

The correct derivation of the relativistic Doppler effect

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Abstract

In the present publication it shall be shown that the derivation of the relativistic Doppler effect is based on the fact that every object at high velocity experiences not only an increase of mass compared to its resting state, but also an increase of the emitted light. This topic, which has already been treated in source [01], is deepened in the present publication.

Keywords: Relativistic Doppler Effect, Relativistic Mass, Time Dilation

1. Introduction

It is known that a moving mass m_b compared to its rest position m_0 is relativistically increased:

$$m_b = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (01)$$

The frequencies of light emission on this moving mass must be increased to the same extent. This can be understood at the example of an emitting star as follows: The energy difference between two energy levels of electrons must be increased in the same degree as the relativistic energy of the mass. The light emission of the transition must now take place at a higher frequency. Because it is known $\Delta E = hf$:

$$f_b = \frac{f_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (02)$$

This approach is to be tested with time dialectics. As is known:

$$f = \frac{1}{T} \quad (03)$$

Each time span t can be calculated from a certain number of n of periods T composed of:

$$t = nT \quad (04)$$

Equations (02), (03) and (04) are easily followed by equations (05) and (06):

$$T_b = T_0 \sqrt{1 - \frac{v^2}{c^2}} \quad nT_b = nT_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (05)$$

$$t_b = t_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (06)$$

We see with equation (06) the known time dilation. However, this can be valid only if the velocity of the mass object is increased compared with its rest state or that of a comparison object. Then on this and only on this moved object the time passes more slowly. That means the time span t_b is reduced compared to the time span t_0 of the object at rest.

It is to be explained here again at the example of two identical atomic clocks under neglect of the gravitational effect, why a first clock on a moving object goes slower opposite to a second one on a resting object.

Both atomic clocks are compared with each other. One is a clock at rest with the time span t_0 as comparison and the other is a clock moving at high speed with the time span t_1 .

Each time span consists of n seconds with a period length of one second T_{1s} .

Consequently, for the clock at rest $n_0 T_{1s0} = t_0$ and for the moving clock $n_1 T_{1s1} = t_1$ must be valid.

Since t_0 is the standard of comparison, the seconds of both atomic clocks must be counted in this time span. Therefore $t_1 = t_0$ is valid.

The moving clock is identified as slower if it shows fewer seconds. The number of seconds of the moving clock must be smaller than that of the stationary one: $n_1 < n_0$.

Therefore, from $t_1 = t_0$ it follows $T_{1s1} > T_{1s0}$. On the moving clock, the second must be longer.

What makes the second longer? The number n_{Cs} of oscillations of the cesium transition frequency of the atomic clock determines the length of a second. The more oscillations occur, the longer is a second. Consequently, the number n_{Cs1} of oscillations of the Cs-transition frequency must be larger in the moving higher-energy state than the number n_{Cs0} in the resting state. Consequently, the resonance in the atomic clock between the microwave frequency f_p and the transition frequency f_{Cs} can only occur at a higher frequency. According to equation (02), the resonance frequency increases with the speed of the atomic clock.

A connection of observers to massless resting and fast moving inertial systems is unscientific. From this can result the already often criticized nonsense that of two observers on one inertial system each the clock of the other one follows.

Every moving object experiences an increase of mass and a dilation of time compared to its own resting state or to the resting state of a reference object.

2. The derivation of the relativistic Doppler effect

Sound and light waves propagate in the form of spherical waves. Depending on the movement of source and observer away from or towards each other, successive periods of sound or light oscillation are stretched or compressed. The relations found by Christian Doppler are shown by equations (07), (09), (12) and (14). Of course, these equations are applicable to light, provided the source and observer are not moving at high velocities. From the Doppler effect it is known that the sound frequency of the observer changes f_{obs} decreases at its distance from the source with f_0 decreases as follows [02]:

$$f_{obs} = f_0 \left(1 - \frac{v}{c}\right) = f_0 \frac{c-v}{c} \quad (07)$$

Thereby c is the speed of sound in the medium. Electromagnetic waves propagate with the speed of light c (same symbol). Also here a frequency increase occurs with approach and a reduction with distance of the observer. The observer moving away with velocity v at frequency f_{obs} is more energetic by a factor of $1/\sqrt{1 - v^2/c^2}$.

