

On Intuitionistic Fuzzy Almost Regular α Generalized Continuous Mappings

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Abstract: The purpose of this paper is to introduce and study the concept of intuitionistic fuzzy almost regular α generalized continuous mapping in intuitionistic fuzzy topological spaces and study some of their properties.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological space, Intuitionistic fuzzy regular α generalized closed set, Intuitionistic fuzzy regular α generalized continuous mappings, Intuitionistic fuzzy almost regular α generalized continuous mappings.

1. INTRODUCTION

The Intuitionistic fuzzy sets was introduced by Atanassov [1]. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological space. In 2010, K. Sakthivel [8] introduced intuitionistic fuzzy α generalized continuous mappings and intuitionistic fuzzy regular α generalized continuous mappings was introduced by Nivetha M and Jayanthi D [7]. In this paper we introduce the notion of intuitionistic fuzzy almost regular α generalized continuous mappings and study their behaviour and properties in intuitionistic fuzzy topological spaces. Also we obtain some interesting theorems.

2. PRELIMINARIES

Definition 2.1:[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. The IFS $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFS in X satisfying the following axioms:

- $0 \sim, 1 \sim \in \tau$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4: [2] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Corollary 2.5:[2] Let $A, A_i (i \in J)$ be intuitionistic fuzzy sets in X and $B, B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \rightarrow Y$ be a function.

Then

- a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
- b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
- c) $A \subseteq f^{-1}(f(A))$ [If f is injective , then $A = f^{-1}(f(A))$]
- d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective , then $B = f(f^{-1}(B))$]
- e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
- f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
- g) $f^{-1}(0 \sim) = 0 \sim$
- h) $f^{-1}(1 \sim) = 1 \sim$
- i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.6:[3] An IFS A in an IFTS (X, τ) is said to be an

- (i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
- (ii) intuitionistic fuzzy α closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (iii) intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$
- (iv) intuitionistic fuzzy regular closed set (IFRCS in short) if $\text{cl}(\text{int}(A)) = A$

Definition 2.7:[5] An IFS A of an IFTS (X, τ) is called intuitionistic fuzzy regular α generalized closed set (IFR α GCS in short) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

Definition 2.8:[6] An IFS A of an IFTS (X, τ) is called intuitionistic fuzzy regular α generalized open set (IFR α GOS in short) if $\alpha \text{int}(A) \supseteq U$ whenever $A \supseteq U$ and U is an IFRCS in X .

The family of all IFR α GOSs of an IFTS (X, τ) is denoted by IFR α GO(X).

Definition 2.9:[3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is

said to be an intuitionistic fuzzy continuous (IF continuous in short) mapping if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.10:[4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is

said to be an

- (i) intuitionistic fuzzy semi continuous (IFS continuous in short) mapping if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$
- (ii) intuitionistic fuzzy α continuous (IF α continuous in short) mapping if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$
- (iii) intuitionistic fuzzy pre continuous (IFP continuous in short) mapping if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$

3. INTUITIONISTIC FUZZY ALMOST REGULAR α GENERALIZED CONTINUOUS MAPPINGS

In this section we introduce intuitionistic fuzzy almost regular α generalized continuous mapping and investigate some of its properties.

Definition 3.1: A mapping $f : X \rightarrow Y$ is said to be an intuitionistic fuzzy almost regular α generalized continuous (IFaR α G continuous in short) mapping if $f^{-1}(A)$ is an IFR α GCS in X for every IFRCS A in Y .

