

# Improving the Trigonometric Functions Learning Concept with Dynamic Mathematics Software

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## Abstract

Since the 21st century, the new model of Internet+Education has gradually penetrated into the field of mathematics education and has become a hot topic in the current research on mathematics education, and education in formatization has become the main development trend. Therefore, the purpose of this research is to help students and teachers how to use Hawgent dynamic mathematics software to help students learn trigonometric functions and help teachers practice trigonometric functions. The research method used is the research and development (R&D) method and this research was done at Chongqing Zhongxian Senior Middle School for the period month of July to August 2021. The model has adaptive guiding principles that can help researchers develop ICT based learning media in the five stages of analysis, design, development, implementation, and evaluation. This research is based on the Hawgent dynamic mathematics software to design a trigonometric function integrable ware design model, dynamically presenting the concept of trigonometric function and its image and properties, and then use integrable ware to assist the teaching of trigonometric functions, and verify it through experimental teaching. The results show that the learning of trigonometric functions assisted by Hawgent dynamic integrable ware design can help improve students' understanding and interest in trigonometric functions. In the future, other researchers can study the influence of information technology on the teaching of mathematical formulas, theorems, and mathematical problem solving, and explore SEM models of influencing factors.

## 1. Introduction

Since the 21st century, the new model of "Internet + Education" has gradually speared into mathematics education, and educational information has become the main trend of education development. The "(General High School Mathematics Curriculum Standards (2017 Edition)[1]" pointed out that "Teachers should pay attention to the in-depth integration of information technology and mathematics learning to achieve effects that are difficult to achieve with traditional teaching methods". It can be seen that information technology[2] has a profound impact on mathematics education. The application of information technology to assist teaching, changing the way teachers teach, students learn mathematics and improving the effectiveness of mathematics teaching achievements has become an emerging topic in mathematics teaching research[2], [3]. A famous philosopher Marx pointed out: "A science can only be truly perfected when it uses mathematics successfully[4], [5]. It can be seen that the development of mathematics is of great significance to the rapid development of science and technology, the improvement of people's production and living standards and the implementation of the task of training mathematics talents carried by mathematics education, considered a key to promoting the development of mathematics. Due to its difficult abstract and complex features, mathematics has become a difficult subject to learn. The abstraction and complexity of mathematics itself are challenging to change. On the conflicting, it is a feasible way to study teaching methods to help students learn more easily by changing teaching methods and learning methods.

Mathematics is the science of studying quantity relations and three-dimensional forms. According to the information processing theory of the human brain, the "number" in mathematics is mainly the verbal information such as words and symbols in indexology. In contrast, the "shape" in mathematics is mainly the visual information such as figures and tables of the exponential objects. The combination of number and shape is a method of mathematical thinking and an effective means of mathematics learning. That is, presenting the text language, symbol language, and graphic language of the same mathematical objects simultaneously helps students learn more easily.

With the development of information technology and the Internet, modern mathematics teaching software has become more effective and convenient. Hawgent Dynamic Mathematics Software[6], [7] is a mathematics software independently developed by Guangzhou Hawgent Education Technology Co.Ltd. That integrates dynamic, intuitive and mathematical characteristics. At the same time, their software's comes with a large number of mathematics teaching resources and series of courses. Unique characteristics. Hawgent dynamic mathematics software has fast calculation functions, rich with graphics presentation, production functions and a large amount of data processing functions which show unique advantages in mathematics teaching and provide interactive learning and research[8], [9]. These functions provide mathematics students with an experimental and inquiry environment.

Research shows that Hawgent dynamic mathematics software can dynamically present abstract and static mathematical knowledge and its measurement function can realize the multi-dimensional presentation of the "number" or "shape" of the same mathematical objects with Fun and interesting, dynamic and intuitive[10]–[12]. It helps students discover the connection center of knowledge, see through the essence of mathematics, enhance their interest in mathematics learning, and improve their academic performance.