In equation (07) the speed of light takes the place of the speed of sound and the equation has to be extended in relativistic consideration. If the velocity of the observer is high, for his frequency f_{obs} according to equation (02) the relativistic frequency increase compared to the resting source with the frequency f_0 is taken into account:

$$f_{obs} = \frac{f_0}{\sqrt{1-\frac{v^2}{c^2}}} \frac{c-v}{c} = \frac{f_0}{\sqrt{\frac{c^2-v^2}{c^2}}} \sqrt{\frac{(c-v)^2}{c^2}} = \frac{f_0}{\sqrt{\frac{(c+v)(c-v)}{c^2}}} \sqrt{\frac{(c-v)^2}{c^2}} = f_0 \sqrt{\frac{c-v}{c+v}} \quad (08)$$

Equation (07) can therefore no longer be applied if the observer is moving at very high speed.

If the source moves away from the observer, the Doppler effect applies first:

$$f_{obs} = f_0 \frac{1}{1+\frac{v}{c}} = f_0 \frac{c}{c+v} \quad (09)$$

Here the source is more energetic because of its movement with velocity v . So, for relativistic observation with high speed of the source for its frequency must be f_0 according to equation (02) the relativistic frequency increase compared to the resting observer with the frequency f_{obs} must be taken into account:

$$\frac{f_{obs}}{\sqrt{1-\frac{v^2}{c^2}}} = f_0 \frac{c}{c+v} \quad (10)$$

$$f_{obs} = \sqrt{1-\frac{v^2}{c^2}} f_0 \frac{c}{c+v} = f_0 \sqrt{\frac{(c+v)(c-v)}{c^2}} \sqrt{\frac{c^2}{(c+v)^2}} = f_0 \sqrt{\frac{c-v}{c+v}} \quad (11)$$

Equation (08) and (11) agree, consequently it does not matter who moves away.

To be on the safe side, the approach shall be considered. If the observer approaches the source, his frequency increases f_{obs} . At first the known Doppler formula is valid:

$$f_{obs} = f_0 \left(1 + \frac{v}{c}\right) = f_0 \frac{c+v}{c} \quad (12)$$

In equation (12) the speed of light takes the place of the speed of sound and the equation must be extended again in relativistic consideration. If the velocity of the observer is high, for its frequency f_{obs} according to equation (02) the relativistic frequency increase compared to the resting source with the frequency f_0 is taken into account:

$$f_{obs} = \frac{f_0}{\sqrt{1-\frac{v^2}{c^2}}} \frac{c+v}{c} = \frac{f_0}{\sqrt{\frac{c^2-v^2}{c^2}}} \sqrt{\frac{(c+v)^2}{c^2}} = \frac{f_0}{\sqrt{\frac{(c+v)(c-v)}{c^2}}} \sqrt{\frac{(c+v)^2}{c^2}} = f_0 \sqrt{\frac{c+v}{c-v}} \quad (13)$$

Equation (12) can consequently no longer be applied if the observer is moving at very high speed.

If the source approaches the observer, on the other hand, the following applies for the frequency f_{obs} according to Doppler:

$$f_{obs} = f_0 \frac{1}{1-\frac{v}{c}} = f_0 \frac{c}{c-v} \quad (14)$$

Now, because of the motion of the source, its frequency must be f_0 must again become relativistically energy richer compared to the observer with frequency f_{obs} become:

$$\frac{f_{obs}}{\sqrt{1-\frac{v^2}{c^2}}} = f_0 \frac{c}{c-v} \quad (15)$$

$$f_{obs} = f_0 \sqrt{1-\frac{v^2}{c^2}} \frac{c}{c-v} = f_0 \sqrt{\frac{(c+v)(c-v)}{c^2}} \sqrt{\frac{c^2}{(c-v)^2}} = f_0 \sqrt{\frac{c+v}{c-v}} \quad (16)$$

The equations (13) and (16) are in agreement, consequently it does not matter here who moves towards the other. It all fits with the observations of nature. And it has nothing at all to do with the special relativity theory which explains the relativistic connections completely wrong.

For sound, the formulas (08) and (13) are of course no longer applicable, because the speed of sound cannot be united with the speed of light in the relativistic expressions. Otherwise, one would have to strictly distinguish both velocities.

3. Conclusions

The known relativistic formulas for the Doppler effect of light are correct. However, the usual derivation via inertial systems is wrong. The correct derivation is obtained, if one applies the known Doppler formulas and includes the relativistic frequency increase of moving objects.

It must be noted that the Doppler frequency change occurs as a measurement phenomenon between the source and the observer. Therefore also the expression $c + v$ is admissible, which pretends that there can be a superluminal velocity. Because for example also with sound the observer measures a higher frequency at approach of the sound source although the source has no higher frequency. The energy supply leading to a high object velocity, however, increases the energy content of the object and leads to the frequency increase of the radiation emitted by it.

4. Summary

The formulas of the relativistic Doppler effect at distance and approach of two objects with the relativistic frequency increase were derived and the previous derivation with the inertial frames used by Einstein was falsified.

5. References

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