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# Economic Order Model With Quadratic Backorder Costs

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**ABSTRACT**

The paper considers backorder costs as a quadratic cost depending on the length of time of the backorder. The quantity ordered is Q and the reorder level is R. We assume that the backorder cost $C_\beta(t)$ is $b_1 + b_2 t + b_3 t^2$. Where $b_1$, $b_2$ and $b_3$ are the coefficients with respect to t, $t^1_e$ and $t^2$,

The equations for the linear cost is derived first in section 1 by deriving the expected backorder cost per cycle using the assumption that demand follows the normal distribution.

We extend the method of analysis to derive the backorder costs when the backorder costs is a quadratic function of the length of time of a backorder. The additional costs to the incurred by the linear costs to the costs that is not time dependent is.**ECONOMIC ORDER MODEL WITH QUADRATIC BACKORDER COSTS.**

## INTRODUCTION

In this paper, the cost depending upon the length of time for which the backorder exists is taken as a quadratic cost. Without inventories customers would have to wait until their orders were filled from a source or were manufactured. The time lag could be a quadratic cost. The longer it takes to meet the order the more severe the backorder costs.
The paper considers the economic model (Q,R).

## LITERATURE REVIEW

The simple models of economic backorder inventory control were extensively dealt with by Hatdley and Whitin (1) Uthayakumar and Parvathi (2) investigates a continuous review inventory model to reduce lead time, yield variability and set up costs simultaneously through capital investments. The backorder rate is depending on the lead time through the amount of shortage.
Zhang G.U. and B.E. Dathwu (3) develops a hybrid inventory system with a time limit in backorders.

## BASIC MATHEMATICS ( Well dealt with in Ref (1) (Hatley and Whitin)

Let $x \sim N(0, 1)$ and g(x) be the probability density function.

$$g(x) = \frac{esp - \frac{x^2}{2}}{\sqrt{2\pi}} \quad -\infty < x < \infty$$

and $$f(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty esp - \frac{v^2 dv}{2}$$

Integrating by parts

$$\int_k^\infty x^n F(x)\, dx$$

$$= \left[ \frac{x^{x+1}}{n+1} F(x) \right]_k^\infty + \int_k^\infty \frac{x^{n-1}}{n+1} g(x) dx$$

If n = 0

$$\int_k^\infty F(x)\, dx = -kF(k) + g(k)$$

$$= g(k) - kF(k)$$

B.1

If n = 1

$$\int_k^\infty xF(x)\, dx = \tfrac{1}{2}\left(\left(1 - k^2\right)F(k) + kg(k)\right).$$

B.2

## 1. LINEAR BACKORDER COST

Let $C_B(t)$ be the cost of a backorder which has been outstanding for time t.

If a backorder is incurred at time z, z< L then L- z is the time for which a backorder exists.

Let R+y, 0< y<Q be the inventory level at time 0, then if the system is out of stock in the time interval z, to Z+dz after the reorder point R is reached then R+y was demanded in time Z and a demand occurred in time d~~z.~~

<u>This probability is</u>

$\frac{D}{\sqrt{2\pi\sigma^2 L}} \exp \frac{-1}{2}\left(\frac{R+y-Dz}{\sqrt{\sigma^2 L}}\right)^2 dz$
1.1

Hence the probability that t= L- Z,

Length of time of a backorder

$\frac{D}{\sqrt{2\pi\sigma^2 L}} \exp \frac{-1}{2}\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right)^2 dz \qquad 0 < z < L$

Giving an inventory level R +y at time 0

Expected cost of backorder

$C_B(L-z)$
$= probability\ of\ there\ been\ a\ backorder\ lasting\ L - \cancel{z}$

$=\frac{DC_B(L-z)}{\sqrt{2\pi\sigma^2 L}} \exp \frac{-1}{2}\left(\frac{R+Y-D\cancel{z}}{\sqrt{\sigma^2 L}}\right) d\cancel{z}$

