18^+$ are 0.08, 0.83, 1.26, 1.63, 2.61, 2.07, 2.52, 2.86, 2.96 and 2.62 in percentage respectively. Similarly, the experimental data of yrasts levels $2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+, 16^+, 18^+$ in ^{172}Hf nucleus are 95.22(4), 309.24(5), 628.33(7), 1037.47(8), 1521.22(10), 2064.67(13), 2654.11(16), 3277.2(19), and 3919.4(3) in keV respectively. The associate theoretical calculation of energy level in ^{172}Hf is 95.251, 310.264, 630.848, 1040.813, 1524.973, 2069.895, 2663.296, and 3292.998 in keV respectively. The deviation of both data in ^{172}Hf is 0.03, 0.33, 0.4, 0.32, 0.25, 0.35, and 0.48 in percentage respectively. We note that the percentage of deviation between calculated and experimental values are very small in $^{170, 172}$Hf isotope. The theoretical calculation using semi-empirical formula for rotational energy of both nuclei is consistent with experimental data.

Table 1: Previous experimental (Baglin, 2002; Singh, 1995) and present theoretical energies of GSB for even-mass ^{170,172}Hf .

A = 170				A = 172			
J^{π}	Exp. $E(J^{\pi})$ [KeV]	Calculated $E(J^{\pi})$ [KeV]	Deviation %	J^{π}	Experiment $E(J^{\pi})$ [KeV]	Calculated $E(J^{\pi})$ [KeV]	Deviation %
0^+	0	0	0	0^+	0	0	0
2^+	100.80(17)	100.883	0.08	2^+	95.22(4)	95.251	0.03
4^+	321.99(20)	324.677	0.83	4^+	309.24(5)	310.264	0.33
6^+	642.9(30)	651.006	1.26	6^+	628.33(7)	630.848	0.4
8^+	1043.1	1060.134	1.63	8^+	1037.47(8)	1040.813	0.32
10^+	1505.2	1536.319	2.07	10^+	1521.22(10)	1524.973	0.25
12^+	2016.1	2066.919	2.52	12^+	2064.67(13)	2069.895	0.25
14^+	2567	2640.387	2.86	14^+	2654.11(16)	2663.296	0.35
16^+	3151.3	3244.545	2.96	16^+	3277.2(19)	3292.998	0.48
18^+	3766.5	3865.336	2.62	18^+	3919.4(3)	3945.876	0.68

To ease the presentation of data, numerical results extracted from nuclear data sheets are presented as red dots in a graph of spin state against excitation energy in KeV scale. The black squares represent calculated energy values based on Sood's semi-empirical formula..

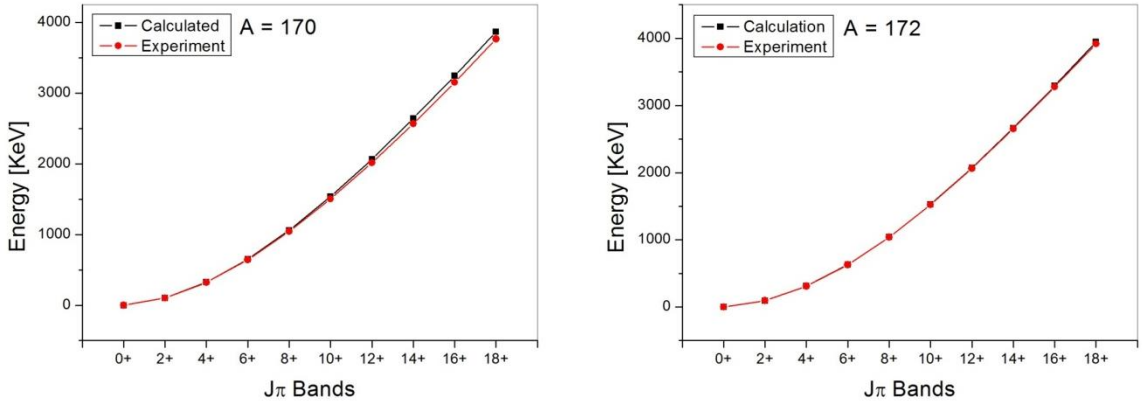

Figure 1: Comparison of the calculated and observed ground state rotational bands of even- $^{170\text{-}172}$Hf Isotopes. The experimental data are taken (Baglin, 2002; Singh, 1995).

