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Mitigation of Short-Term Climate Risks and Uncertainties through Insurance and Self-Protection: Theory and Application

Abdelmoumine Traore

Abstract

This paper investigates the interplay between short-term insurance and self-protection strategies in mitigating weather-related risks and uncertainties amidst rising global temperatures. We explore the decision-making process behind these strategies, focusing on whether the choice between insurance and self-protection depends on the type of stochastic loss encountered, distinguishing between risk and uncertainty.

Existing research highlights the context-specific nature of the relationship between insurance and self-protection. While a significant portion of the literature has concentrated on understanding long-term dynamics, the sudden occurrence of weather-related events requires a closer examination of short-term decision-making processes. This paper contributes by providing a theoretical framework for analyzing how weather stochastics influence producers' decisions regarding insurance and self-protection in the short term.

Simulation outcomes reveal distinct responses of farmers to risk and uncertainty. Under risk, farmers without irrigation systems tend to increase their reliance on crop insurance as precipitation risks heighten, while those with irrigation systems adopt a nuanced approach, adjusting their insurance purchases based on the severity of precipitation risks. This suggests that irrigation serves as both a substitute and complement to crop insurance, depending on the level of risk. Conversely, under uncertainty, farmers exhibit a general trend of decreased crop insurance purchases regardless of their self-protection measures. Addressing uncertainty within agricultural loss mitigation frameworks is crucial for safeguarding against potential food insecurity and increasing investment to mitigate climate-related disasters.

Policy implications underscore the need to consider producers' level self-protection and the type of stochastics faced in climate policy design. Additionally, reducing uncertainty in weather forecasts is imperative to mitigate farmers' vulnerability and promote agricultural resilience.

Keywords: self-protection, insurance, expected utility, robust optimization, short-term risk vs. short-term uncertainty, risk aversion, climate change

1 Introduction

This paper aims to investigate the dynamics between short-term insurance and self-protection as strategies for mitigating weather-related risks and uncertainties over a short horizon. In the context of rising global temperatures and the consequential shifts in weather patterns, economic agents are compelled to devise effective coping mechanisms. Typically, these strategies involve either purchasing financial instruments such as derivatives or insurance policies, or implementing self-protective measures to minimize the likelihood of losses. We delve into the decision-making process concerning these strategies, particularly focusing on whether the choice between insurance and self-protection for short-term loss mitigation depends on the type of stochastic loss encountered, distinguishing between risk and uncertainty.

Decision-makers operate within the domain of risk when they possess knowledge of the stochastic process generating outcomes and can estimate the probabilities associated with each potential outcome. Conversely, uncertainty arises when decision-makers lack awareness of the stochastic process but have subjective knowledge regarding potential outcomes (Knight, 1921; Luce and Raiffa, 1957; Raiffa, 1968, Heath and Twersky, 1991; Hertwig et al., 2004; Ulkumen et al., 2016). It is important to note that deep uncertainty exists when decision-makers are unable to agree on the system model or the probability distributions to place over inputs (Lempert et al., 2003). Our inquiry revolves around understanding how economic agents navigate these different settings of risk and uncertainty in the face of weather fluctuations.

Existing research highlights the ambiguous and context-specific nature of the relationship between insurance and self-protection, which varies across industries, geographic locations, types of stochastic events, and temporal frames. While much of the literature has focused on the long-term dynamics of this relationship, the context of climate-related stochastic events necessitates a closer examination of short-term decision-making processes, given the sudden and unpredictable nature of weather patterns. For instance, studies have shown that individuals with certain types of insurance are more inclined to engage in proactive measures aimed at mitigating risks, thereby reducing their dependence on long-term care services. Cutler et al. (2008) demonstrates positive correlations between the acquisition of various insurance products, such as term life insurance and long-term care insurance, and the adoption of risk reduction strategies among individuals in the United States. However, empirical investigations have also revealed significant variations in the relationship between insurance coverage and the adoption of risk mitigation measures, emphasizing the need for a nuanced understanding of decision-making processes in response to weather-related uncertainties (Cohen and Siegelman, 2010). Findings by Einav et al. (2013) suggest that while certain individuals exhibit moral hazard behavior, leading to increased demand for health insurance, highly risk-averse individuals may opt out of such behavior, yet display a strong willingness to invest in insurance coverage, particularly in response to perceived health risks.

This paper contributes to the existing literature by providing a theoretical framework for analyzing how weather stochastics influence producers' decisions regarding insurance and self-protection in the short term. Under conditions of risk, our analysis is grounded in the expected utility framework, where producers maximize expected utility subject to constraints imposed by their marketing and production environments (Feder, 1980; Pope, 1982; Bard and Barry, 2001; Kumbhakar, 2002; Flaten et al., 2005; Hao Aimin, 2010; Cao et al., 2011; Sulewski et al., 2014; Ullah and Ali, 2015; Vollmer et al., 2017; Iyer et al., 2020). On the other hand, under conditions of uncertainty, we adopt a robust optimization approach to capture decision-making in the face of ambiguity (Ben-Tal, 1985; Ben-Tal and Teboulle, 1987; Artzner et al., 1999; Follmer and Schied, 2002; Frittelli and Gianin, 2002; Ruszczy'nski and Shapiro, 2006; Lesnevski et al., 2007; Ben-Tal et al., 2010; Choi et al., 2011; Shapiro et al., 2013; Laeven and Stadje, 2014). By delineating these decision-making processes and conducting simulations based on our theoretical models, we offer insights into farmers' choices between self-protection and insurance when confronted with risks versus uncertainties.

2 Theoretical Model of Loss Mitigation through insurance vs self-protection

2.1 Salient Features of the Self-Protection vs. Insurance Problem

Our model explains the choices of a producer seeking to mitigate stochastic economic losses through self-protection and insurance. We consider this problem across a variety of specifications to consider particular salient features of the problem:
(i) The producer faces two types of stochastic losses: risk and uncertainty.
(ii) In the case of risk, the producer is risk-averse.
(iii) The producer is assumed to be hedonic, ie. the producer is only motivated by economic profit.

2.2 Basic Assumptions

(i) Consider producer *i* seeking to mitigate stochastic economic losses likely to occur during a single period t. A short-termist could be considered myopic when relevant future events may not impact current period performance.
(ii) In the short-term, the producer cannot alter their level of self-protection. Self-protection is considered as a state condition. For example, in the case of farming, if a farmer does not have an irrigation system, it is unlikely that they will acquire one in the short-term due to the substantial sunk costs associated with technology adoption, including expenses for field design, equipment, and labor (Feder, Just, and Zilberman (1985), Dinar and Yaron (1992),Lee (2005),Koundouri, Nauges, and Tzouvelekas (2006)).

2.3 The Self-Protection vs. Insurance Choice Problem for a Producer with Hedonistic Preferences

Let's consider producer i facing a weather event $\phi_{i,t}$ that could significantly affect production in period t. We begin with the specification of how outcomes are generated. We define the flow of generation of external weather factors ($\phi_{i,t}$) as a vector affecting production as follows:

$$\phi_{i,t} \sim g(\Gamma_{\phi_{i,t}}). \quad (1)$$

If $\phi_{i,t}$ can be described probabilistically, g is defined as a density function with a mean ($\mu_{\phi_{i,t}}$) and deviation ($\sigma_{\phi_{i,t}}$). If $\phi_{i,t}$ is uncertain and cannot be described probabilistically, g is defined as the uncertainty set (Φ) from which $\phi_{i,t}$ is generated.

Atmospheric science tells us that $\phi_{i,t}$ is related to the aggregate stock of carbon emissions (V_t) in the atmosphere (IPCC, 2013; Vose et al., 2017; Hayhoe et al., 2018; National Academy of Sciences, 2020). Although hedonistic agents do not track their carbon emissions ($S_{i,t}$), the EPA announces the aggregate stock of carbon emissions publicly (V_t) in the atmosphere up to period t. All producers feel V_t through its impact on $\phi_{i,t}$, which we can consider being temperature or precipitation, for example. Therefore, we assume that at any given time, V_t is known by all producers in the economy. V_t shifts the mean and the spread of g defined in eq.(1) when $\phi_{i,t}$ can be described probabilistically. If $\phi_{i,t}$ is uncertain and cannot be described probabilistically, V_t affects the bounds of the set g over which $\phi_{i,t}$ is defined. Therefore, $\Gamma_{\phi_{i,t}}$ defined in eq.(1) is a function of V_t:

$$\begin{aligned} \Gamma_{\phi_{i,t}} &\equiv \Gamma_{\phi_{i,t}}(V_t), \\ \phi_{i,t} &\sim g(\Gamma_{\phi_{i,t}}(V_t)). \end{aligned} \quad (2)$$

Together, eq.(2) defines weather dependance on a stock, state condition V_t. The signs of $\frac{\partial \Gamma_{\phi_{i,t}}}{\partial V_t}$ and $\frac{\partial^2 \Gamma_{\phi_{i,t}}}{\partial V_t^2}$, the first and second derivatives of $\Gamma_{\phi_{i,t}}$ with respect to V_t depend on the definition of $\phi_{i,t}$. For example, if $\phi_{i,t}$ is defined as temperature, global climate models and historical climate data show that there is a simple linear relationship between total cumulative emissions and temperature change ($\phi_{i,t}$) (Valone.T (2021)). Suppose $\phi_{i,t}$ is defined as rainfall, then the relationship between rainfall and the stock of carbon emissions varies by region[1]

2.3.1 Self-Protection as a Strategy to Mitigate Weather Changes

As weather fluctuations are due to the increase of V_t over time, producer i may decide to use their stock of self-protection $\vartheta_{i,t}$ to impact production. $\vartheta_{i,t}$ reduces the effect of V_t on the parameters of $\Gamma_{\phi_{i,t}}$. As an illustration, a farmer having an irrigation system can reduce the effect of drought on his field during period t. Therefore, the specification of $\phi_{i,t}$ needs to include $\vartheta_{i,t}$.

$$\phi_{i,t} \sim g(\Gamma_{\phi_{i,t}}(V_t|\vartheta_{i,t}). \quad (3)$$

[1]Kooperman et al. (2018) suggests that South American forests may be more vulnerable to rising CO2 than Asian or African forests. They found that the Amazon rainforest is most at risk of drought and forest mortality due to rising CO2. With Amazon releasing less water vapor into the atmosphere and fewer clouds forming over the forest, water vapor from the Atlantic Ocean will not have pre-existing clouds to bond with and will blow over the forest to the Andes. However, increased CO2 and reduced moisture impact will differ entirely in other tropical forests, especially in Africa and on islands in Malaysia, Papua New Guinea, and Indonesia. These forests could see increased rainfall as lack of moisture will lead to a huge increase in surface temperature compared to the surrounding ocean air, thus pulling in greater moisture from ocean systems.