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ where $\mu_a=0.6, \mu_b=0.7, \nu_a=0.4, \nu_b=0.2$ and $G_2 = \langle x, (0.2, 0.2), (0.7, 0.7) \rangle$ where $\mu_a=0.2, \mu_b=0.2, \nu_a=0.7, \nu_b=0.7$ and $G_3 = \langle y, (0.6, 0.7), (0.3, 0.1) \rangle$ where $\mu_u=0.6, \mu_v=0.7, \nu_u=0.3, \nu_v=0.1$ and $G_4 = \langle y, (0.1, 0.1), (0.8, 0.8) \rangle$ where $\mu_u=0.1, \mu_v=0.1, \nu_u=0.8, \nu_v=0.8$. Then $\tau = \{0 \sim, G_1, G_2, 1 \sim\}$ and $\sigma = \{0 \sim, G_3, G_4, 1 \sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3, 0.1), (0.6, 0.7) \rangle$ is an IFRCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.3, 0.1), (0.6, 0.7) \rangle$ where $\mu_a=0.3, \mu_b=0.1, \nu_a=0.6, \nu_b=0.7$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\alpha \text{cl}(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X . Thus f is an IFaR α G continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFaR α G continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping. Let V be an IFRCS in Y . Since every IFRCS is an IFCS, V is an IFCS in Y . Then $f^{-1}(V)$ is an IFCS in X , by hypothesis. Since every IFCS is an IFR α GCS [5], $f^{-1}(V)$ is

an IFR α GCS in X. Hence f is an IFaR α G continuous mapping.

Example 3.4: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5,0.7), (0.4,0.2) \rangle, G_2 = \langle x, (0.2,0.2), (0.7,0.7) \rangle, G_3 = \langle y, (0.6,0.6), (0.3,0.2) \rangle$ and $G_4 = \langle y, (0.2,0.2), (0.8,0.8) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3,0.2), (0.6,0.6) \rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.3,0.2), (0.6,0.6) \rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3^c)) = \langle x, (0.4,0.2), (0.5,0.6) \rangle \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X but not an IFCS in X, since G_3^c is an IFCS in Y but $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$. Therefore f is an IFaR α G continuous mapping but not an IF continuous mapping.

Theorem 3.5: Every IF α continuous mapping is an IFaR α G continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IF α continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IF α CS in X, by hypothesis. Since every IF α CS is an IFR α GCS [5], $f^{-1}(V)$ is an IFR α GCS in X. Hence f is an IFaR α G continuous mapping.

Example 3.6: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.4,0.4), (0.4,0.2) \rangle, G_2 = \langle x, (0.3,0.2), (0.5,0.5) \rangle, G_3 = \langle y, (0.5,0.5), (0.3,0.2) \rangle$ and $G_4 = \langle y, (0.2,0.2), (0.8,0.8) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3,0.2), (0.5,0.5) \rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.3,0.2), (0.5,0.5) \rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X but not an IF α CS in X, since G_3^c is an IFCS in Y but $cl(int(cl(f^{-1}(G_3^c)))) = G_1^c \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFaR α G continuous mapping but not an IF α continuous mapping.

Remark 3.7: IFP continuous mapping and IFaR α G continuous mapping are independent to each other.

Example 3.8: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5,0.4), (0.5,0.3) \rangle, G_2 = \langle x, (0.1,0.2), (0.8,0.8) \rangle, G_3 = \langle y, (0.6,0.6), (0.3,0.3) \rangle, G_4 = \langle y, (0.1,0.1), (0.9,0.9) \rangle$.

Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3,0.3), (0.6,0.6) \rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.3,0.3), (0.6,0.6) \rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X but not an IFPCS in X, since G_3^c is an IFCS in Y but $cl(int(f^{-1}(G_3^c))) = G_1^c \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFaR α G continuous mapping but not an IFP continuous mapping.

Example 3.9: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.8,0.6), (0.2,0.2) \rangle, G_2 = \langle x, (0.2,0.3), (0.6,0.6) \rangle, G_3 = \langle y, (0.7,0.8), (0.1,0.2) \rangle, G_4 = \langle y, (0,0), (0.9,0.9) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.1,0.2), (0.7,0.8) \rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.1,0.2), (0.7,0.8) \rangle$ is an IFS in X. Then G_3^c is an IFCS in Y. Now $cl(int(f^{-1}(G_3^c))) = 0\sim \subseteq f^{-1}(G_3^c)$. Therefore $f^{-1}(G_3^c)$ is an IFPCS in X but not an IFR α GCS in X, since $\alpha cl(f^{-1}(G_3^c)) = G_2^c \not\subseteq G_2$ but $f^{-1}(G_3^c) \subseteq G_2$ where G_2 is an IFROS in X. Therefore f is an IFP continuous mapping but not an IFaR α G continuous mapping.

