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INVENTORY MODEL (M,R,T) CONTINUOUS LEAD TIMES, QUADRATIC BACK ORDER COSTS AND RANDOM SUPPLY (Series 2)

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ABSTRACT

This paper considers the (M,R,T) inventory model in which the backorder costs is a quadratic function of the time of a backorder, lead time is continuous and supply random Results of series 1, which the same model was considered for constant lead times is the basis for deriving this paper's model. The inventory costs when lead time is constant is averaged over the states of lead time in which the distribution of lead time is assumed to be a gamma distribution.

In averaging over the states of lead time extensive use is made of the Bessel function of imaginary argument.

INVENTORY MODEL (M,R,T) CONTINUOUS LEAD TIMES, AND RANDOM SUPPLY

INTRODUCTION

This paper is a continuation of series 1 paper in which the (M,R,T) model considered constant lead times, quadratic backorder costs and random supply.

The lead time is gamma distributed. The results for constant lead times is averaged over the states of the lead times. The demand during the lead time remains a normal distribution.

At review time for the (M, R, T) inventory model, when the quantity or hand is less than or equal to R a quantity is ordered which is sufficient to bring the inventory position or the quantity on hand plus or order up to R.

In series 1, (Omorodion (2013)) we obtained the inventory cost for the (M, R, T) model for the constant lead time, from which we proceeded to obtain the inventory costs for the continuous lead time, quadratic backorder costs and random supply.

LITERATURE REVIEW

Zipkin (2006) treats both fixed and random lead times and examines both stationary and limiting distributions under different assumptions.

Bertismas (1999) in his paper 'Probabilistic service level guarantee in make to-stock', considered both linear and quadratic inventory costs and backorder costs.

Pritibhushan (2008) since in his paper 'A note on Bernoulli Demand inventory model presents a single – item, continuous monitoring inventory model with probabilistic demand for the item and probabilistic lead time of order replacement.

Hadley and Whitin (197 2) extensively developed the inventory model for constant lead time and linear backorder costs.

Equation (21) series 1 from which we have G_{14} (R+Y,T) gives the expected cost of carrying inventory and backorders including the cost of a stock out dependent on the number of stockouts only, for fixed lead times, L

We have

$$G_{14}(R+Y,T) = hcT\left(R+Y-DL-\frac{DT}{2}\right) + b_1(G_5(R+Y,T+L) - G_5(R+Y,L_1))$$

$$+(b_2+hc)(G_2(R+Y,L) - G_2(R+Y,T+1,L)) + b_3(G_{12}(R+Y,T+L) - G_{12}(R+Y,L)) + sG_9(R+Y,T) \quad (1)$$

Where the following is stated

$$G_2(R,T) = \left(\frac{\sigma^4+2D^4T^2}{4D^3} + R\frac{(\sigma^2+2D^2T)}{2D^2} + \frac{R^2}{2D}\right)F\left(\frac{R+DT}{\sqrt{\sigma^2T}}\right)$$

$$+\frac{1}{2}\left(\sigma T^{3/2} - \frac{\sigma^3T^{1/2}}{D^2} - \frac{T^{1/2}R}{D}\right)g\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) - \frac{\sigma^4}{4D^3}esp\left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R+DT}{\sqrt{\sigma^2T}}\right) \quad (2)$$

$$G_5(R,T) = \sqrt{\sigma^2T}\,g\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) - (R-DT)F\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) \quad (3)$$

$$G_{12}(R,T) = D\left(\frac{R^3}{3D^3} + \frac{\sigma^2R^2}{2D^4} + \frac{\sigma^4R}{2D^5} + \frac{\sigma^6}{2D^6} - \frac{\sigma^2T^2}{2D^2} - \frac{T^2R}{D} + \frac{T^3}{3} - \frac{R^2T}{D^2}\right)$$

$$F\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) + \frac{D}{\sqrt{\sigma^2T}}g\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right)\left(\frac{2}{3}\frac{\sigma^2RT^2}{D^2} + \frac{\sigma^2T^3}{3D} + \frac{\sigma^2R^2T}{3D^3} - \frac{\sigma^4RT}{2D^4}\right.$$

$$+\frac{\sigma^4T^2}{6D^3}+\frac{8\sigma^6T}{D^5}\Big)+esp\left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R+DT}{\sqrt{\sigma^2T}}\right)\frac{\sigma^6}{4D^2} \quad (4)$$

We shall exclude the cost dependent on the number of stockouts in determining the inventory costs for (M,R,T).