Eq.(3) shows that self-protection has a locational impact on climate variables $\phi_{i,t}$ through $\vartheta_{i,t}$. In mathematical form:

$$\frac{\partial^2 \Gamma_{\phi_{i,t}}}{\partial \vartheta_{i,t} \partial V_t} /= 0. \tag{4}$$

Again the sign of eq.(4) depends on the definition of $\phi_{i,t}$.

2.3.2 Insurance Purchasing as a Strategy to Mitigate Weather Changes

In addition to self-protection, producer i can purchase an insurance policy to receive an indemnity ($I_{i,t}$) in case their economic performance indicator ($\Omega_{i,t}$) exceeds the trigger for indemnity payment ($\hat{\Omega}_{i,t}$) defined by the insurer. $\Omega_{i,t}$ could be revenue, production yield, or profit. $\hat{\Omega}_{i,t}$ could be the corresponding insurer guarantee revenue, yield, or profit. Let $\rho_{i,t}$ be the total premium paid by the producer to insure his economic performance indicator ($\Omega_{i,t}$). $\rho_{i,t}(cv_{i,t}|\vartheta_{i,t})$ depends on the service flow from the fixed stock of self-protection $\vartheta_{i,t}$ and vary with the coverage level ($cv_{i,t}$) chosen by the economic producer when purchasing insurance. The higher the coverage level chosen by the producer, the higher the premium rate and $\hat{\Omega}_{i,t}$ given $\vartheta_{i,t}$. $\Gamma_{I,i,t}(\rho_{i,t};\Omega_{i,t};\hat{\Omega}_{i,t};I_{i,t};\ c_{i,t})$ is a vector containing the above-mentionned insurance parameters.

Insurance may require a deductible ($d_{i,t}$) to be met before a payment is made. Certification adjusters must verify losses before payments are made, and these payments are subject to audits. The deductible is the amount of loss incurred before insurance coverage begins, determined by subtracting the coverage level percentage chosen from 100 percent ($d_{i,t} = 100 - c_{i,t}$). For example, if the insured elected a 65 percent coverage level, the deductible would be 35 percent (100 % – 65 % = 35 %). Indemnity is paid when the economic outcome $\Omega_{i,t}$ is realised such that $\hat{\Omega}_{i,t} > \Omega_{i,t}$. Otherwise, indemnity is zero. Thus, in mathematical form:

$$\begin{aligned} & I_{i,t} \equiv I_{i,t}(\Omega_{i,t}, \hat{\Omega}_{i}, c_{i,t}), \\ & \text{If } \Omega_{i,t} < \hat{\Omega}_{i,t} \rightarrow L_{i,t} > 0, \\ & \text{Otherwise, If } \hat{\Omega}_{i,t} \leq \Omega_{i,t} \rightarrow L_{i,t} = 0. \end{aligned} \tag{5}$$

In the next section, we write the profit definition for the myopic producer. Let I be a dummy variable such that if $I = 0$, the producer does not purchase an insurance policy, and if $I = 1$, the producer does purchase an insurance policy. Let v be a dummy variable such that if $v = 0$, the producer does not have self- protection, and if $v = 1$, the producer has self-protection.Therefore we write i) the profit definition with no insurance (I=0) and no self-protection (v=0) and (ii) the profit definition with insurance (I=1) and no self-protection (v=0), (iii) the profit definition with no insurance (I=0) but with self-protection (v=1), (iv) the profit definition with both insurance (I=1) and self-protection (v=1).

2.3.3 The Profit Definitions of the Hedonic Producer

Let t be a production time interval. Let's consider agent i producing output vector $Y_{i,t}$ using input vector $X_{i,t}$ during period t. $Y_{i,t}$ is a 1xm vector and $X_{i,t}$ is a 1xj vector. $Y_{i,t}$ is priced at $P_{i,t}$ where $P_{i,t}$ is a 1xm vector, and inputs are priced at $R_{i,t}$ where $R_{i,t}$ is a 1xj vector. Both $P_{i,t}$ and $R_{i,t}$ are stochastic and idiosyncratic. $P'_{i,t}$ and $R'_{i,t}$ are the transpose vectors of $P_{i,t}$ and $R_{i,t}$ respectively.

Both self-protection and insurance affect the producer's profit (Table **??**). Self-protection affects the revenue side of the profit function through $\Gamma_{\phi\, i,t}$ However, the activation and operation of the self-protection generates additional expenditures for the producer. For example, when irrigation is used, it requires electricity to pump the water and in some states like California, the water itself needs to be paid for. Therefore, self-protection affects both the revenue and the costs of the producer. As to insurance, it does not affect the production directly but pays a lump-sum $I_{i,t}$(the indemnity defined in eq.(5)) to the insured on condition that the economic indicator falls below the trigger. At the same time, the producer has to pay a premium during period. The premium paid by the producer could differ depending on their level of self-protection. For example, in crop insurance, farmers with an irrigation system pay a lower premium compared those without.

(i) The contemporaneous profit definition without insurance and no self-protection of the myopic producer is the following:

$$\Pi^{I=0}_{i,v=0,t} \equiv P'_{i,t} * Y_{i,v=0,t} - R'_{i,v=0,t} * X_{i,v=0,t}. \tag{6}$$

Where $Y_{i,v=0,t}$, $R_{i,v=0,t}$, and $X_{i,v=0,t}$ are respectively the production function, the input price vector, and the input quantity vector with no self-protection.

(ii) The contemporaneous profit definition with insurance and no self-protection of the myopic producer is the following:

$$\Pi^{I=1}_{i,v=0,t} \equiv P'_{i,t} * Y_{i,v=0,t} - R'_{i,v=0,t} * X_{i,v=0,t} + I_{i,v=0,t} - \rho_{i,v=0,t}. \tag{7}$$

Where $I_{i,v=0,t}$, $\rho_{i,v=0,t}$ are respectively the indemnity function, and the premium paid by the producer with no self-protection.

(iii) The contemporaneous profit definition with self-protection and no insurance of the myopic producer is the following:

$$\Pi^{I=0}_{i,v=1,t} \equiv P'_{i,t} * Y_{i,v=1,t} - R'_{i,v=1,t} * X_{i,v=1,t}. \tag{8}$$

Where $Y_{i,v=1,t}$, $R_{i,v=1,t}$, and $X_{i,v=1,t}$ are respectively the production function, the input price vector, and the input quantity vector with self-protection.

(iv) The contemporaneous profit definition with self-protection and insurance of the myopic producer is the following:

$$\Pi^{I=1}_{i,v=1,t} \equiv P'_{i,t} * Y_{i,v=1,t} - R'_{i,v=1,t} * X_{i,v=1,t} + I_{i,v=1,t} - \rho_{i,v=1,t}\ . \tag{9}$$

Where $I_{i,v=1,t}$, $\rho_{i,v=1,t}$ are respectively the indemnity function, and the premium paid by the producer with self-protection.

2.3.4 The Multiple Output Production Function of the Hedonic Producer

The producer follows a time-intensive production process initiated at time t and completed at the end of period t. Whether the producer is hedonic or prosocial, they produce a vector of proprietary outputs $y_{i,t}$ and a vector of nonproprietary outputs $S_{i,t}$ using a vector of short-term input controls committed at the beginning of period t $X_{i,t}$. Every production process generates some waste, whether the producer is hedonic or prosocial. Let $Y_{i,t}$ be a vector containing the proprietary outputs ($y_{i,t}$) and the nonproprietary outputs ($S_{i,t}$) such that $Y_{i,t} = (y_{i,t}, S_{i,t})$.

In this theory, we define $S_{i,t}$ as a bad output and $y_{i,t}$ as a good output. Papers on modeling multiple output technologies can be classified into two groups based on the approach to modeling bad outputs. The first group of papers considers a multi-equation representation of polluting technology, while the second group adopts an alternative single-equation specification of the production process in the presence of bad outputs. The multi-equation representation primarily attributed to Fernández et al. (2002, 2005), Forsund (2009), and Murty et al.(2012) rely on the more traditional multiplicative radial formulation of a system of a desirable technology and its accompanying undesirable by-production. In contrast, in the spirit of Chambers et al., the single-equation approach usually formalizes polluting technology as a function under the joint weak disposability of good and bad outputs (Weaver (1996), Chung et al. (1997), and Fare et al. (2005). Let G be the production output possibility set such that:

$$G = \{(Y_{i,t}) : X_{i,t} \text{ can produce } Y_{i,t}\}. \quad (10)$$

As demonstrated by Fare et al. (2005), the directional distance function meets the following standard axioms : (i) The output set is compact for each input vector, (ii) The outputs are weakly disposable, (iii) Jointness needs to be satisfied by G, (iv) Good and bad outputs are null-joint. Figure 2 in Appendix shows an illustration of the directional distance function. Therefore, we focus our attention on the distance function because it allows representing in a single equation the joint production of multi-outputs using multi-inputs when some of the outputs are bad. Let F_G be the directional output distance function defined on G . F_G is a measure of efficiency if:

$$F_G(X_{i,t}, y_{i,t}, S_{i,t} | \vartheta_{i,t}, \phi_{i,t}) = 0. \quad (11)$$

The stochastic nature of $\phi_{i,t}$ leads the producer to make their decision based on subjective perceptions of possible occurrences of $\phi_{i,t}$. Like Weaver (1977), we derive the provisional production function using the expected production function E(F) and the Taylor series expansion of F around $\phi_{i,t}$. for details about the derivation.

$$E(F_G) \equiv F_G(X_{i,t}, y_{i,t}, S_{i,t} | \vartheta_{i,t}, E(\phi_{i,t}), Var(\phi_{i,t})) = 0. \quad (12)$$

In the short-term, the producer can have control of $X_{i,t}$ and $y_{i,t}$ only; $\vartheta_{i,t}$ cannot be controlled in the short-term. By definition, $\phi_{i,t}$ is not directly controlled by the producer. According to Fare et al. (2005), the directional output distance function inherits its properties from the output possibility set $G(X_{i,t})$. These properties include:

$$\frac{\partial F_G}{\partial y_{i,t}} < 0. \quad (13)$$

$$\frac{\partial F_G}{\partial S_{i,t}} > 0. \quad (14)$$

$$\frac{\partial F_G}{\partial X_{i,t}} > 0. \quad (15)$$

The second-order conditions require that F_G be concave around $(y_{i,t}, S_{i,t}) \in G(X_{i,t})$.

2.3.5 The Definition of the Utility Function of the Hedonic Producer

We assume that the producer is risk-averse. When the myopic producer needs to decide on the best alternative between purchasing insurance vs. not, they seek to maximize under risk its expected utility over the probability distribution of $\phi_{i,t}$. However, under uncertainty, he seeks to maximize the worst-case scenario of their utility over the uncertainty set (Φ) of $\phi_{i,t}$. Therefore, before elaborating on the choice problem of the producer, we need to define the utility function of the producer.