Remark 3.10: IFS continuous mapping and IFaR α G continuous mapping are independent to each other.

Example 3.11: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5,0.5), (0.3,0.2) \rangle, G_2 = \langle x, (0.1,0.2), (0.6,0.6) \rangle, G_3 = \langle y, (0.5,0.5), (0.3,0.1) \rangle, G_4 = \langle y, (0.1,0.1), (0.8,0.8) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3,0.1), (0.5,0.5) \rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.3,0.1), (0.5,0.5) \rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X but not an IFSCS in X, since G_3^c is an IFCS in Y but $int(cl(f^{-1}(G_3^c))) = G_2 \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFaR α G continuous mapping but not an IFS continuous mapping.

Example 3.12: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.6,0.7), (0.2,0.2) \rangle, G_2 = \langle x, (0.2,0.2), (0.7,0.7) \rangle,$

$G_3 = \langle y, (0.2,0.2), (0.6,0.7) \rangle, G_4 = \langle y, (0.6,0.6), (0.4,0.4) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.6,0.7), (0.2,0.2) \rangle$ is an IFRCs in Y . Then $f^{-1}(G_3^c) = \langle x, (0.6,0.7), (0.2,0.2) \rangle$ is an IFS in X . Then G_3^c is an IFCS in Y . Now $\text{int}(\text{cl}(f^{-1}(G_3^c))) = G_1 \subseteq f^{-1}(G_3^c)$. Therefore $f^{-1}(G_3^c)$ is an IFSCS in X but not an IFR α GCS in X , since $\alpha \text{cl}(f^{-1}(G_3^c)) = G_2^c \not\subseteq G_1$ but $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Therefore f is an IFS continuous mapping but not an IFaR α G continuous mapping.

Theorem 3.13: A mapping $f : X \rightarrow Y$ is an IFaR α G continuous mapping if and only if the inverse image of each IFROS in Y is an IFR α GOS in X .

Proof: Necessity: Let A be an IFROS in Y . This implies A^c is an IFRCs in Y . Since f is an IFaR α G continuous mapping, $f^{-1}(A^c)$ is an IFR α GCS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFR α GOS in X .

Sufficiency: Let A be an IFRCs in Y . This implies A^c is an IFROS in Y . By hypothesis, $f^{-1}(A^c)$ is an IFR α GOS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFR α GCS in X . Hence f is an IFaR α G continuous mapping.

Theorem 3.14: Let $f : X \rightarrow Y$ be a mapping where $f^{-1}(V)$ is an IFRCs in X for every IFCS in Y . Then f is an IFaR α G continuous mapping but not conversely.

Proof: Let V be an IFRCs in Y . Since every IFRCs is an IFCS, V is an IFCS in Y . Then $f^{-1}(V)$ is an IFRCs in X . Since every IFRCs is an IFR α GCS [5], $f^{-1}(V)$ is an IFR α GCS in X . Hence f is an IFaR α G continuous mapping.

Example 3.15: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.7,0.7), (0.3,0.2) \rangle, G_2 = \langle x, (0.1,0.1), (0.9,0.9) \rangle, G_3 = \langle y, (0.7,0.7), (0.2,0.1) \rangle, G_4 = \langle y, (0,0), (0.9,0.9) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.2,0.1), (0.7,0.7) \rangle$ is an IFRCs in Y . Then $f^{-1}(G_3^c) = \langle x, (0.2,0.1), (0.7,0.7) \rangle$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\alpha \text{cl}(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFR α GCS in X but not an IFRCs in X , since G_3^c is an IFCS in Y but $\text{cl}(\text{int}(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$. Therefore

f is an IFaR α G continuous mapping, but not the mapping as in Theorem 3.14.

Theorem 3.16: Let $f : X \rightarrow Y$ be a mapping. If $f^{-1}(\alpha \text{int}(B)) \subseteq \alpha \text{int}(f^{-1}(B))$ for every IFS B in Y , then f is an IFaR α G continuous mapping.

