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Inventory Model (M, R, T,) Continnuos Lead Times, Exponential Backorder Costs And Random Supply

Series 3

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ABSTRACT

We derived the inventory costs in series 1 (Omorodion 2014) costs for the inventory model (M.R.T) in which the model has constant lead times, quadratic and exponential back order costs and random supply. In series 2 we considered the continuous lead times, random supply and quadratic backorder costs. In this paper series 3, we derive the inventory costs when the backorder cost is exponential and random supply and continuous lead times. The results of series 1 is made use of. The model result for this paper is derived by averaging the results of exponential backorder costs of series 1 over the states of lead time, where lead time is a gamma variate.

INTRODUCTION

In this paper we derive the cost equation for continuous lead by averaging over the states of lead times, the corresponding costs for constant lead times, obtained in series 1, for exponential backorder costs.

The lead time is a gamma variate and demand in the lead time is normally distributed and the supply is a gamma variate.

At review time for the (M, R, T) the inventory model when the quantity on hand in less than or equal to R a quantity is ordered which is sufficient to bring the inventory position or the quantity on hand plus or order up to R.

In series (1) (Omorodion 2014) we obtained the inventory cost for the (M, R, T) model for the constant lead time from which we proceeded to obtain the inventory costs for the continuous lead times exponential backorder costs and random supply.

LITERATURE REVIEW

Emal Drikan (2005), in his thesis assumed three different supply processes. Under the all-or-nothing type supply process and partially available supply process, the structure of optional policy is proved to be a base stock policy. He shows that a simple base stock policy is not optional using binomially distributed supply process.

Mukhopadhyay (2007) indicated that when order fluctuation and backorder fluctuations are observed other costs are incurred. If it is decided to reduce the aggregate inventory, the average production batch size should be decreased in order to maintain a balanced inventory. Over a range, the cost curve may be approximated by quadratic derives, a model where backorder demand ratio is exponentially decreasing with the waiting time.

Hadley and Whitin (1972) discusses the model when lead time is constant extensively.

Equation (28) of series (1) Omorodion 2014) G_{21} (R, L+T) is the expected cost of carrying inventory and backorders including, the cost of a stockout dependent on the number of stock outs only.

$$G_{21}((R+Y,T) = hcT\left(R+Y-DL,\frac{DT}{2}\right) + hc(G_2(R+Y,T+L) - G_2(R+Y,L) + G_{19}(R+Y,T+L) -$$

$$G19R+Y,L+sG9(R+Y,T) \qquad (32)$$

We shall exclude the cost dependent on the number of stockouts in determining the inventory of cost of (M,R,T)

$$Let\ G_{30}(R+Y,T) = \int_0^\infty H\,(L)(G_{21}(R+Y,T) - sG_9(R+Y,T)dL$$

Substituting for G_2 (R+Y,T) we have

$$C_{30}(R+Y,T) = \int_0^\infty H\,(L)\ \left[hcT\left(R+Y-DL-\frac{DT}{2}\right) + hc\ (G_2(R+Y,L+T) - G_2(R+Y,T) \qquad (33)\right.$$

$$\left.+G_{19}(R+Y,T+L) - G_{19}(R+Y,T))\right]dy$$

In addition to equation (7)to(12) we require

$$G_{31}(R) = \int_0^\infty G_{19}\ \ (R,L)H(L)dL$$

and

$$G_{32}(R) = \int_0^\infty G_{19}\ \ (R,L+T)dL$$

From equation (20) of series (1)

$$G_{19}(R,L) = \frac{2Db_1}{b_2(\sigma^2 b_2^2 + 2D^2 b_2)} esp\left[L\left(\frac{\sigma^2 b^2 + 2D^2 b_2}{2D^2}\right) - \frac{b_2 R}{D}\right] F\left(\frac{R - L\left(D + \frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 L}}\right)$$

$$+\frac{b_1}{b_2}\left(\frac{R-DL}{D\sqrt{\sigma^2 L}}\right) g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) \qquad (34)$$

$$\frac{-b_1}{b_2 D}\ F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) - \frac{\sigma_2^2 b^2 b_1 esp\left(\frac{2DR}{\sigma^2}\right)}{Db_2(\sigma^2 b_2^2 + 2D^2 b_2)}\ F\ \left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right)$$

$$Multiplying\ by\ H(L) where\ H(L) = \frac{\alpha^k L^{k-1} esp - \alpha L}{\Gamma(k)}$$

$$H(L)\ G_{19}(R,L) = \frac{2Db_1\alpha^k}{b_2(\sigma^2 b^2 + 2D^2 b_2)\ \Gamma(k)}\ esp\left[L\frac{(\sigma^2 b_2^2 + 2D^2 b_2 - \alpha)}{2D^2} - \frac{b_2 R}{D}\right]$$