$$substituting\ for\ G_{14}(R+Y,T) \quad (5)$$

$$G_{29}(R+Y,T)=\int_0^a H(L)\left(hcT\left(R+Y-DL-\frac{DT}{2}\right)\right.$$

$$+b_1(G_5(R+Y,T+L)-G_5(R+Y,L))+b_2(G_2(R+Y,T+L)$$

$$-G_2(R+Y,L))+b_3(G_{12}(R+Y.T+L)-G_{12}(R+Y,L))\,dl \quad (6)$$

Let

$$G_{23}(R)=\int_0^a H\,(L)\;G_5(R,L)\;dL \quad (7)$$

$$G_{24}(R)=\int_0^a H\,(L)\;G_2(R,L)\;dL \quad (8)$$

$$G_{25}(R)=\int_0^a H\,(L)\;G_{12}(R,L)\;dL \quad (9)$$

$$G_{26}(R)=\int_0^a H\,(L)\;G_5(R,T+L)\;dL \quad (10)$$

$$G_{27}(R)=\int_0^a H\,(L)\;G_2(R,T+L)\;dL \quad (11)$$

$$G_{28}(R)=\int_0^a H\,(L)\;G_{12}(R,T+L)\;dL \quad (12)$$

Stated – equation (3)

$$G_5(R,L)=\sqrt{\sigma^2L}\;g\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)-(R-DL)F\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)$$

$$Multiplying\ by\ H(L) where\ H(L) = \frac{\propto^k\ esp\ (-\propto L)L^{k-1}}{\Gamma(k)}$$

Hence

$$H(L)G_5(R,L) = \frac{\propto^k\ esp\ \propto L}{\Gamma(k)}\left[\sigma L^{k-1/2} g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right.$$

$$\left.-(RL^{k-1} - DL^k)\ F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right) \qquad (13)$$

Noting that

$$\int_0^\infty H(L)\frac{1}{\sqrt{\sigma^2 L}}\ g\left(\frac{x-DL}{\sqrt{\sigma^2 L}}\right)dL = \int_0^\infty exp\ (-\propto L)\frac{L^{k-1}\ \propto^k}{\sqrt{\sigma^2 L}\ \ \Gamma(k)} g\left(\frac{x-DL}{\sqrt{\sigma^2 L}}\right)dL$$

$$= \frac{\propto^k}{\sigma\sqrt{2\pi}\ \ \Gamma(k)}\int_0^\infty L^{k-3/2}\ exp\left(\frac{Dx}{\sigma^2}\right) exp\ \left(\frac{-x^2}{2\sigma^2 L} - L\left(\frac{2\propto\sigma^2 + D^2}{2\sigma^2}\right)\right)dL$$

$$= \frac{\propto^k}{\sigma\sqrt{2\pi}\ \ \Gamma(k)} exp\left(\frac{Dx}{\sigma^2}\right)\left[2\left(\frac{x^2}{2\propto\sigma^2 + D^2}\right)^{\frac{1}{2}\left(k-\frac{1}{2}\right)} K_{k-\frac{1}{2}}^{\left(\frac{x}{\sigma^2}(2\propto\sigma^2+D^2)^{\frac{1}{2}}\right)}\right]$$

If k is an integer then

$$K_{k-\frac{1}{2}}(z) = K_{\frac{1}{2}}(z)\sum_{j=0}^{k-1}\frac{(k+j-1)!}{j!\,(k-j-i)!}(2z)^{-j}$$

Where

$$K_{\frac{1}{2}}\ (z) = \frac{\sqrt{\pi}}{\sqrt{2}}\ Z^{-\frac{1}{2}}\exp(-\mathrm{Z})$$

Hence

$$K_{k-\frac{1}{2}}(z) = \sqrt{\pi}\ \sum_{j=0}^{k-1}\frac{(k+j-1)!}{j!(k-j-1)!}(2z)^{-j-\frac{1}{2}}\exp(-z)$$

$$Hence \int_{0}^{a} H(L)G_5(R,L)dL \ applying\ equation\ 14$$

We have

$$G_{23}(R) = \propto^k \ esp\left(\frac{DR}{\sigma^2}\right)\left[2\left(\frac{R}{\theta}\right)^{k+1/2} K_{k+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right] - \frac{\propto^k \ esp\left(\frac{DR}{\sigma^2}\right)}{2\sigma\sqrt{2\pi}\ \Gamma(k)}\left[R\sum_{z=1}^{k}\frac{(k-1)!}{\propto^2\ (k-z)!}\right.$$

$$\left.\left(2D\left(\frac{R}{\theta}\right)^{k-z+1/2} K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-z-1/2} K_{k-z-1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)\right] + D\sum_{z=1}^{k+1}\frac{k!}{\propto^z\ (k+1-z)!}$$

$$\left(2D\left(\frac{R}{\theta}\right)^{k-z+3/2} K_{k-z+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-z+1/2} K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)$$