We use a quadratic utility function to express the expected utility as a mean-risk variance model. This expected utility function was used by Weaver and al. (2001).

$$U_{i,v,t}(\Pi_{i,v,t}) \equiv \Pi_{i,v,t} + \psi'_i * (\Pi_{i,v,t})^2. \quad (16)$$

ψ'_i is the negative of ψ_i, the risk aversion coefficient such that $\psi_i \geq 0$.

The first derivative of the utility function with respect to profit must be positive:

$$\frac{\partial U_{i,v,t}}{\partial \Pi_{i,v,t}} > 0. \quad (17)$$

The sign of the first derivative of the utility function with respect to $y_{i,t}$ is:

$$\frac{\partial U_{i,v,t}}{\partial y_{i,t}} = \frac{\partial U_{i,v,t}}{\partial \Pi_{i,v,t}} * \frac{\partial \Pi_{i,v,t}}{\partial y_{i,t}} > 0. \quad (18)$$

The second derivative of the utility function with respect to $\Pi_{i,v,t}$ is the following:

$$\frac{\partial^2 U_{i,v,t}}{\partial \Pi^2_{i,v,t}} = \psi'_i < 0. \quad (19)$$

Therefore, the utility function has a concave shape which makes the utility maximization problem of the producer convex. When $\phi_{i,t}$ can be described probabilistically, the expected utility can be written as follows:

$$E_{\phi_{i,t}}[U_{i,v,t}(\Pi_{i,v,t})] = E_{\phi_{i,t}}(\Pi_{i,v,t}) + \psi'_i * E_{\phi_{i,t}}(\Pi^2_{i,v,t}). \quad (20)$$

During the simulation, the expected value of $\Pi_{i,v,t}$ and $\Pi^2_{i,v,t}$ are computed by sampling $\phi_{i,t}$ from the distributions described in eq. (1)

$$E_{\phi_{i,t}}(\Pi_{i,v,t}) = \int_{\Phi_{i,t}} [\Pi_{i,v,t}] * g(\Gamma_{\phi_{i,t}})\, d\phi_{i,t},$$
$$E_{\phi_{i,t}}(\Pi^2_{i,v,t}) = \int_{\Phi_{i,t}} [\Pi^2_{i,v,t}] * g(\Gamma_{\phi_{i,t}})\, d\phi_{i,t}, \tag{21}$$
$$st.\ [\phi_{min}, \phi_{max}] \in \Phi.$$

2.3.6 The Choice Problem of the Hedonic Producer

In the short term, four (4) alternatives could occur. The first alternative (B_1) is not to purchase insurance under no self-protection, the second alternative (B_2) is to buy insurance under no self-protection. The third alternative (B_3) is not to buy insurance under self-protection, and the fourth alternative (B_4) is to buy insurance under self-protection. The alternatives are mutually independent and exhaustive. Therefore, the economic agent's expected utility function is separable with respect to these alternatives. If the producer does not have self-protection, only alternatives B_1 and B_2 are possible. If the producer has self-protection, then options B_3 and B_4 are chosen.

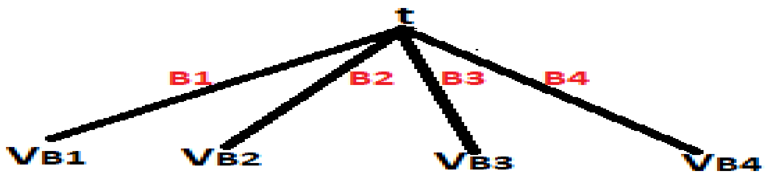

Figure 1: Illustration of the Choice of the Myopic Producer

Figure 1 illustrates the short-term decision making process of the producer at time t. In the short-term, under risk, the economic agent evaluates his optimal expected utility separately across alternatives B_1, B_2, B_3, and B_4 and chooses the alternative providing the highest expected utility. Under uncertainty, the producer chooses the alternative providing the highest utility among alternatives B_1, B_2, B_3, and B_4.

Alternative B_1: The contemporary utility and expected utility without insurance under no self-protection is defined below:

$$U_{i,v,t}(B_1) \equiv U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}])$$
$$EU_{i,v,t}(B_1) \equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}]) \tag{22}$$

Alternative B_2: The contemporary utility and expected utility with insurance under no self-protection is defined below:

$$U_{i,v,t}(B_2) \equiv U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}])EU_{i,v,t}(B_2) \quad \equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}]) \tag{23}$$

Alternative B_3: The contemporary utility and expected utility without insurance under self-protection is defined below:

$$U_{i,v,t}(B_3) \equiv U_{i,v,t}[\Pi^{l=0}_{i,v=1,t}]) \qquad (24)$$
$$EU_{i,v,t}(B_3) \equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=1,t}])$$

Alternative B_4**:** The contemporary utility and expected utility with insurance under self-protection is defined below:

$$U_{i,v,t}(B_4) \equiv U_{i,v,t}[\Pi^{l=1}_{i,v=1,t}]) \qquad (25)$$
$$EU_{i,v,t}(B_4) \equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=1,t}])$$

2.3.7 The Utility Maximization Problem of the Hedonic Producer under Risk

In the short-term (v=0), the myopic producer does not put any value on the future. The short-term choice of the producer under risk consists in maximizing their expected utility of profit by choosing the optimal $X_{i,t}$ and $Y_{i,t}$ given ψ_i, $\Gamma_{l,i,t}$, $\vartheta_{i,t}$, $\Gamma_{\phi i,t}$ ($\mu_{\phi\, i,t}$, $\sigma_{\phi_{i,t}}$), $E(P_{i,t})$, $E(R_{i,t})$. At time t, the producer solves four maximization problems and chooses the alternative that provides him with the highest expected utility.

(i) For alternative B_1 corresponding to the case where the myopic producer does not purchase insurance under no self-protection, the maximization problem is the following for each period t:

$$\max_{\{X_{i,t}, Y_{i,t}\}} E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}]),$$
$$st. F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \Gamma_{\phi_{i,t}}) = 0, \qquad (26)$$
$$Y_{i,t} > 0, X_{i,t} > 0.$$

with $\Pi^{l=0}_{i,v=0,t}$ defined in equation (6) and $E_{\phi_{i,t}}(U_{i,v,t})$ defined in equation (22) and the provisional production function discussed in eq. (12). In Lagrangian form, problem (26) becomes:

$$\max_{\{X_{i,t}, Y_{i,t}\}} E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}]) + \lambda * F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \Gamma_{\phi_{i,t}}), \qquad (27)$$
$\lambda \in R$, The Lagrange Multiplier.

Solving the problem described in eq. 27, we obtain the following optimal solutions:

$$Y^*_{B_1} = Y^*_{B_1}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i),$$
$$X^*_{B_1} = X^*_{B_1}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i). \qquad (28)$$

where $Y^*_{B_1}$ is the optimal output vector under alternative B_1. $X^*_{B_1}$ is the optimal input vector under alternative B_1. The optimal indirect expected utility of the producer under alternative B_1 is the following:

$$EU^*_{i,v,t}(B_1) \equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}(Y^*_{B_1}, X^*_{B_1})]). \qquad (29)$$

(ii) For alternative B_2 corresponding to the case where the myopic producer purchases insurance under no self-protection, the maximization problem is the following:

$$\max_{\{X_{i,t},Y_{i,t},c_{i,t}\}} E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}]),$$
$$F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \Gamma_{\phi_{i,t}}, \psi_i) = 0, \tag{30}$$
$$Y_{i,t} > 0, X_{i,t} > 0.$$

with $\Pi^{l=1}_{i,v=0,t}$ defined in eq. (7) and $E_{\phi_{i,t}}(U_{i,v,t})$ defined in eq. (23). Using the same strategy used to solve case B_1, we obtain the optimal solutions and optimal indirect utility under alternative B_2,

$$\begin{aligned} Y^*_{B_2} &= Y^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\ X^*_{B_2} &= X^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\ c^*_{B_2} &= c^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\ EU^*_{i,v,t}(B_2) &\equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_2}, X^*_{B_2}, c^*_{B_2})]). \end{aligned} \tag{31}$$

(iii) For alternative B_3 corresponding to the case where the myopic producer does not purchase insurance under self-protection, the maximization problem is the following:

$$\max_{\{X_{i,t},Y_{i,t},dK_{i,t}\}} E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=0}_{i,v=1,t}]),$$
$$F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \Gamma_{\phi_{i,t}}, \psi_i) = 0, \tag{32}$$
$$Y_{i,t} > 0, X_{i,t} > 0.$$

with $\Pi^{l=0}_{i,v=1,t}$ defined in eq. (8) and $E_{\phi_{i,t}}(U_{i,v,t})$ defined in eq. (24). Using the same strategy used to solve case B_1, we obtain the optimal solutions and optimal indirect utility under alternative B_3,

$$\begin{aligned} Y^*_{B_3} &= Y^*_{B_3}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\ X^*_{B_3} &= X^*_{B_3}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\ EU^*_{i,v,t}(B_3) &\equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_3}, X^*_{B_3}]). \end{aligned} \tag{33}$$

(iv) For alternative B_4 corresponding to the case where the myopic producer purchase insurance under self-protection, the maximization problem is the following:

$$\max_{\{X_{i,t},Y_{i,t},c_{i,t},dK_{i,t}\}} E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}]),$$
$$F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \Gamma_{\phi_{i,t}}, \psi_i) = 0, \tag{34}$$
$$Y_{i,t} > 0, X_{i,t} > 0.$$

with $\Pi^{l=1}_{i,v=0,t}$ defined in eq. (9) and $E_{\phi_{i,t}}(U_{i,v,t})$ defined in eq. (25). Using the same strategy used to solve case B_1, we obtain the optimal solutions and optimal indirect utility under alternative B_2,

$$
\begin{aligned}
Y^*_{B_4} &= Y^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
X^*_{B_4} &= X^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
c^*_{B_4} &= c^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
EU^*_{i,v,t}(B_4) &\equiv E_{\phi_{i,t}}(U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_4}, X^*_{B_4}, c^*_{B_4})]).
\end{aligned} \tag{35}
$$

Under risk, the myopic producer with no self-protection compares $EU^*_{i,v,t}(B_2)$ and $EU^*_{i,v,t}(B_1)$ and chooses the alternative that provides him with the highest indirect expected utility. Whereas, the myopic producer with self-protection compares $EU^*_{i,v,t}(B_3)$ and $EU^*_{i,v,t}(B_4)$ and chooses the alternative that provides him with the highest indirect expected utility.