Hence the expected backorder cost per cycle

$= D \int_0^Q \int_0^L \frac{C_B(L-z)}{\sqrt{2\pi\sigma^2 L}} \exp \frac{-1}{2}\left(\frac{R+Y-D\cancel{Z}}{\sqrt{\sigma^2 L}}\right)^2 \ dzdy$
1.2

For the linear backorder cost function

$$C_B(t) = \mathrm{b}_1 + \mathrm{b}_2\mathrm{t}$$

When the cost function is linear

$$C_B(L-\cancel{z}) = \mathrm{b}_1 + \mathrm{b}_2(\mathrm{L}-\mathrm{z})$$

and substituting into 1.2, expected backorder cost per cycle

$= \frac{D}{\sqrt{\sigma^2 L}} \int_0^Q \int_0^L (b_1 + b_2 (L-z)) g \left(\frac{R+y-D\cancel{z}}{\sqrt{\sigma^2 \mathrm{L}}}\right) dzdy$
1.3

Simplifying

Expected backorder costs per cycle

$\frac{D}{\sqrt{\sigma^2 L}} \int_0^Q \int_0^L (b_1 + b_2 L)\, g\left(\frac{R+Y-D\cancel{z}}{\sqrt{\sigma^2 L}}\right) dzdy$

$-\frac{Db_2}{\sqrt{\sigma^2 L}} \int_0^Q \int_0^L z\ g\ \left(\frac{R+Y-D\cancel{Z}}{\sqrt{\sigma^2 L}}\right) dzdy$

1.4

Letting v = $\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right)$

Expected backorder costs

$$= - \int_0^Q \int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}} (b_1 + b_2 L)\, g(v) dvdY$$

$$+ b_2 \int_0^Q \int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}} \left(\frac{R+Y-V\sqrt{\sigma^2 L}}{D}\right) g(v) dvdy$$

Simplifying we have

$-\int_0^Q \int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-DL}{\sqrt{\sigma^2 L}}} (b_1 + b_2 L - b_2 \frac{(R+Y)}{D})\ g\ (v) dvdy$

$-b_2 \int_0^Q \int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-DL}{\sqrt{\sigma^2 L}}} v\ g(v) dvdy$

1.5

Integrating and noting that

$$\int_R^\infty Vg(V) = g(R)$$

We have the expected backorder costs equal to

$= \int_0^Q \left(b_1 + b_2 L - b_2 \frac{(R+y)}{D}\right)\left(F\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right) - F\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\right) dy -$

$b_2 \int_0^Q (g\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right) - g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}})\right) dy$

Assume that $\mathrm{F}\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right) = 0$

1.6

Then the expected backorder costs equals

$\int_0^Q \left(b_1 + b_2 L - b_2 \frac{(R+Y)}{D})\right)\ F\ \left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right) \mathrm{dy}$

$+ b_2 \int_0^Q g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right) dy$

1.7

Noting that $b_2 L - b_2\left(\frac{R+Y}{D}\right)$

$= -b_2\left(\frac{R+Y-DL}{D}\right)$

Then expected backorder costs equals

$$\int_0^Q \left(\frac{b_1 - b_2\sqrt{\sigma^2 L}}{D}\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\right) F\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right) dY$$

$+ b_2 \int_0^Q g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right) dY$

Let V = $\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)$

Then expected backorder equals

$$\sqrt{\sigma^2 L}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}\left(\frac{b_1-b_2\sqrt{\sigma^2 L.v}}{D}\right)F(V)dV+$$

$$b_2\sqrt{\sigma^2 L}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} g(V)dV \qquad 1.8$$

using B1 and B.2 equations

and integrating, the expected backorder costs equals

$$-\sqrt{\sigma^2 L}\, b_1(g(V)-V\,F(V))]_{(R-DL)/\sqrt{\sigma^2 L}}^{(R+Q-DL)/\sqrt{\sigma^2 L}}$$

$$-b_2\sigma^2 L\,F(V)]_{(R-DL)/\sqrt{\sigma^2 L}}^{(R+Q-DL)/\sqrt{\sigma^2 L}}$$

$$\frac{+\,b_2\sigma^2 L}{2D}[(1-v^2)F(V)+\ V\,g\,(v)]_{(R-DL)/\sqrt{\sigma^2 L}}^{(R+Q-DL)/\sqrt{\sigma^2 L}}$$