From equation (2)

$$G_2(R,L) = \left(\frac{\sigma^4}{4D^3} + \frac{DL^2}{2} + \frac{R\sigma^2}{2D^2} - RL + \frac{R^2}{2D}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) + \frac{1}{2}$$

$$\left(\sigma L^{3/2} - \frac{\sigma^3 L^{1/2}}{D^2} - \frac{\sigma L^{1/2} R}{D}\right)g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) - \frac{\sigma^4 esp\left(\frac{2DR}{\sigma^2}\right)}{4D^3}F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

Simplifying

$$G_2(R,L) = \left[\left(\frac{\sigma^4}{4D^3} + \frac{\sigma^2}{2D^2} + \frac{R^2}{2D}\right) - RL + \frac{DL^2}{2}\right]F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) + \frac{1}{2}$$

$$\left(-L^{1/2}\left(\frac{\sigma^3}{D^2} + \frac{\sigma^R}{D}\right) + \sigma L^{3/2}\right)g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) - \frac{\sigma^4}{4D^2}esp\left(\frac{2DR}{\sigma^2 L}\right)F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right)$$

$$Multiplying\ by\ H(L) = \frac{\propto^k\ esp(-\propto L)}{\Gamma(k)}$$

$$H(L)G_2(R,L) = \frac{esp(-\propto L)\ \propto^k}{\Gamma(k)}\left[\left(\frac{\sigma^4}{4D^3} + \frac{\sigma^2 R}{2D^2} + \frac{R^2}{2D}\right)L^{k-1} - RL^k + \frac{DL^{k+1}}{2}\right]$$

$$F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right)-\frac{1}{2}\frac{\propto^k\ esp(-\propto L)}{\Gamma(k)}\left[\left(\frac{\sigma^3}{D^2}+\frac{\sigma^R}{D}\right)L^{k-1/2}-\sigma L^{k+3/2}\right]g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

$$-\frac{\sigma^4}{4D^2}esp\ \left(\frac{DR}{4D^2}\right)F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right)\frac{esp(-\propto L)L^k\ \propto^k}{\Gamma(k)} \qquad (16)$$

Hence

$$\int_0^a H\ (L)G_2(R,L)dl\ applying\ equation\ 14$$

We have

$$G_{24}(R)=\frac{\propto^k\ esp\left(\frac{DR}{\sigma^2}\right)\propto L}{2\sigma\sqrt{2\pi}\ \Gamma(k)}\left[\left(\frac{\sigma^4}{4D^3}+\frac{\sigma^2 R}{2D^2}+\frac{R^2}{2D}\right)\sum_{z=1}^{k}\frac{(k-1)!}{\propto^z\ (k-z)!}\right.$$

$$\left(2D\left(\frac{R}{\theta}\right)^{k-z+1/2}K_{k-z+1/2}^{\left(\frac{R9}{\sigma^2}\right)}+2R\left(\frac{R}{\theta}\right)^{k-z-1/2}K_{k-z-1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}-R\sum_{z=1}^{k+1}\frac{k!}{\propto^z\ (k+1-z)!}\right.$$

$$\left(2D\left(\frac{R}{\theta}\right)^{k-z+3/2}K_{k-z+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)}+2R\left(\frac{R}{\theta}\right)^{k-z+1/2}K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)+\frac{D}{2}$$

$$\sum_{z=1}^{k+2}\frac{(k+1)!}{\propto^z\ (k-2-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-z+5/2}K_{k-z+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)}+2R\left(\frac{R}{\theta}\right)^{k-z+3/2}K_{k-z+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)$$

$$-\frac{\propto^k\ esp\left(\frac{DR}{\sigma^2}\right)}{2\sqrt{2\pi}\ \Gamma(k)}\left[2D\left(\frac{\sigma^3}{D^2}+\frac{\sigma R}{D}\right)\left(\frac{R}{\theta}\right)^{k+1/2}K_{k+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}-\sigma\left(\frac{R}{\theta}\right)^{k+3/2}K_{k+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right]$$

$$-\frac{\sigma^4\ \propto^k\ esp\left(\frac{DR}{\sigma^2}\right)}{\sqrt{2\pi}4D^2 2\sigma}\left[\sum_{z=1}^{k}\frac{(k-1)!}{\propto^z\ (k-z)!}\left(2R\left(\frac{R}{\theta}\right)^{k-z+1/2}K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right.\right.$$

$$\left.\left.-2D\left(\frac{R}{\theta}\right)^{k-z-1/2}K_{k-z-1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)\right. \qquad (17)$$