2.3.8 The Utility Maximization Problem of the Hedonic Producer under Uncertainty

The short-term choice of the producer under uncertainty consists in maximizing the worst-case scenario of their utility over uncertainty set $\Phi_{i,t}$. The producer chooses the optimal $X_{i,t}$ and $Y_{i,t}$ given ψ_i, $\Gamma_{l,i,t}$, $\vartheta_{i,t}$, $\Gamma_{\phi_{i,t}}(min(\phi_{i,t}), max(\phi_{i,t}))$, $E(P_{i,t})$, $E(R_{i,t})$. For each period t, the producer solves two robust optimization problems and chooses the alternative that provides him with the highest utility.

(i) For alternative B_1 corresponding to the case where the myopic producer does not purchase insurance and does not do any self-protection, the robust optimization problem is the following for each period t:

$$
\begin{aligned}
&\max_{\{X_{i,t}, Y_{i,t}\}} \min_{\phi_{i,t}\in\Phi_{i,t}} U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}],\\
&st. F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \phi_{i,t}) = 0,\\
&Y_{i,t} > 0, X_{i,t} > 0.
\end{aligned} \tag{36}
$$

For alternative B_1, the optimal solutions and optimal indirect utility of the producer under robust optimization is the following:

$$
\begin{aligned}
Y^*_{B_1} &= Y^*_{B_1}(\vartheta_0, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i),\\
X^*_{B_1} &= X^*_{B_1}(\vartheta_0, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i),\\
U^*_{i,v,t}(B_1) &\equiv U_{i,v,t}[\Pi^{l=0}_{i,v=0,t}(Y^*_{B_1}, X^*_{B_1})].
\end{aligned} \tag{37}
$$

(ii) For alternative B_2 corresponding to the case where the myopic producer purchase insurance but does not do any self-protection, the robust optimization problem is the following for each period t:

$$
\begin{aligned}
&\max_{\{X_{i,t}, Y_{i,t}, c_{i,t}\}} \min_{\phi_{i,t}\in\Phi_{i,t}} U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}],\\
&st. F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \phi_{i,t}) = 0,\\
&Y_{i,t} > 0, X_{i,t} > 0.
\end{aligned} \tag{38}
$$

For alternative B_2, the optimal indirect utility of the producer under robust optimization is the following:

$$
\begin{aligned}
Y^*_{B_2} &= Y^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
X^*_{B_2} &= X^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
c^*_{B_2} &= c^*_{B_2}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
U^*_{i,v,t}(B_2) &\equiv U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_2}, X^*_{B_2}, c^*_{B_2})].
\end{aligned} \tag{39}
$$

(iii) For alternative B_3 the robust optimization problem is the following for t:

$$
\begin{aligned}
&\max_{\{X_{i,t}, Y_{i,t}, c_{i,t}\}} \min_{\phi_{i,t}\in\Phi_{i,t}} U_{i,v,t}[\Pi^{l=0}_{i,v=1,t}],\\
&st. F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \phi_{i,t}) = 0,\\
&Y_{i,t} > 0, X_{i,t} > 0.
\end{aligned} \tag{40}
$$

For alternative B_3, the optimal indirect utility of the producer under robust optimization is the following:

$$
\begin{aligned}
Y^*_{B_3} &= Y^*_{B_3}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
X^*_{B_3} &= X^*_{B_3}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
U^*_{i,v,t}(B_3) &\equiv U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_3}, X^*_{B_3})].
\end{aligned} \tag{41}
$$

(iv) For alternative B_4 corresponding to the case where the myopic producer does purchase insurance and invest in self-protection, the robust optimization problem is the following for each period t:

$$
\begin{aligned}
&\max_{\{X_{i,t}, Y_{i,t}, c_{i,t}\}} \min_{\phi_{i,t}\in\Phi_{i,t}} U_{i,v,t}[\Pi^{l=1}_{i,v=1,t}],\\
&st. F(Y_{i,t}, X_{i,t}|\vartheta_{i,t}, \phi_{i,t}) = 0,\\
&Y_{i,t} > 0, X_{i,t} > 0.
\end{aligned} \tag{42}
$$

For alternative B_4, the optimal indirect utility of the producer under robust optimization is the following:

$$
\begin{aligned}
Y^*_{B_4} &= Y^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
X^*_{B_4} &= X^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
c^*_{B_4} &= c^*_{B_4}(\vartheta_{i,t}, E(P_{i,t}), E(R_{i,t}), \Gamma_{\phi_{i,t}}, \psi_i, \Gamma_{l,i,t}),\\
U^*_{i,v,t}(B_4) &\equiv U_{i,v,t}[\Pi^{l=1}_{i,v=0,t}(Y^*_{B_4}, X^*_{B_4}, c^*_{B_4})].
\end{aligned} \tag{43}
$$

With uncertainty, the myopic producer under no self-protection compares $U^*_{i,v,t}(B_1)$, $U^*_{i,v,t}(B_2)$ and chooses the alternative that provides him with the highest indirect utility under robust optimization. Whereas, under self-protection, the myopic producer compares $U^*_{i,v,t}(B_3)$, $U^*_{i,v,t}(B_4)$

3 Application of the Theoretical Model to Farming

This section examines the applicability of the theory to farming, particularly in navigating the choice between self-protection and insurance amid risk and uncertainty. The tradeoff between investment and insurance is commonly observed in industries exposed to mitigatable risk or uncertainty. For instance, in the renewable energy sector, energy storage can mitigate intermittency issues caused by natural resource variability, while weather-index insurance offers a means to reduce weather-related economic losses (Dowling et al., 2020; Xiao et al., 2019). Similarly, businesses in winter sports, reliant on consistent snowfall, face decisions regarding investment in snowmaking technology or purchasing weather insurance to hedge against low snow levels (Steiger et al., 2021; MSI GuaranteedWeather, 2022).

In the farming context, economic producers seek to mitigate short-term weather-related losses, either through purchasing insurance policies or implementing self-protection measures. In this application, irrigation serves as a primary form of self-protection available to farmers. In regions with arid climates, weather insurance often necessitates irrigation infrastructure. For example, in Arizona, insurers require proof of adequate irrigation facilities and water availability for insured crops (RMA, 2021). Conversely, in regions with less reliance on irrigation, such as parts of Illinois, farmers may opt for non-irrigated practices. Thus, our analysis focuses on a non-arid area where farmers have the flexibility to decide on irrigation practices and the purchase of crop insurance.

3.1 Basic Assumptions

The following assumptions were made in the application of the theory to farming:

(i) At time t, the farmer plans to plant corn in an area without irrigation. The myopic farmer decides whether or not to get insurance and uses only nitrogen (N) as fertilizer.

(ii) We assume that the farmer faces a single source of risks and uncertainties: the farmer casts some doubt on the fluctuation of the precipitation rate during the growing period. We suppose that the farmer has no doubt and trusts the forecasts of the National Weather Service (NWS) regarding the average temperature and precipitation rate during the planting and harvest periods, as well as the forecast of the average temperature during the growing period.

(iii) The farmer evaluates the last ten years' average precipitation rate during the growing period ($\tilde{\mu}_{W_g}$) and the last ten years' variance of the precipitation rate during the growing period ($\tilde{\sigma}_{W_g}$). $\tilde{W}_g^{min}$ ($\tilde{W}_g^{max}$) is the last ten years' average minimum (maximum) precipitation rate during the growing period. The farmer believes that $\tilde{\mu}_{W_g}$, $\tilde{\sigma}_{W_g}$, $\tilde{W}_g^{min}$, $\tilde{W}_g^{max}$ have been at adequate levels over the past ten years. Therefore, he treats $\tilde{\sigma}_{W_g}$, $\tilde{\mu}_{W_g}$, $\tilde{W}_g^{min}$, $\tilde{W}_g^{max}$ as reference points for irrigation.

(iii) Let $\mu_{W_{g,t}}$, $\sigma_{W_{g,t}}$, $W_{g,t}^{min}$, and $W_{g,t}^{max}$ be respectively the mean, the variance, the minimum, and the maximum precipitation rate at time t. When the weather can be described probabilistically, we assume that the farmer anticipates that the standard deviation of precipitation ($\sigma_{W_{g,t}}$) during the growing time increases with V_t. Similarly, the farmer anticipates that at time t, the mean precipitation ($\mu_{W_{g,t}}$)

during the growing time decreases with V_t. Whereas, when the weather is uncertain and cannot be described probabilistically, the farmer anticipates that the range of the precipitation during the growth period increases with V_t, which means $W_{g,t}^{min}$ gets lower and $W_{g,t}^{max}$ gets higher with V_t.

(iv) Under risk, irrigation has a goal to maintain the mean water rate on the field ($\mu_{W_{tot}}$) within a range of 10 % compared to $\tilde{\mu}_{W_g}$, and the variance of the total rate on the field ($\sigma_{W_{tot}}$) within a range of 10 % compared to the reference level $\tilde{\sigma}_{W_g}$. Under uncertainty, irrigation has for goal to maintain the minimum total water rate on the field at time t (W_{tot}^{min}) within a range of 10 % compared to the reference level $\tilde{W}_g^{min}$, and the maximum total water rate on the field at time t (W_{tot}^{max}) within a range of 10 % compared to the reference level $\tilde{W}_g^{max}$.

(v) The farmer purchases yield insurance. The yield guarantee and premium rate depend on the coverage level chosen by the farmer and is set by the RMA.

(vi) To capture the effect of stochastic climate variables, we assume that the average input and output prices are fixed for the period under study.

3.2 Functional Form Specifications

3.2.1 The Production Function Without Irrigation

Using the multiple output production defined in the theory and assuming that $S_{i,t} = 0$ as the farmer is purely hedonic, we estimate the quadratic directional output distance production function[2].