A trigonometric function is also a basic and important function[13], [14]. It is a mathematical model that depicts periodic changes in the real world. "a trigonometric angle function" is a generalization of acute-angle trigonometric functions and is an important part of the main line of high school functions. From the acute-angled trigonometric function in junior high school to the trigonometric function at any angle in high school, the complexity of the concept of the trigonometric function itself[15], [16]. The abstractness of the symbol and the variability of the form have led to flexibility and comprehensiveness for students in answering. The requirements are very high, so there are many difficulties. It is difficult for students to imagine how the concept of trigonometric function is established in any angle, so it is even more difficult to explore the next content of trigonometric function-the learning of images and properties[13], [16]. However, the traditional "blackboard + chalk" drawing method is static and not vivid enough. On the contrary, it creates obstacles to students' comprehension and memory, and it is difficult to highlight key points and solve difficult points.

Yin Yu (2015) used Geogebra software specifically used to integrate high school trigonometric function learning. Experimental research shows that "Geogebra software has a good teaching effect in high school trigonometric function teaching. This software can be promoted and used in teaching and information technology that can provide better teaching service[17]–[19].

Li Xin (2018) uses modern educational technology and takes "the image of function  $y = A \sin(\omega x + \psi)$ " as a typical example to explore the impact of information-based teaching methods in high school mathematics classrooms on students' learning classroom content[20]–[22].

Zhang Ruijie (2019) combed the history of trigonometric functions and designed a framework for the teaching design of trigonometric functions from the perspective of the history of mathematics. Finally, experiments show that the teaching of trigonometric functions in the history of mathematics is effective for breaking through the difficulties of thinking and can improve students' interest in learning. Students can comprehend the cultural value and application value of mathematics[23]–[26].

Kang Liyi (2021) uses information technology to improve trigonometric functions in high schools based on the perspective of APOS theory[27]–[30].

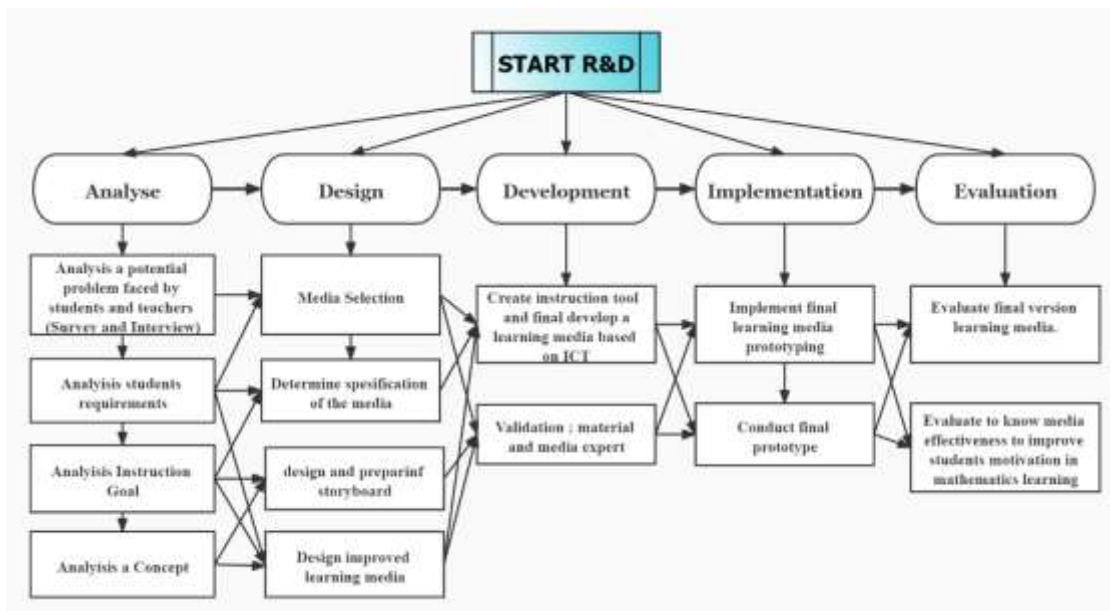
How to effectively use information technology, integrate mathematics teaching and improve the "trigonometric function" teaching methods and students' learning methods is a question worthy of research. This article attempts to base on the "Angle Trigonometric Function" integrable ware model designed by

Hawgent, and then through experimental teaching methods, to verify that the learning of trigonometric functions assisted by this integrable ware helps to improve students' understanding of trigonometric functions.