Simplifying G_{12} (R,L) in equation (14) we have

$$G_{12}(R,L) = \left[\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{2D^4} + \frac{\sigma^2 R}{2D^5} + \frac{\sigma^6}{4D^6}\right) - \frac{R^2 L}{D} + L^2\left(\frac{R}{D} - \frac{\sigma^2}{2D^2}\right) - \frac{L^3}{3}\right] D.\, F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

$$+\frac{D}{\sqrt{\sigma^2}}\, g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{\sigma^2 R^2}{3D^3} + \frac{\sigma^4 R}{2D^4}\right) L^{1/2} + \frac{8\sigma^6}{D^5} L^{1/2} + L^{3/2}$$

$$\left[\left(-2\frac{\sigma^2 R}{3D^2} + \frac{\sigma^2}{6D^3}\right) + \frac{\sigma^2 L^{5/2}}{3D}\right] + \frac{\sigma^6}{4D^6} esp\ \left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) \qquad (18)$$

Hence H(L) G_{12} (R,L)

$$\frac{\propto^k\ esp(-\propto L)}{\Gamma(k)}\left[\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{2D^4} + \frac{\sigma^4 R}{2D^5} + \frac{\sigma^6}{4D^6}\right) L^{k-1} - \frac{R^2 L^k}{D^2} + L^{k+1}\left(\frac{R}{D} - \frac{\sigma^2}{2D^2}\right)\right.$$

$$\left.-\frac{L^{k+2}}{3}\right] F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) + \frac{1}{\sigma} g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left[\left(\frac{\sigma^2 R^2}{3D^3} + \frac{\sigma^4 R}{2D^4}\right) L^{k-1/2} + \frac{8\sigma^6}{D5} L^{k+1/2}. + L^{k+\frac{1}{2}} + \frac{8\sigma^6}{D^5}\left(\frac{\sigma^4}{6D^3} - \frac{3}{2}\frac{2\sigma^2 R}{D^2}\right) +\right.$$

$$\left.\frac{\sigma^2}{3D} L^{k+3/2}\right] \frac{esp(-\propto L)\propto^k}{\Gamma(k)} + \frac{\sigma^6}{4D^6} esp\left(\frac{2DR}{\sigma^2}\right)$$

$$\frac{\propto^k\ esp(-\propto L) L^{k-1}}{\Gamma(k)} F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) \qquad (19)$$

Hence G_{25} (R, L) $= \int_0^\infty H$ (L)G_{12} $(R, L) dL$ applying equation 14

$$G_{25}\ (R) = \frac{a^k\ esp\ \left(\frac{DR}{\sigma^2}\right)}{2\sigma\ \Gamma(k)\sqrt{2\pi}}\left[\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{3D^3} + \frac{\sigma^4 R}{4D} + \frac{\sigma^6}{4D^6}\right)\sum_{z=1}^{k}\frac{(k-1)!}{\propto^z\ (k-z)!}\right.$$

$$\left(2D\left(\frac{R}{\theta}\right)^{k-z+1/2}\ K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-z-1/2}\ K_{k-z-1/2}^{\left(\frac{R\theta}{\sigma^2}\right)-} - \frac{R^2}{D^2}\right.$$

$$\sum_{z=1}^{k+1}\frac{k!}{\propto^z\ (k+1-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-z+3/2}\ K_{k-z+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-z+1/2}\ K_{k-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right.$$

$$+\left(\frac{M}{D} - \frac{\sigma^2}{2D^2}\right)\sum_{z=1}^{k+2}\frac{(k-1)!}{\propto^z\ (k+2-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-z+5/2}\ K_{k-z+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\right.$$

$$\left.\left(\frac{R}{\theta}\right)^{k-z-3/2}\ K_{k-z-3/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right) - \frac{1}{3}\sum_{z=1}^{k+3}\frac{(k+1)!}{\propto^z\ (k+3-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-z+7/2}\ K_{k-z+7/2}^{\left(\frac{R\theta}{\sigma^2}\right)} +\right.$$

$$\left.\left.\left(\frac{R}{\theta}\right)^{k-z+5/2}\ K_{k-z+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)\right] + \frac{esp\left(\frac{DR}{\sigma^2}\right)\propto^k}{\sqrt{2\pi\sigma^2}\ \Gamma(k)}\left[2\left(\frac{\sigma^2 R^2}{3D^3} + \frac{\sigma^4 R}{4D^4} + \frac{8\sigma^6}{D^5}\right)\right.$$