During the growth period, the inputs available to the farmer to grow corn are (i) X_1 = N (Nitrogen), (ii) $X_2 = T_g$ (Growing Time Temperature), (iii) $X_3 = W_g$ (Precipitation during growing time). The output is corn yield (Y).

$$\begin{aligned} Y &= \beta_0 + \beta_1(N) + \beta_2(W_g) + \beta_3(N)^2 + \beta_4(W_g)^2 + \beta_5[N * (W_g)] + \beta_6(T_p) + \\ &\beta_7(T_p)^2 + \beta_8(T_g) + \beta_9(T_g)^2 + \beta_{10}(T_g * W_g) + \beta_{11}(T_g * N) + \beta_{12}(T_h) + \\ &\beta_{13}(T_h)^2 + \beta_{14}(W_p) + \beta_{15}(W_p)^2 + \beta_{16}(W_h) + \beta_{17}(W_h)^2 + \epsilon \end{aligned} \tag{44}$$

Eq. (44) is a direct specification[3] to estimate a yield curve as a function of inputs and climate factors such as temperature and precipitation. Past papers that have estimated corn yield response to nitrogen have used a quadratic yield function (Llewelyn and Featherstone,1996; Bert et al., 2007; Thorp et al., 2008; Paz et al., 1999, Batchelor et al., 2002, Link et al., 2006, Dogan et al., 2006, Miao et al., 2006). Researchers have found quadratic forms to be more suitable than linear response functions for modeling corn yield response to N (Bullock and Bullock, 1994; Cerrato and Black-

[2]The specification of the multiple output function in a quadratic form can be found in the appendix

[3]For indirect estimation of the crop production function, this can be achieved through the specification of appropriate dual formulations, such as the cost or profit functions (Blackborby, Primont, and Russell; Diewert 1971, 1974; Jorgenson and Lau, 1974). The indirect production function is dependent on the input prices (r), the profit functions (Π), the fixed capital (ϑ), and time t, i.e., $y(r, \pi, K, t)$. The production function can then be econometrically estimated using a translog, CES, or Lewbel (Hilmer et Holt, 2005). Since we do not have farm-level profit data, the indirect estimation will not be used for our simulation.

mer, 1990; Bullock and Bullock, 1994; Roberts et al., 2002; Boyer et al. 2013; Laila Puntel et al., 2016). Boyer and al. (2013) and Lailai Puntel (2016) used only nitrogen rates (N) applied to corn and (N^2) in their estimation of corn yield response to nitrogen. Llewelyn et al. (1996) estimated corn yield using nitrogen rates, water rates, and the square and interaction terms of nitrogen and water rates. Long-term field experiments on corn have been undertaken in Missouri (Sandborn Field), Nebraska (Knorr-Holden), and Illinois (Morrow's plot) (Scofield Holden., 1927; Aref Wander., 1997; Bijesh et al., 2021). Yield is affected by climatic conditions at planting and harvest, therefore we included T_p and W_p to eq. (44). Similarly, we added T_h and W_h to capture the effect of soil conditions at harvest on yield. We focus on county-level data as representative of actual rather than experimental practice. We estimate county-level yield response to nitrogen and weather as specified in the following equation for an area with low to no irrigation (Illinois, Indiana, Ohio, and Pennsylvania):

$$Y_{i,v=0,t} = \hat{\beta}_0 + \hat{\beta}_1(N_{i,t}) + \hat{\beta}_2(W_{g,t}) + \hat{\beta}_3(N_{i,t}))^2 + \hat{\beta}_4(W_{g,t})^2 + \hat{\beta}_5[(N_{i,t}) * (W_{g,t})] + \hat{\beta}_6(T_{p,t}) + \hat{\beta}_7(T_{p,t})^2 + \hat{\beta}_8(T_{g,t}) + \hat{\beta}_9(T_{g,t})^2 + \hat{\beta}_{10}(T_{g,t} * W_{g,t}) + \hat{\beta}_{11}(T_{g,t} * N_{i,t})) + \hat{\beta}_{12}(T_{h,t}) + \hat{\beta}_{13}(T_{h,t})^2 + \hat{\beta}_{14}(W_{p,t}) + \hat{\beta}_{15}(W_{p,t})^2 + \hat{\beta}_{16}(W_{h,t}) + \beta_{17}(W_{h,t})^2 + \epsilon \quad (45)$$

Since we do not have data on irrigation rates for corn at the county level, we focus our study on major corn producers located in counties with very low to no irrigation. Those counties are within the states of Illinois, Ohio, and Pennsylvania. Precipitation is the water rate applied to corn in counties with no irrigation. Precipitation and temperature data are available from the Prism database of Oregon University.

$Y_{i,v=0,t}$ is the county-level corn yield from 1987-2012 for the low to no irrigation area. $Y_{i,v=0,t}$ is available in the quick stat database of the USDA/NASS. $N_{i,t}$ is the nitrogen rate used by each county for producing corn from 1987-2012. The county-level nitrogen rate was estimated using the procedure described by Yushu et al. (2021). They use a top-down area-based approach that allocates Nitrogen fertilizer inputs into corn-producing areas by combining state-level crop-specific nitrogen fertilizer application rates (NASS) and percentage of the area receiving N fertilizer (NASS/USDA) with the county-level proportion of crop-specific planted area (USGS).

$W_{p,t}$, $W_{g,t}$ and $W_{h,t}$ are the average precipitation rate in the area during the planting season, the growing season, and the harvest season respectively. $T_{p,t}$, $T_{g,t}$ and $T_{h,t}$ are the average temperature during the planting, the growing, and the harvest season, respectively. We included $T_{p,t}$ and $W_{p,t}$ because soil conditions at planting are affected by temperature and precipitation. Similarly, we added $T_{h,t}$ and $W_{h,t}$ to capture the effect of soil conditions at harvest on yield. Precipitation and temperature data are available from the Prism database of Oregon University. The summary statistics of the empirical variables are available in Table 1.

Results of the econometrics estimation

Equation (45) was estimated using a fixed effect model at the year and state level. The results show that the nitrogen and weather variables significantly impact county-level yield (Table 2). Results suggest with 99 % confidence that nitrogen, temperature, and rainfall negatively affect yield. That means the relationship is positive for low values of nitrogen and rainfall, but the relationship becomes negative for high values. The model accounts for the properties of the quadratic form that imposes non-zero elasticity of substitution among factors.

However, to evaluate the effects of variation in these point estimates, we treat these parameters as random with mean equal to point estimate and variance based on estimated variance. We assume that each $\hat{\beta}$ are drawn from a normal distribution with mean $\mu_{\hat{\beta}}$ and standard deviation $\sigma_{\hat{\beta}}$. ie $\hat{\beta} \sim N(\hat{\beta}, \sigma_{\hat{\beta}})$.

3.2.2 The Production Function with Irrigation

The total rate of water on the field ($W_{tot,t}$) is the sum of the rainfall rate ($W_{g,t}$) plus the irrigation rate ($W_{i,t}$). The farmer chooses an optimal irrigation rate $W^*_{i,v,t}$ to maintain the variance and the mean of the water rate on the field within a range of 10 % with respect to their reference levels $\sigma_{\tilde{W}_g}$ and $\mu_{\tilde{W}_g}$. The yield function($Y_{i,v=1,t}$) defined in Equation (45) becomes with irrigation:

$$Y_{i,v=1,t} = \hat{\beta}_0 + \hat{\beta}_1(N_{i,t}) + \hat{\beta}_2(W_{tot,i,t}) + \hat{\beta}_3(N_{i,t}))^2 + \hat{\beta}_4(W_{tot,i,t})^2 + \hat{\beta}_5[N_{i,t} * W_{tot,i,t}] + \hat{\beta}_6(T_{p,t}) + \hat{\beta}_7(T_{p,t})^2 + \hat{\beta}_8(T_{g,t}) + \hat{\beta}_9(T_{g,t})^2 + \hat{\beta}_{10}(T_{g,t} * W_{tot,i,t}) + \hat{\beta}_{11}(T_{g,t} * N_{i,t}))+ \hat{\beta}_{12}(T_{h,t}) + \hat{\beta}_{13}(T_{h,t})^2 + \hat{\beta}_{14}(W_{p,t}) + \hat{\beta}_{15}(W_{p,t})^2 + \hat{\beta}_{16}(W_{h,t}) + \beta_{17}(W_{h,t})^2 + \epsilon \quad (46)$$

3.2.3 The Stochastic Distributions and the Uncertainty Sets

The Distribution of the Precipitation during Growing Time without Irrigation

The farmer has doubts about the precipitation rate during the growing period. The precipitation rate during the growing season (W_g) is stochastic with distribution $T(\Gamma_{W_g})$. Following Weaver et al. (2001), we specify the distribution of W_g as a normal distribution:

$$T(\Gamma_{W_{g,t}}|V_t) = \frac{1}{\sigma_{W_{g,t}} * \sqrt{2 * \pi}} * e^{-\frac{(W_{g,t} - \mu_{W_{g,t}})^2}{2 * \sigma_{W_{g,t}}}}. \quad (47)$$

We note that $\sigma_{W,g,t}$ ($\mu_{W,g,t}$) increases (decreases) over time due to the increasing stock of carbon emissions (V_t) over each period.

$$\mu_{W_{g,t+1}} = \mu_{W_{g,t}} - \kappa_\mu * V_t, \quad \sigma_{W_{g,t+1}} = \sigma_{W g,t} + \kappa_\sigma * V_t \quad (48)$$

where κ_μ (κ_σ) represents the rate of decrease (increase) of $\mu_{W\,g,t}$ ($\sigma_{W g,t}$) as the stock of carbon emission (V_t) increases over time. $\mu_{W\,g,0}$ is the current average rainfall rate, and $\sigma_{W\,g,0}$ is the current rainfall variance in the area under study.

The Distribution of the Precipitation during Growing Time with Irrigation

The distribution remains normal, but the mean and the variance of the water rate are replaced by $\mu_{W\,tot,t}$, and $\sigma_{W\,tot,t}$. $\vartheta_{v=1}$ is the efficiency of the irrigation technology.

$$T(\Gamma_{W_{tot,t}}) = \frac{1}{\sigma_{W_{tot,t}} * \sqrt{2*\pi}} * e^{-\frac{(W_{tot,t}-\mu_{W_{tot,t}})^2}{2*(\sigma_{W_{tot,t}})^2}}. \quad (49)$$

Where $W_{tot,t}$, $\mu_{tot,t}$, $\sigma_{tot,t}$ are defined as follow:

$$\begin{aligned}
&(\text{i})\, W_{tot,t} = W_{g,t} + W_{i,t} * \vartheta_{v=1},\\
&(\text{ii})\, \mu_{W_{tot,t}} = \mu_{W_{g,t}} + W_{i,t} * \vartheta_{v=1},\\
&(\text{iii})\, \sigma_{W_{tot,t}} = \sigma_{W_{g,t}} - W_{i,t} * \vartheta_{v=1},\\
&(\text{iv})\, 0.9 * \tilde{\mu}_{W_g} \leq \mu_{W_{tot,t}} \leq 1.1 * \tilde{\mu}_{W_g},\\
&(\text{v})\, 0.9 * \tilde{\sigma}_{W_g} \leq \sigma_{W_{tot,t}} \leq 1.1 * \tilde{\sigma}_{W_g},
\end{aligned} \quad (50)$$

From condition (50), we can deduce that the rate of irrigation of the farmer ($W_{i,t}$) is bounded as follows:

$$\begin{aligned}
&(\text{i})\, \frac{0.9 * \tilde{\mu}_{W_g} - \mu_{W_{g,t}}}{\vartheta_{v=1}} \leq W_{i,t} \leq \frac{1.1 * \tilde{\mu}_{W_g} - \mu_{W_{g,t}}}{\vartheta_{v=1}},\\
&(\text{ii})\, \frac{-1.1 * \tilde{\sigma}_{W_g} + \sigma_{W_{g,t}}}{\vartheta_{v=1}} \leq W_{i,t} \leq \frac{-0.9 * \tilde{\sigma}_{W_g} + \sigma_{W_{g,t}}}{\vartheta_{v=1}}
\end{aligned} \quad (51)$$

The two inequalities in equation (51) can be combined as follows:

$$\frac{0.9 * \tilde{\mu}_{W_g} - 1.1 * \tilde{\sigma}_{W_g} - \mu_{W_{g,t}} + \sigma_{W_{g,t}}}{2 * \vartheta_{v=1}} \leq W_{i,t} \leq \frac{1.1 * \tilde{\mu}_{W_g} - 0.9 * \tilde{\sigma}_{W_g} - \mu_{W_g} + \sigma_{W_g}}{2 * \vartheta_{v=1}}. \quad (52)$$

$\mu_{W_{tot,t}}$ be the post-irrigation mean total water rate, and $\sigma_{W_{tot,t}}$ be the post-irrigation total water rate variance. As discussed in the assumption section, irrigation has for goal, under risk, to maintain the mean water rate on the field within a range of 10 % from $\tilde{\mu}_{W_g}$, and the variance within a range of 10 % from $\tilde{\sigma}_{W\,g}$.