## 2. Method

The research method used is the research and development (R&D) method[8], [31], [32]. This model has adaptability guideline that helps researchers in develop ICT-based learning media in five stages: analysis, design, development, implementation, and evaluation. The mathematics topic in this learning media is trigonometric function and taught in senior high school. This research was done at Chongqing Zhongxian Senior Middle School for the period month of July to August 2021. Therefore in the implementation stage, researchers tried the learning media to 53 students from Chongqing Zhongxian Senior Middle School, and saw the students' response towards the learning media. This research used random sampling. An evaluation was carried out initially and at the end of the study using a formative evaluation method.

The first stage an analysis of the potential faced by student and teacher, student requirement, instruction goal and analysis of concept. The second stage of design is when a researcher selects media, determines media specifications, prepares story board, and designs improved learning media. The flowchart made can be seen in Figure 1 below.



**Figure 1.** ADDIE Research and development framework[14], [33]

In the development stage, a researcher creates tool and instruction and final development of the learning media and invited material and media expert to evaluate the learning media after the next stage or implementation stage. The implementation stage, researchers try to implement and conduct learning media prototype. The last stage is the learning material and media expert to evaluate the final version of learning media and know the effectiveness of improving student motivation in mathematics learning. The learning media is made by Hawgent dynamic mathematics software. The learning media were validated by 3 validators consisting of 1 expert media validator, 1 material expert validator, and 1 colleague validator (students).

Based on the research and development stages above, the research and development stages involves several materials and media experts to validate the instrument at the implementation stage. Thus, for more clarity, we can refer to the following table.

**Table 1.** Material and media expert validation instrument[11]

Validators	Aspect Measured
Material and Media Expert	The material relevant to the competency that students must master;

	Material presented coherently from beginning to end; The accuracy of the use of media in accordance with the objectives and learning materials; Can be used as learning media; Grove structured and organized learning media; Systematic, coherent and clear media system; Simple and interesting learning media; The storyboard easy to understand;
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### 3. Results and discussion

#### 3.1 Analyse stage

Trigonometric function is the first section of Chapter 5 of the compulsory book of Chinese general high school textbooks. Undertake the previous study of arbitrary angle and radian system. Mathematics comes from life, and finally, it is applied to life. Therefore, the study of this lesson lays the foundation for the application of trigonometric functions in real life. Therefore, the focus of this lesson is on the concept of trigonometric functions and their images and properties. From the acute-angled trigonometric functions learned in junior high school to the arbitrary-angled trigonometric functions learned in this lesson, the leap of thinking often hinders students' learning difficulties. Where. In addition, as we all know, a trigonometric function is an abstract concept. A triangle with any angle is unimaginable in students' cognition, which leads to low interest in learning. Most teachers often only use simple blackboard drawing and language description when explaining trigonometric functions. It is difficult to establish the concept of trigonometric function in students' minds, and it is even more difficult to learn the content of trigonometric function images and properties.

#### 3.2 Design Stage

Aiming at the problem of abstract trigonometric functions, the researchers used Hawgent dynamic mathematics software to design dynamic, easy-to-understand, and easy-to-learn integrable ware to assist trigonometric function learning.