$$L^{k-1}F\left(\frac{R - L(D + \sigma^2 D b_2)}{\sqrt{\sigma^2 L}}\right) + \frac{b_1}{Db_2}\frac{esp(-\alpha L)}{\sqrt{\sigma^2 L}}\ (RL^{k-i} - DL^k)g\ \left(\frac{R - DL}{\sqrt{\sigma^2 L}}\right)$$

$$-\frac{b_1}{b_2 D}\ esp(-\alpha L)L^{k-1}F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) - \frac{\sigma^2 b^2 b_1}{Db_2(\sigma^2 b_2^2 + 2D^2 b_2)\ \Gamma(k)} esp\left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) \tag{35}$$

$$\int_0^\infty H\ (L)G_{19}(R,L)dL\ applying\ equation\ 14$$

$$Let\ \theta^2 = 2\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2 - 2D^2\alpha}{2D^2}\right)$$

Then

$$G_{31}(R) = \frac{2Db_1\alpha^k esp\left(R\left(D + \frac{\sigma^2 b_2}{D}\right) - \frac{b_2 R}{D}\right)}{2\sigma(b_2^2\sigma^2 + 2D^2 b_2)\ \Gamma(k)\sqrt{2\pi}} \sum_{z=1}^k \frac{(k-1)!}{\left(\frac{\sigma^2 b_2^2 + 2b_2 - 2D^2\alpha}{2D^2}\right)!}\ (k-z)!$$

$$\left(2\left(D + \frac{\sigma^2 b_2}{D}\right)\left(\frac{R}{\theta}\right)^{k-z+\frac{1}{2}} K_{k+1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right)^{k-z-\frac{1}{2}} K_{k+1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)}\right.$$

$$-\frac{b_1 esp\left(\frac{DR}{\sigma^2}\right)}{\sigma\sqrt{2\pi}b_2\ \Gamma(k),D}\left[2R\left(\frac{R}{\theta}\right)^{k+1/2} K_{k-1/2}\left(\frac{R\theta}{\sigma^2}\right) - 2D\left(\frac{R}{\theta}\right)^{k+1/2} K_{k+1/2}\left(\frac{R\theta}{\sigma^2}\right)\right] + \frac{1}{\sqrt{2\pi}}\frac{b_1 esp\left(\frac{DR}{\sigma^2}\right)}{b_2 D\sigma\ \Gamma(k)}$$

$$\sum_{z=1}^k \frac{(k-1)!}{\alpha^z(k-z)!}\left[2D\left(\frac{R}{\theta}\right)^{k-z+1/2} K_{k-z+1/2}\left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\sigma}\right) K_{k-z+1/2}\left(\frac{R\theta}{\sigma^2}\right)\right]$$

$$-\frac{\sigma^2 b_2 b_1 esp\left(\frac{DR}{\sigma^2}\right)\alpha^k}{Db_2^2(\sigma^2 b_2^2 + 2D^2 b_2)\ \Gamma(\mathrm{k})} \sum_{z=1}^k \frac{(k-1)!}{\alpha^z\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2 - 2D^2\alpha}{2D^2}\right)!}(k-z)!$$

$$\left(2D\left(\frac{R}{\theta}\right)^{k-z+\frac{1}{2}} K_{k-2+\ 1/2}{}^{\left(\frac{R\theta}{\sigma^2}\right)} + 2R\left(\frac{R}{\theta}\right) K_{k-z-\frac{1}{2}}{}^{\left(\frac{R\sigma}{\sigma^2}\right)}\right) \tag{37}$$

From equation (4) substituting L+T for L we have

$$G_{19}(R, L+T) = \frac{2Db_1}{b_2(\sigma^2 b_2^2 + 2D^2 b_2)} esp\left[L\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2Db_1}\right) - \frac{b_{2R}}{D}\right]$$

$$F\left[R - \frac{(L+T)\left(D + \frac{(\sigma^2 b_2)}{D}\right)}{\sqrt{\sigma^2(L+T)}}\right] + \frac{b_1}{b_2}\ \frac{((R-DT)-DL)}{D\sqrt{\sigma^2}(L+T)}\ g\ \left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{b_1}{Db_2} F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

$$-\frac{\sigma^2 b_2 b_1 esp\left(\frac{2DR}{\sigma^2}\right)}{Db_2(\sigma^2 b_2^2 + 2D^2 b_2)} F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \tag{38}$$