From figure 1 we notice that for heavier even Hafnium $^{170\text{-}172}$Hf isotopes, the phenomenological Sood's semi-empirical formula fits the observed energies to an extent of remarkable agreement up to spin state 18^+ which relate to data in table 1. The maximum deviation is 2.96 % in ^{170}Hf at 16^+ level and 0.68 % in ^{172}Hf at 18^+ level. The deviations from J(J+1) rule in rotational energy bands are interpreted as the compression of energy levels, or in another arguments, an increase in nuclear moment of inertia as the nucleus undergoes changing rotational angular velocity.

Conclusions

Sood's semi-empirical formula is successfully applied to calculate rotational spectrum of even-even $^{170\text{-}172}$Hf isotopes. The theoretical calculation of yrast levels of ^{170}Hf at 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+, 16^+, 18^+ are 100.883, 324.677, 651.006, 1060.134, 1536.319, 2066.919, 2640.387, 3244.545, and 3865.336 keV respectively. The associate theoretical calculation of energy level of ^{172}Hf is 95.251, 310.264, 630.848, 1040.813, 1524.973, 2069.895, 2663.296, and 3292.998 in keV respectively. The results agreed well with previous experimental values of ^{170}Hf [Baglin, 2002] and ^{172}Hf [Sing, 1995] within the expectations of simple phenomenological fitting. The maximum deviation is 2.96 % in ^{170}Hf at 16^+ levels and 0.68 % in ^{172}Hf at 18^+ level.

Ethics approval and consent to participate

"Not applicable" in this section.

Data Availability
A data can be found from this research articles. Moreover reader can contact with corresponding author to get details data.

Conflicts of Interest
There are no conflicts of interest.

Funding Statement
There is no research fund.

Authors' contributions
All authors are equally contributed to this work. All authors read and approved the final manuscript.

Acknowledgments
We thank Physics Education Department, faculty of education, Tishk International University, Erbil-Iraq, Physics department, Rabigh college of Science, King Abdulaziz University and Department of Physics, University of Kerbala, for supporting this work.

Supplementary Materials
"Not applicable" in this section.

References

1. A. Bohr and B. R. Mottelson,(1953) "Interpretation of Isomeric Transitions of Electric Quadrupole Type**"** *Physics. Review* **89**, 316
2. A. Bohr, B. R. Mottelson. "Collective Nuclear Motion and the Unified Model, Beta- and Gamma-Ray Spectroscopy" https://cds.cern.ch/record/212241/files/cern-55-07.pdf
3. M. C. Baglin ,(2002) "Nuclear Data for A= 170" *Nuclear Data. Sheets*, 96, 611-873
4. E. Feenberg, (1947) Semi-empirical Theory of the Nuclear Energy Surface, *Reviews of Modern Physics* **19**, 233.
5. A.E.S. Green and N.A. Engler,(1953) "Mass Surfaces", Phys. Rev. 91, 40-45
6. A. Kumar A and Gunye MR (1980). The Yrast States of the Even Hafnium Isotopes $^{166-172}$Hf, J. Phys. G: Nucl. Phys. **6**,229 – 240.
7. M. A Preston, R. K Bhaduri (1975). Structure of the Nucleus, Addison-Wesley Publishing Company.
8. B, Sing (1995), "Nuclear Data for A= 172". Nucl. Data. Sheets, **75**,199-376
9. K. Singh and H.S. Sahota (1985), "Absolute Pair Production Cross Sections for Z = 13, 26, 29, 50 and 82 at (1.173+1.333) MeV", Ind. J. Phys. A, **59**,157.
10. S. A. Saethre, A. Hjorth, S. Johnson, S. Jägare, H. Ryde and Z. Szymański. (1973), "Angular Velocity Expansions of Nuclear Rotational Energies" *Nucl. Phys. A*. 207,486-512.
11. F. S. Stephens, J.R.M. Diamond and I. Perlman (1959), "Multiple Coulomb Excitation in Th-232 and U^{238}" *Phys. Rev. Lett.* 3, 435.
12. P.C. Sood (1968), "Comparative Study of Two Parameter Models for Predicting Rotational Energies in even-even Nuclei". *Nuclear Data Sheet A*, 4, 281-300.
13. R. Wallace and J.A. Welch, (1960) "Beta Spectra of the Mirror Nuclei", *Physical Review*, 117, 1297.