1.9

Simplifying we have, the expected backorder costs

B (Q, R) = $b_1\sqrt{\sigma^2 L}\left[\left(g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\right.\right.$

$gR+Q-DL\sigma 2L-R+Q-DL\sigma 2LFR+Q-DL\sigma 2L$

$+\frac{b_2\sigma^2 L}{2D}\left[\left(\left(1+\frac{R-DL}{\sqrt{\sigma^2 L}}\right)^2\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right]$

$-\left(\left(1+\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)^2\right)F\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)$

$-\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)g\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\Big]\Big]$

1.10

## 2. QUADRATIC BACKORDER COST

We shall extend this method of analysis to derive the backorder costs when the backorder cost is a function of the length of time of a backorder

$$C_B(t)=\ b_1+b_2t+b_3t^2 \qquad 2.1$$

Expected backorder costs per cycle and applying equation 1.3

$$C_B(t)=\ \frac{D}{\sqrt{\sigma^2 L}}\int_0^Q\int_0^L\left(b_1+b_2(L-z)+b_3(L-z)^2\right)g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right)dzdY$$

Simplifying, expected backorder cost per cycle

$=\frac{D}{\sqrt{\sigma^2 L}}\int_0^Q\int_0^L(b_1+b_2L+b_3\,L^2-z(b_2+2b_3L)+$

$b_3z^2)\ g\ \left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right)dzdy$

Let v = $\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}$

Then we have

-

$\int_0^Q\int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}}\Big[\ (b_1+b_2L+b_3L^2)-\left(\frac{R+Y-\sqrt{\sigma^2 LV}}{D}\right)$

$(b_2+2b_3L)+b_3\left(\frac{R+Y-\sqrt{\sigma^2 LV}}{D}\right)^2\Big]g(v)dvdY$

Simplifying we have

$-\int_0^Q\int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-DL}{\sqrt{\sigma^2 L}}}(b_1+b_2L+b_3L^2-\frac{(R+Y)(b_2+2b_3L)}{D}+$

$b_3(R+Y)^2$ g(v)dvdY

= -

$\int_0^Q\int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-DL}{\sqrt{\sigma^2 L}}}\sqrt{\sigma^2 L}\left(\frac{V}{D}(b_2+2b_3L)-\frac{2b_3(R+Y)V}{D^2}\right)g\ (v)dvdY$

$-\quad b_3\frac{\sigma^2 L}{D^2}\int_0^Q\quad\int_{\frac{R+Y}{\sqrt{\sigma^2 L}}}^{\frac{R+Y-DL}{\sqrt{\sigma^2 L}}}V^2\,g(v)dvdY$

2.2

Integrating with respect to v

And applying B1 and B.2

$-\int_0^Q\left(b_1+b_2L+b_3L^2-\frac{(R+Y)(b_2+2b_3L)}{D}\right)+$

$b3(R+Y)2D2)*$

$$\left(F\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)-F\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\right)dY$$

$-\frac{\sqrt{\sigma^2 L}}{2}\int_0^Q\left(\left(\frac{b_2+2b_3L}{D}\right)-\frac{2b_3(R+Y)}{D^2}\right)(g\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)-$

$g\ \left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\Big)dY$

$-\frac{b_3\sigma^2 L}{D^2}\int_0^Q\left(F\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)+\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)g\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)*\right.$

$-FR+Y-DL\sigma 2L+$

$R+Y-DL\sigma 2LgR+Y-DL\sigma 2LdY$

2.3

Assuming that F$\left(\frac{R+Y}{\sqrt{\sigma^2 L}}\right)=0\ then\ we\ have$

$=\int_0^Q\left(b_1+b_2L+b_3L^2-\frac{(R+Y)(b_2+\,2b_3L)}{D}+\right.$

$b3\,R+Y2D2$

*F$\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)dY$

$+\ \sqrt{\sigma^2 L}\int_0^Q\frac{(b_2+2b_3L)}{D}-\frac{2b_3(R+Y)}{D^2})g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)dY$