The Uncertainty Set of Precipitation during Growing Time without Irrigation

We consider that precipitation during growing time is within the irrigation uncertainty set $\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}]$. We assume that the increase in the stock of carbon emission (V_t) widens the box uncertainty set over time.

$$\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}] = [W^{min}_{g,t-1} - \kappa_{min} * V_t, W^{max}_{g,t-1} + \kappa_{max} * V_t]. \quad (53)$$

κ_{min} (κ_{max})is the rate of decrease (increase) of $W^{min}_{g,t}$ ($W^{max}_{g,t}$) as V_t increases.

The Uncertainty Set of Precipitation during Growing time with Irrigation

Irrigation can reduce the size of the uncertainty set of rainfall during growing time

$$\begin{aligned} &\Phi_t^{ir} = [W_{tot,t}^{min}, W_{tot,t}^{max}],\\ &W_{tot,t}^{min} = W_{g,t-1}^{min} - \kappa_{min} * V_t + \mathrm{W}_{i,t} * \vartheta_{i,t},\\ &W_{tot,t}^{max} = W_{g,t-1}^{max} + \kappa_{max} * V_t - \mathrm{W}_{i,t} * \vartheta_{i,t},\\ &st.\ 0.9 * \tilde{W}_g^{min} < W_{tot,t}^{min} < 1.1 * \tilde{W}_g^{min},\\ &0.9 * \tilde{W}_g^{max} < W_{tot,t}^{max} < 1.1 * \tilde{W}_g^{max}. \end{aligned} \tag{54}$$

From condition (54), we can deduce that the rate of irrigation of the farmer ($W_{ir,i,t}$) is bounded under uncertainty as follows:

$$\begin{aligned} &(i)\ \frac{0.9 * \tilde{W}_g^{min} - W_{g,t}^{min}}{\vartheta_{i,t}} \le W_{i,t} \le \frac{1.1 * \tilde{W}_g^{min} - W_{g,t}^{min}}{\vartheta_{i,t}},\\ &(ii)\ \frac{-1.1 * \tilde{W}_g^{max} + W_{g,t}^{max}}{\vartheta_{i,t}} \le W_{i,t} \le \frac{-0.9 * \tilde{W}_g^{max} + W_{g,t}^{max}}{\vartheta_{i,t}}. \end{aligned} \tag{55}$$

The two inequalities in equation (55) can be combined as follows:

$$\begin{aligned} &\frac{0.9 * \tilde{W}_g^{min} - 1.1 * \tilde{W}_g^{max} - W_{g,t}^{min} + W_{g,t}^{max}}{2 * \vartheta_{i,t}} \le \mathrm{W}_{i,t} \le \ldots\\ &\frac{1.1 * \tilde{W}_g^{min} - 0.9 * \tilde{W}_g^{max} - W_{g,t}^{min} + W_{g,t}^{max}}{2 * \vartheta_{i,t}}. \end{aligned} \tag{56}$$

Replacing $W_{g,t}^{min}$ and $W_{g,t}^{max}$ by their definitions in eq. (53), the inequality in (56) becomes:

$$\begin{aligned} &\frac{0.9 * \tilde{W}_g^{min} - 1.1 * \tilde{W}_g^{max} - (W_{g,t-1}^{min} - \kappa_{min} * V_t) + (W_{g,t-1}^{max} + \kappa_{max} * V_t)}{2 * \vartheta_{i,t}} \le \mathrm{W}_{i,t} \le \ldots\\ &\frac{1.1 * \tilde{W}_g^{min} - 0.9 * \tilde{W}_g^{max} - (W_{g,t-1}^{min} - \kappa_{min} * V_t) + (W_{g,t-1}^{max} + \kappa_{max} * V_t)}{2 * \vartheta_{i,t}}. \end{aligned} \tag{57}$$

3.2.4 The Insurance Premium Curve and the Yield Guarantee

When a farmer chooses to purchase yield insurance, he faces a premium rate schedule and a yield guarantee schedule. The RMA determines these schedules as a function of the coverage level ($c_{i,t}$) chosen by the farmer. The crop insurance decision tool allows us to find the premium rate schedule for yield insurance and the guarantee yield (Farmdoc, 2020).

We fit an exponential curve in the schedules to establish a smooth relationship between the premium rate vs. the coverage level ($c_{i,t}$) (Figure 3, see appendix), the guarantee yield vs. the coverage level ($c_{i,t}$) (Figure 4, see appendix). The relationship between the yield insurance premium rate and coverage level for corn in the area under study has the following form:

$$\rho_{i,t} \equiv \rho_y(cv_{i,t}) = 0.0139 * e^{0.074 * cv_{i,t}} \text{ with } R^2 = 0.911. \tag{58}$$

The relationship between the yield guarantee vs. coverage level for corn in the area under study is:

$$y_g(cv_{i,t}) = 2768 * e^{0.0151 * cv_{i,t}} \text{ with } R^2 = 0.8244. \quad (59)$$

3.2.5 The Profit of the Myopic Farmer

The Profit with No Insurance and no Irrigation of the Myopic Farmer (Alternative B_1)

If farmer i does not insure his field during period t, but does not have an irrigation system, $\Pi^{I=0}_{i,v=0,t}$ is the myopic farmer's profit without insurance and without irrigation obtained during period t. We assume the farmer plants corn on his field. He uses nitrogen as an input, where $N_{i,t}$ is the nitrogen rate used by the farmer. r_N is the nitrogen price. We assume the input price is fixed over time. E($P_{i,t}$) is the subjective price expectation defined as a 10-year Simple Moving Average.

$$\Pi^{I=0}_{i,v=0,t} = A_i * [E(P_{i,t}) * Y_{i,v=0,t} - R_{i,N} * N_{i,t}]. \quad (60)$$

where A_i is the planted area of the field. $y_{i,t}$ is the production per unit acre.

The Profit with Insurance and no Irrigation of the Myopic Farmer (Alternative B_2)

If the myopic farmer is insured during period t and does not have an irrigation system, $\Pi^{I=1}_{i,v=0,t}$ is the farmer's profit with insurance obtained from planting some acres of the crop during period t. $I_{i,t}$ is the indemnity received by the farmer for insuring a unit acre of crop during period t, and $\rho_{i,t}$ is the total premium that is supposed to be paid by the farmer to insure a unit acre of crop during period t. As it is known, $W_{g,t}$ is the exogenous event vector that can cause loss and thus is a focus on insurance. However, $W_{g,t}$ is not directly insured. Instead, the yield is insured. With yield insurance, the farmer gets indemnified when the actual yield is lower than the guaranteed yield. In that case, the farmer has to pay a deductible ($d_{i,t}$) and get reimbursed for the rest of the loss. As defined in the theory, $d_{i,t} = 1 - c_{i,t}$. The guaranteed yield (y_g) is given in eq. (59).

$$\begin{aligned} &\Pi^{I=1}_{i,v=0,t} = A_i * [(E(P_{i,t}) * Y_{i,v=0,t} - R_N * N_{i,t} + I_{i,t} - \rho_{i,t}], \\ &\text{if } y_g(c_{i,t}) > y_{i,t} \implies I_{i,t} \equiv (1 - d_{i,t}) * E(P_{i,t}) * (y_g(c_{i,t}) - y_{i,t}) > 0, \\ &\text{if } y_g(c_{i,t}) < y_{i,t} \implies I_{i,t} = 0. \end{aligned} \quad (61)$$

The Profit of the Myopic Farmer with No Insurance but with the Usage of Irrigation (Alternative B_3)

With irrigation, the farmer has to consider the cost of pumping water. The farmer has to select the irrigation water rate ($W_{ir,i,t}$) during period t.

$$\Pi^{I=0}_{i,v=1,t} \equiv A_i * [E(P_{i,t}) * y_{i,t} - R_N * N_{i,t} - R_W * W_{i,t}] \quad (62)$$

The Profit of the Myopic Farmer with Insurance but with the Usage of

Irrigation (Alternative B_4)

$$\Pi^{l=1}_{l,i,v=1,t} = A_i * [E(P_{i,t}) * y_{i,t} - R_N * N_{i,t} - R_W * W_{i,t} + I_{i,t} - \rho_{i,t}]$$
$$\text{if } y_g(c_{i,t}) > y_{i,t} \implies I_{i,t} \equiv (1 - d_{i,t}) * E(p) * (y_g(c_{i,t}) - y_{i,t}) > 0, \quad (63)$$
$$\text{if } y_g(c_{i,t}) < y_{i,t} \implies I_{i,t} = 0,$$

3.3 The Decision-Making Process of the Farmer

3.3.1 Under Risk

The expected utility maximization problem of the farmer without insurance and no irrigation:

$$\max_{N_{i,t}} EU(\Pi^{l=0}_{i,v=0,t})$$
$$st. Y_{i,v=0,t} \equiv Y_{i,v=0,t}(N_{i,t}|E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta}) \quad (64)$$
$$N_{min} \leq N_{i,t} \leq N_{max}$$

The expected utility maximization problem of the farmer with yield insurance and no irrigation:

$$\max_{N_{i,t}, c_{i,t}} EU(\Pi^{l=1}_{i,v=0,t})$$
$$st. Y_{i,v=0,t} \equiv Y_{i,v=0,t}(N_{i,t}, c_{i,t}|E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta}) \quad (65)$$
$$N_{min} \leq N_{i,t} \leq N_{max}$$
$$50 \leq c_{i,t} \leq 90$$