$$\left(\frac{R}{\theta}\right)^{k+1/2} K_{k+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2\left(\frac{\sigma^4}{6D^3} - \frac{2\sigma^2 R}{3D^2}\right)\left(\frac{R}{\theta}\right)^{k+3/2} K_{k+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + \frac{2\sigma^2}{3D}$$

$$\left(\frac{R}{\theta}\right)^{k+5/2} K_{k+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\Bigg] + \frac{\sigma^6}{2\sigma^4 D^6}\frac{esp\left(\frac{DR}{\sigma^2}\right)}{\Gamma(k)}\frac{\propto^k}{\sqrt{2\pi}}\left[\sum_{z=1}^{k}\frac{(k+1)!}{\propto^z\,(k-z)!}\right.$$

$$\left(-2D\left(\frac{R}{\theta}\right)^{k-z+1/2} + 2R\left(\frac{R}{\theta}\right)^{k-z-1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)}\right] \qquad (20)$$

From equation 3substitution L+T for L

$$G_5(R, L+T) = \sigma(L+T)^{1/2} g\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - (R - D(L+T))$$

$$F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \qquad (21)$$

$Simplifying$

$$G_5(R, L+T) = \sigma(L+T)^{1/2} g\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - [(R - DT) - DL]$$

$$F\left(\frac{M - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \qquad (22)$$

Multiplying by H(L)

$$H(L)G_5(R, L+T) = \frac{\sigma\propto^k esp(-\propto L)}{2\pi\,\Gamma(k)} L^{k-1}(L+T)^{1/2} g\left(\frac{R-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)\frac{\propto^k esp-\propto L}{\Gamma(k)}$$

$$[(R - DT)L^{k-1} - DL^k]\, F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \qquad (23)$$

$Hence\ G_{26}(R,T) = \int_0^\infty H(L)G_5(R, L+T)\, dL\ applying\ equation\ 14$

$$G_{26}(R,T) = \frac{\sigma}{\sqrt{2\pi}}\frac{esp\left(\propto T + \frac{DR}{\sigma^2}\right)}{\Gamma(k)}\propto^k \sum_{j=0}^{k-1}(-T)^j\binom{k-1}{j}\left(2\left(\frac{R}{\theta}\right)^{k-j+1/2} K_{k-j+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right.$$

$$+\frac{1}{2}\frac{\propto^{k}}{\sigma\sqrt{2\pi}}esp\left(\propto T+\frac{DR}{\sigma^{2}}\right)\left[\sum_{j=0}^{k-1}(-T)^{j}\binom{k-1}{j}\frac{(k-1-j)!}{\propto^{z}\ (k-j-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j+1/2}\right.\right.$$

$$K_{k-j-z+1/2}^{\left(\frac{R\theta}{\sigma^{2}}\right)}-2R\left(\frac{R}{\theta}\right)^{k-j-z-1/2}\ K_{k-j-z-1/2}^{\left(\frac{R\theta}{\sigma^{2}}\right)}\Bigg)(R-DT)-D\sum_{j=0}^{k}(-T)^{j}\binom{k}{j}$$

$$\sum_{z=1}^{k+1-j}\frac{(k-j)!}{\propto^{z}\ (k+1-j-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j-z+3/2}\ K_{k-j-z+3/2}^{\left(\frac{R\theta}{\sigma^{2}}\right)}+2R\left(\frac{R}{\theta}\right)^{k-j-z+1/2}\right.$$

$$K_{k-j-z+1/2}^{\left(\frac{R\theta}{\sigma^{2}}\right)}\Bigg) \qquad (24)$$

From equation (4) substituting L+T for and simplifying

$$G_{12}(R,L+T)=-\left[\left(\frac{R^{3}}{3D^{3}}+\frac{\sigma^{2}R^{2}}{2D^{4}}+\frac{\sigma^{4}R}{4D^{5}}+\frac{\sigma^{6}}{4D^{6}}-\frac{R^{2}T}{D^{2}}\right)-\frac{R^{2}L}{D^{2}}+\sum_{j=0}^{2}\binom{2}{i}T^{i}L^{2-i}\right.$$

$$\left(\frac{R}{D}-\frac{\sigma^{2}}{D^{2}}\right)-\frac{1}{3}\sum_{i=0}^{3}\binom{3}{i}T^{i}L^{3-i}\Bigg]\ F\left(\frac{R-DL}{\sqrt{\sigma^{2}L}}\right)+\frac{1}{\sigma}\ g\left(\frac{R-D(L+T)}{\sqrt{\sigma^{2}(L+T)}}\right)$$