##### (1) Design principles and steps of integrable ware parts

Based on the rules of Hawgent dynamic software and the definition of trigonometric functions, in the unit circle, the sine function  $y=\sin(x)$ , With the help of the formula, the arcs of the unit circle centre angle  $\alpha$  and the arcs are equal in value. Therefore, use the arc length of the arc against the central angle  $\alpha$  as the independent variable, the same is true,  $\cos(\alpha)=$ ,  $\tan(\alpha)=$  First make a unit circle, and then use a variable ruler to control the moving point of the unit circle to move it counter clockwise on the unit circle, and draw the arc of the point's motion trajectory; then, measure on the unit circle The arc length of the arc for the central angle  $\alpha$  is used as the independent variable  $x$ , and finally the trajectory of the point  $(\sin(x), \sin(x))$  is tracked.

##### Step 1: Draw the unit circle

First, click to display the "coordinate system", select the coordinate system in the edit mode, click the right button to call up the "attribute edit box" of the coordinate axis, execute the command "display", "x-axis in  $\pi$  as the unit, change "false" To "true", "draw the scale, change "false" to "true"; then execute the command "drawing", "coordinate points, input (0,0) and (-1,0)"; then press and hold "Ctrl" key select point B, point A in turn, execute the command "construction", "circle", you can get the unit circle. As shown in figure 3 bellow.



**Figure 2.** Key steps and renderings of drawing the unit circle

### Step 2: Draw an arc

First, select the circle and execute the command "Drawing" | "Free Point"; then, select Free Point C and execute the command "Construction" "Variable Ruler: minimum value 0, maximum value 360, with frequency 200" (this variable ruler Meaning: a moving point C on the unit circle moves on the circle at a frequency of 200 steps/circle); then select the circle, point A, point C in turn, and execute the command "drawing" "arc" "on the circle The arc". As shown in Figure 3 below.



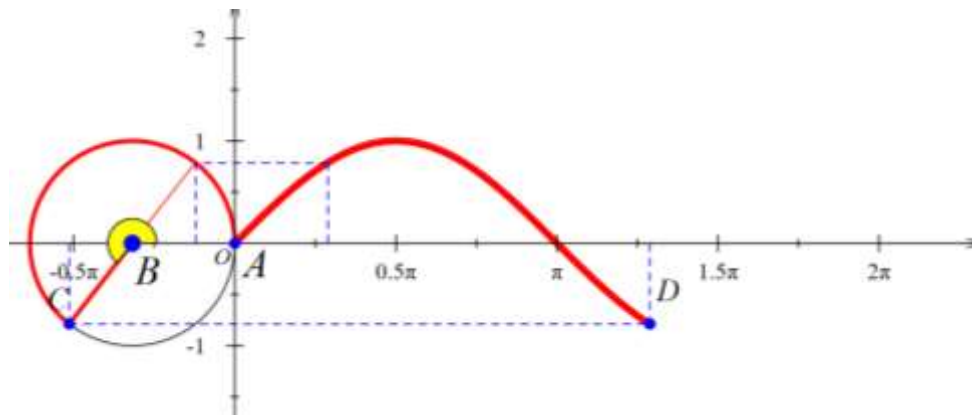
**Figure 3.** Key steps and effect diagrams of drawing arcs

### Step 3: Draw a trigonometric function image

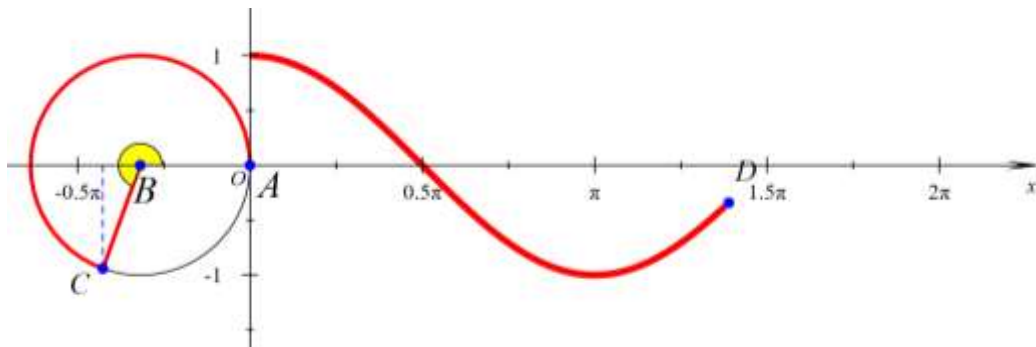
First, select the arc AC, execute the command "Construction" "Length" to get the length of the arc AC is  $u000$  (you can view its variable name in its property edit box), and then execute the command "Drawing" "Coordinate point: D ( $u000, \sin(u000)$ )", select point D and point C in turn, execute the command "construct" and "track"; select the track, right-click in the edit mode to call up the "attribute edit box" of the track, and execute the command "General" "Maximum: "Click OK.