The expected utility maximization problem of the farmer with no insurance but with irrigation:

$$\max_{N_{i,t}, W_{i,t}} EU(\Pi^{l=0}_{i,v=1,t})$$
$$st. Y_{i,v=1,t} \equiv Y_{i,v=1,t}(N_{i,t}, W_{i,t}|\vartheta_{v=1}, E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta})$$
$$N_{min} \leq N_{i,t} \leq N_{max}$$
$$\frac{0.9 * \tilde{\mu}_{W_g} - 1.1 * \tilde{\sigma}_{W_g} - \mu_{W_{gt}} + \sigma_{W_{gt}}}{2 * \vartheta_{v=1}} \leq W_{i,t} \leq \frac{1.1 * \tilde{\mu}_{W_g} - 0.9 * \tilde{\sigma}_{W_g} - \mu_{W_g} + \sigma_{W_g}}{2 * \vartheta_{v=1}}. \quad (66)$$

The expected utility maximization problem of the farmer with yield insurance and with irrigation:

$$\max_{N_{i,t}, W_{i,t}, c_{i,t}} EU(\Pi^{l=1}_{i,v=1,t})$$
$$st. Y_{i,v=1,t} \equiv Y_{i,v=1,t}(N_{i,t}, W_{i,t}, c_{i,t}|\vartheta_{v=1}, E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta})$$
$$N_{min} \leq N_{i,t} \leq N_{max}$$
$$50 \leq c_{i,t} \leq 90$$
$$\frac{0.9 * \tilde{\mu}_{W_g} - 1.1 * \tilde{\sigma}_{W_g} - \mu_{W_{gt}} + \sigma_{W_{gt}}}{2 * \vartheta_{v=1}} \leq W_{i,t} \leq \frac{1.1 * \tilde{\mu}_{W_g} - 0.9 * \tilde{\sigma}_{W_g} - \mu_{W_g} + \sigma_{W_g}}{2 * \vartheta_{v=1}}. \quad (67)$$

3.3.2 Under Uncertainty

The robust utility maximization problem of the farmer without insurance and no irrigation:

$$\max_{\{N_{i,t}\}} \min_{W_{g,t}\in\Phi} U_{i,v,t}(\Pi^{I=0}_{i,v=0,t})$$
$$st. Y_{i,v=0,t} \equiv Y_{i,v=0,t}(N_{i,t}|E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta}) \tag{68}$$
$$\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}]$$
$$N_{min} \le N_{i,t} \le N_{max}$$

The robust utility maximization problem of the farmer with yield insurance and no irrigation:

$$\max_{N_{i,t}, c_{i,t}} \min_{W_{g,t}\in\Phi} U_{i,v,t}(\Pi^{I=1}_{i,v=0,t})$$
$$st. Y_{i,v=0,t} \equiv Y_{i,v=0,t}(N_{i,t}, c_{i,t}|E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta})$$
$$\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}] \tag{69}$$
$$N_{min} \le N_{i,t} \le N_{max}$$
$$50 \le c_{i,t} \le 90$$

The robust utility maximization problem of the farmer with no insurance but with irrigation:

$$\max_{N_{i,t}, W_{i,t}} \min_{W_{g,t}\in\Phi} U_{i,v,t}(\Pi^{I=0}_{i,v=1,t})$$
$$st. Y_{i,v=1,t} \equiv Y_{i,v=1,t}(N_{i,t}, W_{i,t}|\vartheta_{v=1}, E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta})$$
$$\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}]$$
$$N_{min} \le N_{i,t} \le N_{max}$$
$$\frac{0.9 * \tilde{W}^{min}_{g} - 1.1 * \tilde{W}^{max}_{g} - W^{min}_{g,t} + W^{max}_{g,t}}{2 * \vartheta_{i,t}} \le W_{i,t} \le \frac{1.1 * \tilde{W}^{min}_{g} - 0.9 * \tilde{W}^{max}_{g} - W^{min}_{g,t} + W^{max}_{g,t}}{2 * \vartheta_{i,t}}. \tag{70}$$

The robust utility maximization problem of the farmer with yield insurance and with irrigation:

$$\max_{N_{i,t}, W_{i,t}, c_{i,t}} \min_{W_{g,t}\in\Phi} U_{i,v,t}(\Pi^{I=1}_{i,v=1,t})$$
$$st. Y_{i,v=1,t} \equiv Y_{i,v=1,t}(N_{i,t}, W_{i,t}|\vartheta_{v=1}, E(T_{p,t}), E(T_{g,t}), E(T_{h,t}), E(W_{p,t}), E(W_{h,t}), \Gamma_{W_{g,t}}, \hat{\beta})$$
$$\Phi_t = [W^{min}_{g,t}, W^{max}_{g,t}]$$
$$N_{min} \le N_{i,t} \le N_{max}$$
$$50 \le c_{i,t} \le 90$$
$$\frac{0.9 * \tilde{W}^{min}_{g} - 1.1 * \tilde{W}^{max}_{g} - W^{min}_{g,t} + W^{max}_{g,t}}{2 * \vartheta_{i,t}} \le W_{i,t} \le \frac{1.1 * \tilde{W}^{min}_{g} - 0.9 * \tilde{W}^{max}_{g} - W^{min}_{g,t} + W^{max}_{g,t}}{2 * \vartheta_{i,t}}. \tag{71}$$

3.4 Algorithms

The simulation was conducted in Matlab (Version R2020a). We consider eight (8) cases to understand the behavior of farmers with different characteristics in the face of risk vs. uncertainty. The characteristics considered is the possession of an irrigation system as means of self-protection (v_i), and the availability of crop insurance for the farmer (I_i):

Case 1: Farmer does not possess an irrigation system for self-protection (v_i=0) and does not have the option to purchase crop insurance (I_i=0) in the face of precipitation risk during the growing period .

Case 2: Farmer does not possess an irrigation system for self-protection (v_i=0) but has the option to purchase crop insurance (I_i=1) in the face of precipitation risk during the growing period.

Case 3: Farmer possesses an irrigation system for self-protection (v_i=1) but does not have the option to purchase crop insurance (I_i=0) in the face of precipitation risk during the growing period.

Case 4: Farmer possesses an irrigation system for self-protection (v_i=1) and has the option to purchase crop insurance (I_i=1) in the face of precipitation risk during the growing period.

Case 5: Farmer does not possess an irrigation system for self-protection (v_i=0) and does not have the option to purchase crop insurance (I_i=0) in the face of uncertainty during the growing period .

Case 6: Farmer does not possess an irrigation system for self-protection (v_i=0) but has the option to purchase crop insurance (I_i=1) in the face of uncertainty during the growing period .

Case 7: Farmer possesses irrigation for self-protection (v_i=1), but does not purchase crop insurance (I_i=0) in the face of uncertainty during the growing period.

Case 8: Farmer possesses irrigation for self-protection (v_i=1), but does not purchase crop insurance (I_i=1) in the face of uncertainty during the growing period.

Four algorithms are established to evaluate the above cases: (i) The probability of the myopic farmer without irrigation to purchase crop insurance under risk, (ii) The probability of the myopic farmer with irrigation to purchase crop insurance under risk, (iii) The probability of the myopic farmer without irrigation to purchase crop insurance under uncertainty, (iv) The probability of the myopic farmer with irrigation to purchase crop insurance under uncertainty (see algorithms in the appendix) .

3.5 Hypothesis

The algorithms will be used to verify the following hypotheses:

Hypothesis I: The myopic farmer with irrigation will tend to purchase crop insurance less than the farmer without irrigation in the face of risk.

Hypothesis II: The myopic farmer with irrigation will tend to purchase crop insurance less than the farmer without irrigation in the face of uncertainty.

Hypothesis III: The farmer has a different behavior under risk vs uncertainty.

3.6 Numerical Simulation Results

The Behavior of the Myopic Farmer in the face of Risk

After conducting the simulation, the findings indicate that farmers without irrigation systems are inclined to increase their reliance on crop insurance when facing heightened precipitation risks during the growing season. Specifically, a 1 cm rise in precipitation risk standard deviation corresponds to an average 1.9% increase in the probability of crop insurance purchase. Notably, the probability of purchase exceeds 50%, indicating a tendency among non-irrigating farmers to opt for insurance (Figure 5).

Conversely, farmers equipped with irrigation systems adopt a dual approach. In fact, the t-test shows that there is a significant difference at the 99 % confidence level in the behavior of farmer with irrigation vs not under risk (Table 4). In instances where the standard deviation ranges between 2.5 and 5 cm, they tend to diminish their reliance on crop insurance by 8% for each 1 cm increase in standard deviation. This reduction is feasible due to the farmers' reliance on irrigation to mitigate climate risks. However, when precipitation standard deviation exceeds 5 cm, irrigating farmers elevate their crop insurance purchases by 24% for every 1 cm increase in precipiation risk (Figure 5) .

This implies that irrigation serves as a substitute for crop insurance in mitigating low to moderate levels of precipitation risk, yet acts as a complement at higher risk levels. There exists an optimal level of precipitation risk at which crop insurance transitions from being a substitute to irrigation to becoming a complement.

Considering that farmers with irrigation systems are inherently less risky compared to those without self-protection measures, all else being equal, the Risk Management Agency (RMA) may encounter adverse selection in instances of low to moderate precipitation risks, as the applicant pool will more likely be comprised of farmers lacking self-protection. However, as risk levels heighten, all farmers, including those without prior self-protection measures, will purchase insurance to bolster their protection levels creating a more diverse pool of insurance suscribers.

The Behavior of the Myopic Farmer in the face of Uncertainty

The simulation outcomes elucidate the response of farmers to uncertainty, revealing a consistent trend of decreased crop insurance purchases among both those employing self-protection measures and those without such measures. For instance, a 1 cm expansion in the uncertainty set corresponds to a 3.5% reduction in the likelihood of purchasing crop insurance for the farmer without irrigation (Figure 6). Moreover, the t-test shows that there is a significant difference at the 99 % confidence level in the behavior of farmer with irrigation vs not under uncertainty (Table 5). This decline in insurance uptake underscores the impact of uncertainty on farmers' loss mitigation strategies. Consequently, heightened uncertainty results in a diminished subscriber base for the Risk Management Agency (RMA), fragilizing the crop insurance program. In scenarios characterized by heightened uncertainty, farmers' reduced inclination to mitigate climate change risks leaves them more susceptible to the adverse consequences of disasters. This susceptibility not only poses immediate threats to agricultural productivity but also raises concerns regarding food security, particularly during extreme events characterized by heightened uncertainty. Thus, addressing uncertainty within agricultural loss mitigation frameworks is imperative for safeguarding against potential food insecurity.