$+\frac{b_3\sigma^2 L}{D^2}\int_0^Q\left(F\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)+\right.$

*R+Y−DLσ2LgR+Y−DLσ2LdY*

2.4

Simplifying we have

$$\int_0^Q (b_1 + b_2L + b_3L^2 - \left(\frac{R+Y}{D}\right)\ (b_2 + 2b_3L)$$
$$+\ b_3\left(\frac{R+Y}{D^2}\right)2$$
$$+\ \frac{b_3\sigma^2 L}{D^2})F\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)dY$$
$$+\int_0^Q\ \sqrt{\sigma^2 L}\left(\left(\frac{b_2+2b_3L}{D}\right.\right.$$
$$\left.-\frac{2b_3(R+Y)}{D^2}\right)$$
$$+\ \frac{\sigma^2 L b_3}{D^2}\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\ \ \right)\Bigg)g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)dY$$

2.5

Noting that $b_1 + b_2L + b_3L^2 - (R+Y)(b_2 +$ *2 b3L+ b3R+Y2D2+b3σ2LD2*

=

$$b_1 + b_3\sigma^2 L + \frac{b_3\sigma^2 L}{\sqrt{\sigma^2 L}}\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right) - \left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\frac{b_2\sqrt{\sigma^2 L}}{D}$$

And noting that

$$\sqrt{\sigma^2 L}\left(\frac{b_2}{D}+\frac{2b_3L}{D}\right)-\frac{2b_3(R+Y)}{D^2}\sqrt{\sigma^2 L}$$
$$+\frac{\sigma^2 L b_3}{D^2}\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)$$

$$=\frac{\sqrt{\sigma^2 L}b_2}{D}-\frac{b_3\sigma^2 L}{D^2}\left(\frac{R+Y-DL}{\sigma^2 L}\right)$$

Then substituting into 5

We have

$$\int_0^Q\left(b_1+\frac{b_3\sigma^2 L}{D^2}+\frac{b_3\sigma^2 L}{D^2}\left(\frac{R+Y-DL}{\sigma^2 L}\right)2\right.$$
$$\left.-\frac{b_2}{D}\sqrt{\sigma^2 L}\left(\frac{R+Y-DL}{\sigma^2 L}\right)\right)$$

$$F\left(\frac{R+Y-DL}{\sigma^2 L}\right)dY$$

$$+\int_0^Q\left(\frac{\sqrt{\sigma^2}Lb_2}{D}-b_3\frac{\sigma^2 L}{D^2}\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)\right)g\left(\frac{R+Y-DL}{\sqrt{\sigma^2 L}}\right)dY$$

2.6

$$let\ V=\frac{R+Y-DL}{\sqrt{\sigma^2 L}}$$

Then we have re-arranging

$$\sqrt{\sigma^2 L}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}\left(\frac{b_1+\ b_3\sigma^2 L}{D^2}\right)F(V)dV\,\frac{-b_2\sigma^2 L}{Q}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} V$$

+

$$\frac{b_3\sigma^3 L^{3}/_{2}}{D^2}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} V^2F\ (V)dV+$$

$$\frac{b_{2\ \sigma^2 L}}{Q\ D}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} g(V)dV$$

$$-\quad\frac{\sigma^3L^3/_2 b_3}{D^2}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} V\ g\ (V)dV$$

2.7

Integrating and applying B1 and B.2 we have

$$-\sqrt{\sigma^2 L}\frac{(b_1+b_3\sigma^2 L)}{D^2}[g(V)-\ V\ F(V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

$$+\ b_2\sigma^2 L[(1-V^2)F(V)+\ Vg(V)]_{\frac{R+-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

$$-\ \frac{b_3\sigma^2 L^3/_2}{3D^2}[(V^2+\ 2)g(V)-V^3f\ (V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

$$-\ \frac{b_2\sigma^2 L}{D}[F(v)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}+\frac{\sigma^2 L^3/_2\,b_3}{D^2}.g(V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