Multiplying by H(L)

$$H(L)G_{19}(R, L+T) = \frac{2Db_1\alpha^k L^{k-1}}{b_2(\sigma^2 b_2^2 + 2D^2 b_2)\ \Gamma(k)} esp\left[T\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) - \frac{b_2 R}{D}\right] g\ \left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

$$-\frac{b_1}{Db_2} esp\frac{(-\alpha L)L^{k-1}\alpha^k}{\Gamma(k)} F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{\sigma^2 b_2^2 b_1 esp(-\propto L)L^{k-1}\propto k}{Db_2(\sigma^2 b_2^2 + 2D^2 b_2)\ \Gamma(k)} F\left(\frac{R - D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) \tag{39}$$

$$G_{32}(R,T) = \int_0^\infty H\ (L)G_{19}(R, L+T)dL\ \ appling\ equation\ 14$$

$$G_{32}(R,T) = \frac{2Db_1\alpha^k esp(T\left(\frac{\sigma^2 b_2^2+2D^2 b_2}{2D^2}-b_2R+\alpha T\right)}{2\sigma\,\Gamma(k) b_2(\sigma^2 b_2^2+2D^2 b_2)^2} esp\left(\frac{DR}{\sigma^2}\right)$$

$$\sum_{i=0}^{k-1}(-T)^i\binom{k-1}{j}\sum_{z=i}^{k-1}\frac{(k-1-j)!}{\left(\frac{\sigma^2 b_2^2+2D^2 b_2}{2D_2}-\alpha\right)^z}-\frac{1}{(k-z)!}$$

$$\left(2\left(D+\frac{\sigma^2 b_2}{D}\right)\left(\frac{R}{\theta}\right)^{k-1-z+1/2} K_{k-1-2+1\frac{1}{2}}\left(\frac{R\theta}{\sigma^2}\right) + 2R\left(\frac{R}{\theta}\right)^{k-1-z+1/2} K_{k-i-2+\frac{1}{2}}\left(\frac{R\theta}{\sigma^2}\right)\right)$$

$$+\frac{2b_1\alpha^k esp\left(\frac{DR}{\sigma^2}\right)esp(\alpha T)}{Db_2\,\Gamma(k)\sqrt{2\pi}\sigma^2}\sum_{i-0}^{k-1}(-T)^{\frac{i}{2}}\binom{k-1}{j}\left(\frac{R}{\theta}\right)^{k-j+1/2} K_{k-j+1\frac{1}{2}}\left(\frac{R\sigma}{\sigma^2}\right)\sum_{z=0}^{k-1}\frac{(k-1-j)!}{\alpha^z(k-j-z)!}$$

$$\left(2D\left(\frac{R}{\sigma}\right)^{k-1-z+1/2} K_{k-1-z+\frac{1}{2}}\left(\frac{R\theta}{\sigma^2}\right)\right)$$

$$-\frac{\sigma^2 b_2^2 b_1\alpha^k esp\left(\frac{DR}{\sigma^2}+T\right)}{2\sigma b_2\left(\sigma^2 b_2^2+2D^2 b_2\right)\sqrt{2\pi}\,\Gamma(k)}\sum_{i-0}^{k-1}(-T)^{i}\binom{k-1}{j}\frac{(k-1-j)}{\alpha^z\,(k-j-z)!}(-20)\left(\frac{R}{\theta}\right)^{k-j-z+\frac{1}{2}} K_{k-j-z+\frac{1}{2}}$$

$$+2R\left(\frac{R}{\theta}\right)^{k-1-z+1/2} K_{k-1-2+\frac{1}{2}}\left(\frac{R\sigma}{\sigma^2}\right)\Bigg) \qquad (40)$$

Applying the above results

$$G_{30}(R+Y,T) = hcT\left(R+Y-\frac{DR}{\alpha}-\frac{DT}{2}\right)+hc\big(G_{27}(R+Y,T)-G_{24}(R+Y)\big)$$

$$+\big(G_{32}(R+Y,T)-G_{31}(R+Y)\big) \qquad (41)$$

Hence the inventory cost for model (M,R,T) is obtained by replacing $G_{21}(R+Y,T)$ by $G_{30}(R+Y,T)$ in equation (29) of series 1

$$C=\frac{\frac{Rc}{T}+S+\sum_{n=1}^{\infty}\int_0^{M-R} G_{31}(R+Y,T)g(M-R-Y,DT)dY+G_{30}(M.T)}{T\left[F\left(\frac{M-R-DT}{\sqrt{\sigma^2 T}}\right)+\sum_{n-1}^{\infty}\int_0^{M-R} n\, g^{n-1}(M-R-Y,DT)F\left(\frac{Y-DT}{\sqrt{\sigma^2 T}}\right)dy\right]} \qquad (42)$$

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