### Step 4: Beautify of the interface

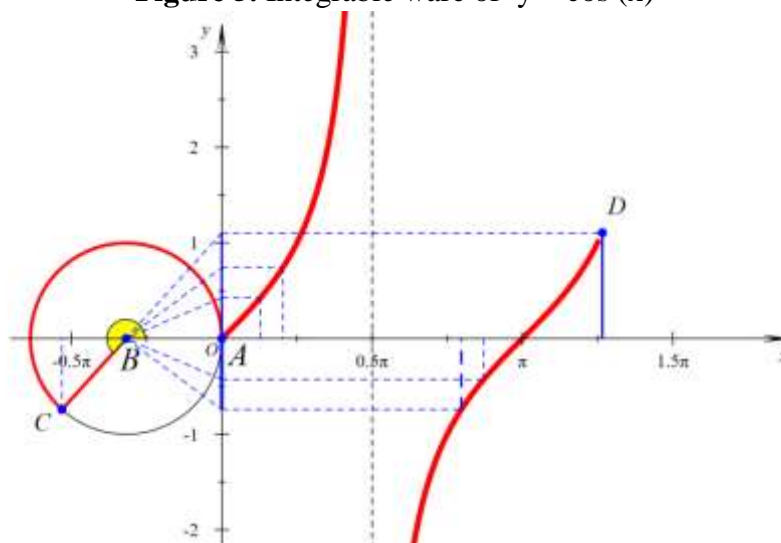
First, select the brush, connect BC, AB; switch to selection mode, select point C, line segment AB in turn, execute the command "drawing", "vertical foot", connect BE; select the line segment BE to design its colour as red and the line shape as a dashed line; Then, select point A, point B, point C in turn, execute the command "construction", "mark"; select arc AC, track in turn, design its colour to purple; finally, hide irrelevant objects. Drag the variable ruler to get the process of sine function image dynamic formation. The final result of integrable ware parts is shown in following Figure



**Figure 4.** Integrable ware of  $y = \sin(x)$



**Figure 5.** Integrable ware of  $y = \cos(x)$



**Figure 6.** Integrable ware of  $y = \tan(x)$

## (2) Application strategy of integrable ware in teaching

### 1) Teachers use integrable ware to promote students' understanding of the nature of mathematics

First of all, teachers can use the unit circle in Figure 7 to help students understand any angle: drag point C to guide students to observe the coordinate change of point C, the size change of  $\angle ABC$ , and the change of  $\text{Rt}\triangle BCD$ . The dynamic demonstration helps students use the coordinates of point C on the unit circle to describe the trigonometric function of any angle.

Then, drag point C in Figure 4. When  $\triangle ABC$  is an acute triangle, guide the students to observe and calculate the value of  $\sin\angle ABC$ . The teacher can perform calculation verification through the software. Through the observation and verification of students, when  $\triangle ABC$  is an acute triangle, the y coordinate of the value point C of  $\sin\angle ABC$  is obtained. Ask the students further: "I have just verified the value of  $\sin\angle ABC$  in an acute triangle. When  $\angle ABC$  is an obtuse angle, a straight angle, a circumferential angle, or an arbitrary

angle, do you have the same conclusion? "Next, we will learn the initiative give it to students and let them explore by themselves. In this process, with the help of information technology, the dynamic and intuitive research path from "special" to "general" expands the research on the definition of sine function.