2.8

Simplifying

$$=-\sqrt{\sigma^2 L}\,b_1[(g(V)-V\ F(V))]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

$$+b_2\frac{\sigma^2 L}{2D}[(1-V^2-2)F(V)+Vg\ (V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

Simplifying

$$=-\sqrt{\sigma^2 L}\,b_1[g(V)-V\ F(V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

$$-\frac{b_2\sigma^3 L}{2D}\ [(1+v^2)F(V)-Vg(V)]_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}$$

-

$\frac{b_3\sigma^2 L^{3/2}}{3D}[(V^2+2)g(V)-V(3+V^2)F(V)]\,\frac{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}}{\frac{R-DL}{\sqrt{\sigma^2 L}}}$

Substituting $\propto(v) and\ \beta(v)$

2.9

letting

$\propto(V) = \frac{\sigma^3 L^{3/2}}{3}((V^2+2)g(V)-V(3+V^2)F(V))$ 2.10

$$\beta(V) = \frac{\sigma^2 L}{2}\left[\left(1+v^2\right)F(v)-vg(v)\right]$$

Then we have expected backorders per cycle to be

$$b_1\left(\propto\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\propto\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)+\frac{b_2}{D}\left(\beta\frac{(R-DL}{\sqrt{\sigma^2 L}}\right)-\beta\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)$$

$+\frac{b_3}{D^2}\left(\propto\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\propto\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)$

2.11

$$number\ of\ cycles = \frac{D}{Q}$$

Hence expected backorder costs per year

$=\frac{b_1 D}{Q}\left(\propto\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\propto\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)+\ b_2((\beta\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$ $-\beta\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)\frac{+b_3}{DQ}\left(\propto\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\propto\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right)$

2.12

Hence from 2.6.8 the inventory costs for model (Q, R) with quadratic backorder cost terms and

C = $\frac{DS}{Q}+\frac{Qhc}{2}+hc\ k\sqrt{\sigma^2 L}+\ \frac{Db_1}{Q}(\propto(k)-\propto\left(\frac{K+Q}{\sqrt{\sigma^2 L}}\right)$

$+\frac{(b_2+hc)}{Q}(\beta(k)-\beta+\left(\frac{k+Q}{\sqrt{\sigma^2 L}}\right)\frac{b_3}{DQ}(\propto\ (k)-\propto$ $\left(k+Q/\sqrt{\sigma^2 L}\right))$

$+\frac{DS}{Q}(\propto(k)-\propto\left(k+Q/\sqrt{\sigma^2 L}\right)$

2.13

## CONCLUSION

With the only additional cost to the linear cost $C_\beta = b_1 + b_2 t$ being the $b_2$ factor:

$$\frac{b_2}{DQ}\left(\alpha(k)-\alpha\left(k+Q/\sqrt{\tau^2}L\right)\right)$$

And the additional cost of the linear to the cost without length of time of shortage is

$$\frac{Db_1}{Q}\left(\alpha(k)-\alpha\left(k+\frac{Q}{\sqrt{\tau^2 L}}\right)\right)$$

Since $\frac{b_2}{DQ}<\frac{Db_1}{Q}$, the additional costs of the quadratic cost to the linear cost is far less than the additional cost of the linear cost to the cost that is not time dependent.

## REFERENCES


1.Hadley G and Whitin T.M. *Analysis of Inventory Systems*, Prentice-Hall International, Inc, Englewood Cliffs, N.T. 1963.

2.Uthayakumar R and Parvathi P. *Continuous Review Inventory Model* International Journal of System Science Vol 10 Issue 3, 7, 2009

3.Zhang G.P, Patuno B.E.(*A Hybrid Inventory System with a Time Limit or Backorder)*, Transactions of Industrial Engineers, 2003.