$$\left[\left(\frac{\sigma^{2}M^{2}}{3D^{3}}+\frac{\sigma^{2}R}{2D^{4}}+\frac{8\sigma^{6}}{4D^{5}}\right)\ (L+T)^{1/2}+\left(\frac{\sigma^{4}}{6D^{3}}-\frac{2\sigma^{2}R}{3D^{2}}\right)(L+T)^{3/2}+\sigma^{2}(L+T)^{1/2}\right.$$

$$\sum_{i=0}^{2}\binom{2}{i}T^{i}L^{2-i}\Bigg]+\frac{\sigma^{6}}{4D^{6}}\ esp\ \left(\frac{2R}{\sigma^{2}}\right)F\left(\frac{R+DL}{\sqrt{\sigma^{2}L}}\right) \qquad (25)$$

Multiplying by H(L) we have

$$H(L)G_{12}(R,L+T)=\frac{\propto^{k}esp(-\propto L)}{\Gamma(k)}\left[\left(\frac{R^{3}}{3D^{3}}+\frac{\sigma^{2}R^{2}}{2D^{4}}+\frac{\sigma^{4}R}{4D^{5}}+\frac{\sigma^{6}}{4D^{6}}-\frac{R^{2}T}{D^{2}}\right)L^{k-1}+\frac{R^{2}L^{k}}{D^{2}}\right.$$

$$+\sum_{i=0}^{2}\binom{2}{i}T^{i}L^{k+i-j}\left(\frac{R}{D}-\frac{\sigma^{2}}{2D^{2}}\right)-\frac{1}{3}\ \sum_{i=0}^{3}\binom{3}{i}T^{i}L^{k-i+1/2}\Bigg]F\left(\frac{R-DL}{\sqrt{\sigma^{2}L}}\right)+\frac{\propto^{k}}{\sigma}$$

$$\frac{\propto^{k}\ esp(-\propto L)}{\sigma\ \Gamma(k)}(L+T)^{1/2}\ g\left(\frac{R-D(L+T)}{\sqrt{\sigma^{2}(L+T)}}\right)\left[\left(\frac{\sigma^{2}R^{2}}{3D^{3}}+\frac{\sigma^{2}R}{2D^{4}}+\frac{8\sigma^{6}}{D^{5}}\right)+\frac{\sigma^{4}T}{6D^{3}}-2\right.$$

$$\frac{\sigma^2 RT}{3D^2}\Bigg] L^{k-1} + \left(\frac{\sigma^4}{6D^3} - \frac{2\sigma^2 R}{3D^2}\right) L^k + \sigma^2 \sum_{i=0}^{2} \binom{2}{i} T^i L^{k-i+1}\Bigg] + \frac{\sigma^6}{4D^6}\ esp\left(\frac{2DR}{\sigma^2}\right)$$

$$\frac{\propto^k\ esp(-\propto L) L^{k-1}}{\Gamma(k)} F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) \qquad (26)$$

$\int_0^\infty H\,(L)\ G_{12}(R, L+T) dl$ *applying equation* 14

$$G_{17}(R,T) = \frac{\propto^k\ esp\ \left(\propto T + \frac{DR}{\sigma^2}\right)}{\sqrt{2\pi}\quad \Gamma(k)\ 2\sigma}\left[\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{2D^4} + \frac{\sigma^2 R}{2D^5} + \frac{\sigma^6}{4D^6} - \frac{R^2 T}{D^2}\right)\right.$$

$$\sum_{j=0}^{k-1} \binom{k-1}{j} (-T)^i \sum_{z=1}^{k-j} \frac{(k-1-z)!}{\propto^z\ (k-j-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j-z+1/2}\ \left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\theta}\right)^{k-j-z-1/2}\right.$$

$$\left. K_{k-j-z-1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right) + \frac{R^2}{D^2}\sum_{j=0}^{k-1}\binom{k-1}{j}(-T)^i \sum_{z=1}^{k+1-j} \frac{(k-j)!}{\propto^z\ (k+1-j)!}\left(2R\left(\frac{R}{\theta}\right)^{k-j-z+3/2}\right.$$

$$\left. K_{k-j-z+3/2} + 2R\left(\frac{R}{\theta}\right)^{k-j-z+1/2} K_{k-j-z+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right) + \left(\frac{R}{D} - \frac{\sigma^2}{2D^2}\right)\sum_{i=0}^{2}\binom{2}{i} T^i$$