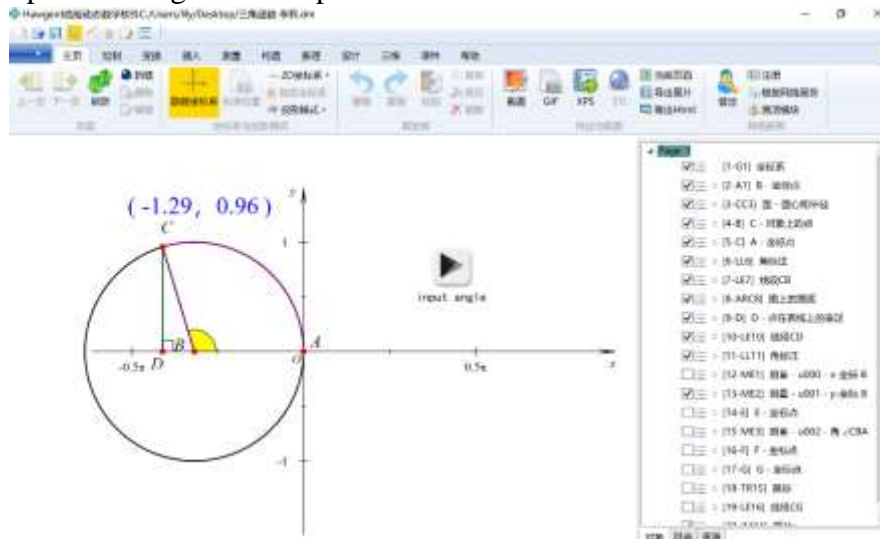


Figure 7. Integrable ware of Angle

2) Students use integrable ware to increase their interest in mathematics learning

When the angle  $\angle ABC$  is an obtuse angle, a right angle, a circular angle or an arbitrary angle, does the y coordinate of point C also have the value of  $\sin\angle ABC$ ? Students use iPad and click the button to input the angles of  $\angle ABC$  as  $180^\circ$ ,  $270^\circ$  and  $360^\circ$  respectively to obtain  $\sin\angle ABC=0$ ,  $\sin\angle ABC=-1$ ,  $\sin\angle ABC=0$ . It is initially confirmed that  $\angle ABC$  is the y-coordinate of point C. Finally, students can drag point  $\angle$  to move it on the circle. During the movement, track the ordinate trajectory of point C, the size is  $\angle ABC$ , and finally get the function image of  $y=\sin\angle ABC$ , as shown in Figure (5). With the exploratory experience defined by  $y=\sin(x)$ , classroom activities can be designed for the learning of  $y=\cos(x)$  and  $y=\tan(x)$ , and students form a learning interest group to conduct independent learning and exploration.

Through the teacher's multimedia presentation and guidance, students use Hawgent dynamic mathematics software to perform operations, guesses and verifications, which dynamically and intuitively help students establish the concept of trigonometric functions in their minds. The use of information technology has created conditions for students to operate and study independently. Not only can students intuitively "see" the image of trigonometric functions, with the help of hawgent measurement function, students can also explore trigonometric functions from a "number" perspective.

3.3 Development stage

At the development stage, researcher invited validation expert and testing learning media in a one sample class. The following are some aspects of expert evaluation, along with their level of validation and interpretation. It can be seen in the table below.