Proof: Let B be an IFROS in Y . By hypothesis, $f^{-1}(\alpha \text{int}(B)) \subseteq \alpha \text{int}(f^{-1}(B))$. Since B is an IFROS, it is an IF α OS in Y . Therefore $\alpha \text{int}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\alpha \text{int}(B)) \subseteq \alpha \text{int}(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = \alpha \text{int}(f^{-1}(B))$. This implies $f^{-1}(B)$ is an IF α OS in X and hence $f^{-1}(B)$ is an IFR α GOS[8] in X . Thus f is an IFaR α G continuous mapping.

Remark 3.17: The converse of the above theorem 3.16 is true if B is an IFROS in Y and X is an IF $\tau_\alpha T_{1/2}$ space.

Proof: Let f be an IFaR α G continuous mapping. Let B be an IFROS in Y . Then $f^{-1}(B)$ is an IFR α GOS in X . Since X is an IF $\tau_\alpha T_{1/2}$ space, $f^{-1}(B)$ is an IF α OS in X . This implies $f^{-1}(B) = \alpha \text{int}(f^{-1}(B))$. Now $f^{-1}(\alpha \text{int}(B)) \subseteq f^{-1}(B) = \alpha \text{int}(f^{-1}(B))$. Therefore $f^{-1}(\alpha \text{int}(B)) \subseteq \alpha \text{int}(f^{-1}(B))$.

Theorem 3.18: Let $f : X \rightarrow Y$ be a mapping. If $\alpha \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\alpha \text{cl}(B))$ for every IFS B in Y , then f is an IFaR α G continuous mapping.

Proof: Let B be an IFRCs in Y . By hypothesis, $\alpha \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\alpha \text{cl}(B))$. Since B is an IFRCs, it is an IF α CS in Y . Therefore $\alpha \text{cl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\alpha \text{cl}(B)) \supseteq \alpha \text{cl}(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $f^{-1}(B) = \alpha \text{cl}(f^{-1}(B))$. This implies $f^{-1}(B)$ is an IF α CS in X and hence $f^{-1}(B)$ is an IFR α GCS[5] in X . Thus f is an IFaR α G continuous mapping.

Remark 3.19: The converse of the above theorem 3.18 is true if B is an IFRCs in Y and X is an IF $\tau_\alpha T_{1/2}$ space.

Proof: Let f be an IFaR α G continuous mapping. Let B be an IFRCs in Y . Then $f^{-1}(B)$ is an IFR α GCS in X . Since X is an IF $\tau_\alpha T_{1/2}$ space, $f^{-1}(B)$ is an IF α CS in X . This implies $\alpha \text{cl}(f^{-1}(B)) = f^{-1}(B)$. Now $f^{-1}(\alpha \text{cl}(B)) \supseteq f^{-1}(B) = \alpha \text{cl}(f^{-1}(B))$. Therefore $f^{-1}(\alpha \text{cl}(B)) \supseteq \alpha \text{cl}(f^{-1}(B))$.

Theorem 3.20: Let $f : X \rightarrow Y$ be a mapping where X is an IF $\tau_\alpha T_{1/2}$ space. If f is an IFaR α G continuous mapping, then $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\alpha \text{cl}(B))$ for every IFRCs B in Y .

Proof: Let B be an IFRCs in Y . By hypothesis, $f^{-1}(B)$ is an IFR α GCS in X . Since X is an IF $\tau_\alpha T_{1/2}$ space, $f^{-1}(B)$ is an IF α CS in X . This implies $\alpha \text{cl}(f^{-1}(B)) = f^{-1}(B)$. Now $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(B) \cup \text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq$

$\alpha \text{cl}(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\alpha \text{cl}(B))$, as every IFRCS is an $\text{IF}\alpha\text{CS}$. Hence $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\alpha \text{cl}(B))$.

Theorem 3.21: Let $f : X \rightarrow Y$ be a mapping where X is an $\text{IF}_{\alpha}T_{1/2}$ space. If f is an $\text{IF}\alpha\text{R}\alpha\text{G}$ continuous mapping, then $f^{-1}(\alpha \text{int}(B)) \subseteq \text{int}(\text{cl}(\text{int}(f^{-1}(B))))$ for every IFROS B in Y .

Proof: This theorem can be easily proved by taking complement in Theorem 3.20.

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