$$\sum_{j=0}^{k+1-i} (-T)^i \binom{k+1-j}{j} \sum_{z=1}^{k+2-i-j} \frac{(k-1-i-j)!}{\propto^z\ (k+2-i-z)!}\left(2R\left(\frac{R}{\theta}\right)^{k-j-z+5/2} K_{k-j-z+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right.$$

$$\left. +2R\left(\frac{R}{\theta}\right)^{k-j-z+3/2} K_{k-j-z+3/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right) - \frac{M}{3}\sum_{i=0}^{3}\binom{3}{i} T^i \sum_{j=0}^{k+2-i} (-T)^i \binom{k+i-2}{j}$$

$$\sum_{z=1}^{k+3-i-j} \frac{(k+2-i-j)!}{\propto^z\ (k+3-i-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j-z+7/2} K_{k-j-z+7/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-j-z+5/2}\right.$$

$$\left.\left. K_{k-j-z+5/2}^{\left(\frac{R\theta}{\sigma^2}\right)}\right)\right] + \frac{esp\ \left(\propto T + \frac{DR}{\sigma^2}\right)}{\sqrt{2\pi\sigma^2}} \frac{\propto^k}{\Gamma(k)}\left[\sum_{j=0}^{k-1}(-T)^i \binom{k-1}{j}\right.$$

$$\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{2D^4} + \frac{\sigma^4 R}{2D^5} + \frac{\sigma^6}{4D^6} - \frac{R^2 T}{D^2}\right) 2\left(\frac{R}{\theta}\right)^{k-j+1/2} K_{k-j+1/2}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2\left(\frac{\sigma^4}{6D^3} - \frac{2\sigma^2 R}{3D^2}\right)$$

$$\sum_{j=0}^{k}(-T)^i\binom{k}{j}\left(\frac{R}{\theta}\right)^{k-j+3/2}\ K_{k-j+3/2}$$

$$+2\sigma^2\sum_{i=0}^{2}\binom{2}{i}T^i\ \sum_{i=0}^{k-i+1}(-T)^j\ \binom{k-i+1}{j}\left(\frac{R}{\sigma}\right)^{k-i+5/2-j}\ K_{k-i+5/2-j}{}^{\left(\frac{R\theta}{\sigma^2}\right)}\Bigg]$$

$$+\frac{\sigma^6}{4D^6}\frac{\propto^{\mathrm{k}}\ \mathrm{esp}\left(\frac{\mathrm{DR}}{\sigma^2}+\propto\mathrm{T}\right)}{\Gamma(k)2\sigma\sqrt{2\pi}}\sum_{j=0}^{k-1}\binom{k-1}{j}(-T)^i\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\propto^2\ (k-j-z)!}\Bigg(-2\mathrm{D}\left(\frac{\mathrm{R}}{\theta}\right)^{\mathrm{k-j-z+1/2}}$$

$$K_{k-j-z+1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)}+2R\quad K_{k-j-z+1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)}\Bigg] \tag{27}$$

From equation (2), substituting L+T for L

$$G_2(R,L+T)=\left[\left(\frac{\sigma^4}{4D^3}+\frac{\sigma^2R}{2D^2}+\frac{R^2}{2D}-TR\right)-LR+\frac{D}{2}\sum_{i=0}^{2}\binom{2}{i}T^iL^{2-i}\right]$$

$$F\left(\frac{R-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)+\frac{\sqrt{(L+T)}}{2}\left[+\left(\frac{\sigma^3}{D^2}+\frac{\sigma R}{D}-\sigma T\right)+\sigma L\right]g\left(\frac{R-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

$$-\frac{\sigma^4}{4D^3}\ esp\ \left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R+D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \tag{28}$$

Multiplying by H(L) we have

$$H(L)G_2(R,L+T)=\left[\left(\frac{\sigma^4}{4D^3}+\frac{\sigma^2R}{2D^2}+\frac{R^2}{2D}-RT\right)L^{k-1}-L^k-\frac{D}{2}\sum_{i=0}^{2}\binom{2}{i}T^iL^{k+1-i}\right]$$

$$F\left(\frac{R-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)+\frac{1}{2}(L+T)^{1/2}\left(-\frac{\sigma^3}{D^2}+\frac{\sigma R}{D}-\sigma T\right)L^{k-1}+\sigma L^kg\left(\frac{R-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

$$-\frac{\sigma^4}{4D^3}\ \frac{\propto^k\ esp(-\propto L)}{\Gamma(k)}esp\left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R+DL}{\sqrt{\sigma^2L}}\right)$$