Table 2. Material and media expert validation instrument

Validators	Aspect Measured	Percentage	Desicion
Material and Media Expert	The material relevant to the competency that students must master;	87.2%	Valid
	Material presented coherently from beginning to end; The accuracy of the use of media in accordance with the objectives and	79.5%	Valid

learning materials;	88.5%	Valid
Can be used as learning media;	90.0%	Valid
Grove structured and organized learning media;	84.3%	Valid
Systematic, coherent and clear media system;	77.9%	Valid
Simple and interesting learning media;	82.0%	Valid
The storyboard easy to understand;	78.3%	Valid

Analyzing the above table, we can draw the following conclusions: 87.2% of experts believe that the content of "trigonometric functions" in this study is related materials that students must master, that is, "trigonometric functions" are important materials for students to improve their logical reasoning and intuitive imagination. 79.5% of the experts believe that the trigonometric function integrable ware page designed by Hawgent dynamic is presented in a coherent manner from beginning to end, and the special acute-angled trigonometric function is extended to the trigonometric function of any angle. The teaching design idea is logically rigorous and coherent. . 88.5% of the experts believe that the "trigonometric function" integrable ware can be used as a learning material and 84.3% of the experts believe that the integrable ware is interesting, systematic, coherent and clear.

### 3.4 Implementation stage

At the implementation stage the researchers conducted direct interviews with several students and several mathematics teachers, the results of the interviews can be seen in the following table:

**Table 3.** Student's and teacher's statement in using Hawgent dynamic mathematics software

Students and teacher's statement	
1)	Hawgent dynamic software can help me understand the concept of trigonometriy functions
2)	Hawgent dynamic mathematics software is very interesting to create learning media
4)	I like to using Hawgent dynamic to help student in the learning process;
5)	The exploration function of Hawgent dynamic software can help me make guesses and verify the image and properties of trigonometric functions;
6)	Hawgent dynamic software can help me understand the concept of trigonometric functions;
7)	I want to try this mathematics software in other mathematics sub-topics

After using the "trigonometric function" integrable ware designed by Hawgent dynamic mathematics software to help teachers' teaching and students' learning, both teachers and students said that Hawgent software can help them understand the concept of trigonometric functions, and its exploration function can help me guess and verify triangles. The image and nature of the function. They also said that because the Hawgent dynamic software is easy to operate and understand, it is very interesting to create learning media. At the same time, they also like to use this software for learning and hope to try other mathematical knowledge. It can be seen that students and teachers have given high evaluations to Hawgent dynamic software-assisted teaching.

### 4. Conclusion

Combining the information technology with mathematics teaching is one of the ways to improve teaching efficiency. In this paper, the dynamic, easy-to-understand and easy-to-learn integrable ware of "trigonometric function concept" and "trigonometric function image and properties" are designed through Hawgent dynamic software, and integrated into the concept teaching of trigonometric function. Through the



practice of teaching experiments, it can be found that Hawgent dynamic software can integrate the software with the following two advantages:

In short, at the right time, choose the right dynamic mathematics technology, use the correct teaching method to teach, restore the dynamic process of the generation of the trigonometric function image, and highlight the essential concept of the trigonometric function. Appropriate use of this integrable ware can improve the weaknesses of drawing errors, unity, and static display of graphics movements in traditional teaching, help students understand the essence of the concept of trigonometric functions, enhance students' interest in learning, and cultivate students to develop intuitive perception and mathematical models And other core literacy.

In practice, this research explores the application of Hawgent dynamic integrable ware to assist the teaching of trigonometric functions, and provides practical reference for changing students' learning process, learning style, and improving learning effects. Theoretically, according to the core functions and basic principles of Hawgent dynamic software, the principles and create a prototype process of the "trigonometric function" integrable ware design of Hawgent dynamic software are given, so as to provide a theoretical reference for teachers to use Hawgent dynamic software to assist middle school mathematics teaching and research.

In this research, only one class used Hawgent dynamic mathematics software to design the "trigonometric function" integrable ware design for information technology-assisted teaching experiments. There was no comparative experiment between the experimental class and the control class. For example, information technology may have different effects on different types of students, that is, irrelevant variables are not well controlled in the teaching experiment. Future research can be conducted not only from the aspects of the teaching influence of information technology on other types of mathematical knowledge, such as mathematical formulas, mathematical theorems, mathematical problem solving, etc., but also to explore the SEM model of the factors that affect the efficiency of information technology-assisted teaching.

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