$$Hence\ G_{17}(R,T)=\int_0^\infty H(L)G_2(R,L+T)dl\ applying\ equation\ 14$$

$$G_{17}(R,T) = \frac{\propto^k\ esp\left(\propto T + \frac{DR}{\sigma^2}\right)}{2\sigma\ \Gamma(k)\sqrt{2\pi}}\left[\left(\frac{\sigma^4}{4D^3} + \frac{\sigma^2 R}{2D^2} + \frac{R^2}{2D} - TR\right)\sum_{j=0}^{k-1}(-T)^i\binom{k-1}{j}\right.$$

$$\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\propto^z\ (k-j-z)!}\left(2D\left(\frac{M}{\theta}\right)^{k-j-z+1/2} K_{k-j-z+1/2}\left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\theta}\right)^{k-j-z-1/2}\right.$$

$$\left.K_{k-j-z-1/2}\left(\frac{R\theta}{\sigma^2}\right)\right) - R\sum_{j=0}^{k-1}(-T)^i\binom{k}{j}\sum_{z=1}^{k-1-j}\frac{(k-j)!}{\propto^2\ (k+1-j-z)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j-z+3/2}\right.$$

$$\left.K_{k-j-z+3/2}\left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\theta}\right)^{k-j-z-1/2} K_{k-j-z-1/2}\left(\frac{R\theta}{\sigma^2}\right)\right) + \frac{D}{2}\sum_{i=0}^{2}\binom{2}{i}T^i\sum_{j=0}^{k+1-j}(-T)^i$$

$$\binom{k+1-j}{j}\sum_{z=1}^{k+2-i-j}\frac{(k-1-i-j)!}{\propto^2\ (k+2-i-j)!}\left(2D\left(\frac{R}{\theta}\right)^{k-j-z+5/2} K_{k-j-z+5/2}\left(\frac{R\theta}{\sigma^2}\right)\right.$$

$$\left.\left.+2R\left(\frac{R}{\theta}\right)^{k-j-z-i-3/2} K_{k-j-z-i-3/2}\left(\frac{R\theta}{\sigma^2}\right)\right)\right] + \frac{\propto^k\ esp\left(-\propto L + \frac{DR}{\sigma^2}\right)^{-2}}{2\sqrt{2\pi}\ \Gamma(k)}\left[\left(\frac{\sigma^3}{D^2} + \frac{\sigma R}{D} + -\sigma T\right)\right.$$

$$\left.\sum_{j=0}^{k-1}T^j\binom{k-1}{j}\left(\frac{R}{\theta}\right)^{k-j+1/2} K_{k-j+1/2}\left(\frac{R\theta}{\sigma^2}\right) + \sum_{j=0}^{k}T^i\binom{k}{j}\left(\left(\frac{R}{\theta}\right)^{k-j-z+3/2} K_{k-j-z-3/2}\left(\frac{R\theta}{\sigma^2}\right)\right]$$

$$-\frac{\sigma^4}{4D^3}\ \frac{\propto^k\ esp\left(\propto T + \frac{DR}{\sigma^2}\right)}{\Gamma(k)2\sigma\sqrt{2\pi}}\sum_{j=0}^{k-1}(-T)^j\binom{k-1}{i}\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\propto^2\ (k-j-z)!}$$

$$\left.\left(-2D\left(\frac{R}{\theta}\right)^{k-j-z+1/2} K_{k-j-z+1/2}\left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\theta}\right)^{k-j-z-1/2} K_{k-j-z-1/2}\left(\frac{R\theta}{\sigma^2}\right)\right)\right] \qquad (29)$$

Substituting into equation (5) equation (14), (17), (20), (24), (27) we obtain

$$G_{29}(R+Y,T) = hcT\left(R + Y - \frac{Dk}{\propto} - \frac{DT}{2}\right) + b_1(G_{16}(R+Y,T) - G_{13}(R+Y)) + b_2$$

$$(G_{17}(R+Y,T) - G_{17}(R+Y)) + b_3(G_{18}(R+Y,T) - G_{15}(R+Y))$$

The inventory cost for model (M,R,T is obtained by replacing G_{14} (R+Y,T) by G_{19}(R+Y,T) in equation (1)

Hence

$$R = \frac{\frac{Rc}{T} + \left(S + \sum_{n=1}^{\infty} \int_0^{n-R} G_{19}(R+Y,T)\, gn(M-R-Y,DT)dY + G_{29}(M,T)\right)}{T\left[F\left(\frac{M-R-DL}{\sqrt{\sigma^2 L}}\right) + \sum_{n=1}^{\infty} \int_0^{n-R} ng^{n-1}(M-R-Y,DT)F\left(\frac{Y-DT}{\sqrt{\sigma^2 L}}\right)dY\right]} \qquad (31)$$

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