The difference in the behavior of farmers in the face of risk vs uncertainty

The behavior of the myopic farmer under risk and uncertainty demonstrates distinct responses to varying levels of predictability and ambiguity in climate conditions. Under risk, farmers without irrigation systems tend to increase their reliance on crop insurance as precipitation risks heighten, while those with irrigation systems adopt a nuanced approach, adjusting their insurance purchases based on the severity of precipitation risks. This suggests that irrigation serves as both a substitute and complement to crop insurance, depending on the level of risk. Consequently, the Risk Management Agency (RMA) faces challenges of adverse selection under lower risk conditions but experiences a more diverse pool of insurance subscribers as risk levels escalate. Conversely, under uncertainty, farmers exhibit a general trend of decreased crop insurance purchases regardless of their self-protection measures. This reluctance to invest in insurance amid uncertainty diminishes the RMA's subscriber base and renders farmers more vulnerable to the impacts of climate-related disasters, thereby highlighting the critical importance of addressing uncertainty within agricultural loss mitigation frameworks to avoid potential food insecurity and safeguard agricultural productivity.

4 Policy Implications

The need to take into account type the stochastics faced by the producers and their level of self-protection in the design of climate policy

Policymakers need to understand that the decision of an individual to mitigate climate-related losses depend on their level of self-protection and the type of stochastics that they face.The behavior of the myopic farmer under risk and uncertainty demonstrates distinct responses to varying levels of predictability and ambiguity in climate conditions. Under risk, farmers without irrigation systems tend to increase their reliance on crop insurance as precipitation risks heighten, while those with irrigation systems adopt a nuanced approach, adjusting their insurance purchases based on the severity of precipitation risks. This suggests that irrigation serves as both a substitute and complement to crop insurance, depending on the level of risk. Consequently, the Risk Management Agency (RMA) faces challenges of adverse selection under lower risk conditions but experiences a more diverse pool of insurance subscribers as risk levels escalate. Conversely, under uncertainty, farmers exhibit a general trend of decreased crop insurance purchases regardless of their self-protection measures. This reluctance to invest in insurance amid uncertainty diminishes the RMA's subscriber base and renders farmers more vulnerable to the impacts of climate-related disasters, thereby highlighting the critical importance of addressing uncertainty within agricultural loss mitigation frameworks to avoid potential food insecurity and safeguard agricultural productivity. As explained by Ellsberg (1986), people who are "ambiguity averse" will increase the probability of an unfavorable prospect, which is not buying insurance in our case.

The Need to Reduce Uncertainty in Climate Change Forecasts

There is a need to reduce the uncertainty in weather forecasts as it makes producers more vulnerable to climate change. Myopic producers do not take any actions to mitigate climate-related losses under ambiguity. That shows under uncertainty, producers underestimate the effect of climate change on their production activities. Therefore, governments must encourage research to improve climate predictions and reduce the size of weather indicators uncertainty sets.

5 Concluding Remarks

In conclusion, this paper has examined the intricate dynamics between short-term insurance and self-protection strategies in mitigating weather-related risks and uncertainties. Against the backdrop of escalating global temperatures and shifting weather patterns, economic agents face the imperative of devising effective coping mechanisms. These mechanisms typically involve either purchasing financial instruments like derivatives or insurance policies or implementing self-protective measures to minimize potential losses. The study delves into the decision-making processes underlying these strategies, particularly exploring whether the choice between insurance and self-protection for short-term loss mitigation hinges on the type of stochastic loss encountered, distinguishing between risk and uncertainty.

We find that decision-makers operate within the realm of risk when they possess knowledge of the stochastic process generating outcomes and can estimate associated probabilities. Conversely, uncertainty arises when decision-makers lack awareness of the stochastic process but have subjective knowledge regarding potential outcomes. This paper underscores the importance of distinguishing between risk and uncertainty in understanding how economic agents navigate decision-making processes amidst weather fluctuations.

Our inquiry contributes to the existing literature by providing a theoretical framework for analyzing how weather stochastics influence producers' decisions regarding insurance and self-protection in the short term. Under conditions of risk, producers maximize expected utility within the expected utility framework, while under uncertainty, a robust optimization approach captures decision-making in the face of ambiguity. By delineating these decision-making processes and conducting simulations based on our theoretical models, we offer insights into farmers' choices between self-protection and insurance when confronted with risks versus uncertainties.

The behavior of farmers under risk and uncertainty demonstrates distinct responses to varying levels of predictability and ambiguity in climate conditions. Under risk, farmers tend to adjust their reliance on crop insurance based on the severity of precipitation risks, while under uncertainty, there is a general trend of decreased insurance purchases regardless of self-protection measures. It is essential for polycimakers to consider the type of stochastics faced by producers and their level of self-protection in climate policy formulation. Moreover, efforts to reduce uncertainty in climate change forecasts are crucial to mitigate vulnerability and safeguard agricultural productivity in the face of climate-related challenges. Addressing these issues is paramount in ensuring food security and resilience in the agricultural sector.

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Appendix: Algorithm I for the myopic farmer facing risk

Step 1: Set parameters

- Parameters: Generate 1000 estimates of $\beta^{\wedge}$, the parameters of F_G assuming $\beta^{\wedge} \sim N(\hat{\beta}, \sigma_{\hat{\beta}})$ (Table 2); Set the farmer's parameters: A_i, ψ_i, r_i. Set T_p, Th, W_p, W_h to their mean values. Set input price r_N and insurance deductible (d) (see table 3).

Step 2: Functional form specifications:

- Specify $T(\Gamma_{W_{g,t}}|V_t)$, $E(p_t)$, F_G, $\Pi^{l=0}_{i,v=0,t}$, and $\Pi^{l=1}_{i,v=0,t}$, ρ_y, y_g under risk
- Set the bounds for the control variables $N_{i,t}$, $Y_{i,t}$ and $c_{i,t}$.

Step 3: Start a Nested for loop for increasing σ_{W_g} and each set of β (1000 sets)

- Update the atmospheric stock V_t, the standard deviation $\sigma_{W_{g,t}}$, and $\mu_{W_{g,t}}$ for each t
- Update the set of β for the yield function
- Solve the myopic farmer problem using the Matlab NLP solver ”fmincon.”
- Save the choice of the farmer (Insurance vs. no Insurance) for each period t and each set of β.
- End Nested For Loop.

Step 4: Construct a hypothesis test to verify if the farmer's choice is statistically different across the periods.

Algorithm II for the myopic farmer facing uncertainty

Step 1: Set parameters

- Parameters are the same as in Algorithm 1

Step 2: Functional form specifications:

- Specify $\Phi_t(V_t)$, $E(p_t)$, F_G, , $\Pi^{l=0}_{i,v=0,t}$, and $\Pi^{l=1}_{i,v=0,t}$, ρ_y, y_g under uncertainty.
- Set the bounds for the control variables $N_{i,t}$, $Y_{i,t}$ and $c_{i,t}$.

Step 3: Start a Nested for loop for an increasing size of uncertainty sets for W_g and each set of β (1000 sets)

- Update the atmospheric stock V_t and the boundary limits of $W_{g,t}$ for each t, which
- Update the set of β for the yield function
- Solve the myopic farmer problem under uncertainty using the Matlab SFP solver ”fminimax.”
- Save the choice of the farmer (Insurance vs. no Insurance) for each period t and each set of β.
- End Nested For Loop.

Step 4: Construct a hypothesis test to verify if the farmer's choice is statistically different across the periods.

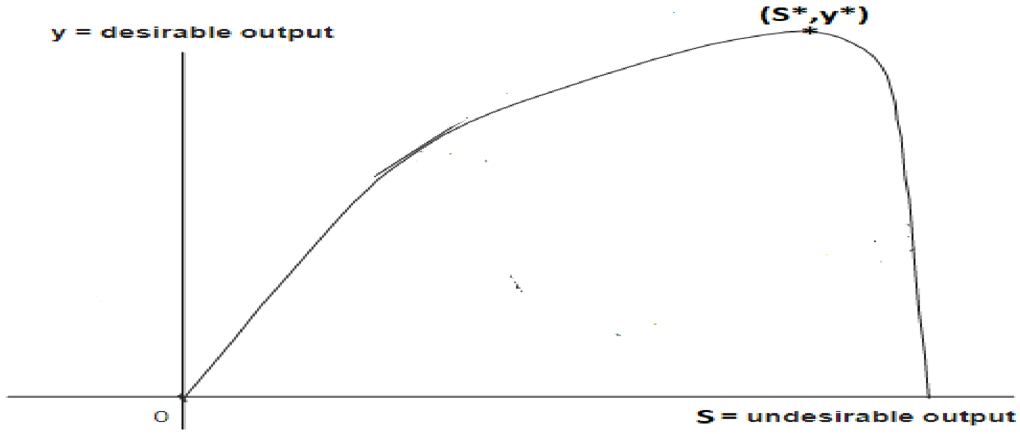

Figure 2: Directional Output Distance Function with Desirable and Undesirable Outputs

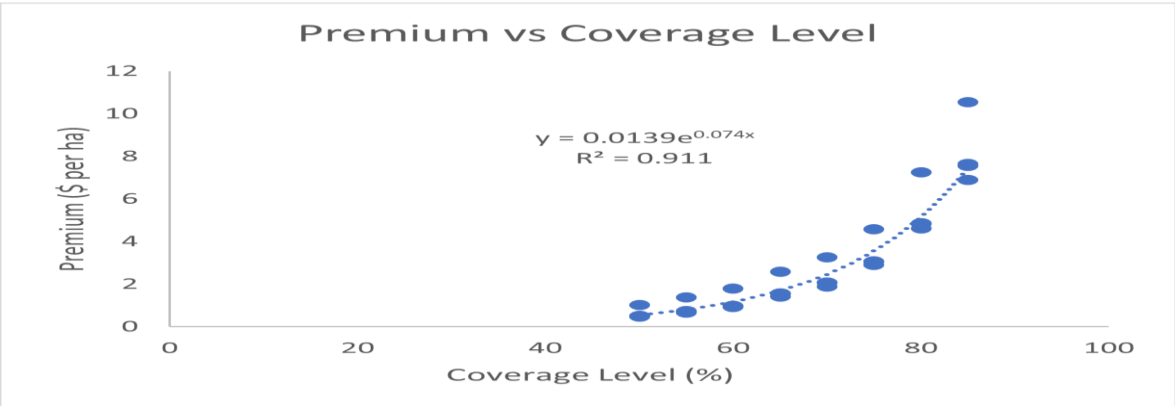

Figure 3: Premium vs. Coverage Level for 25 Counties Randomly Selected in the Area of Pennsylvania, Indiana, Ohio, and Illinois (Crop Insurance Decision Excel Tool, Farmdoc, Illinois, 2022)

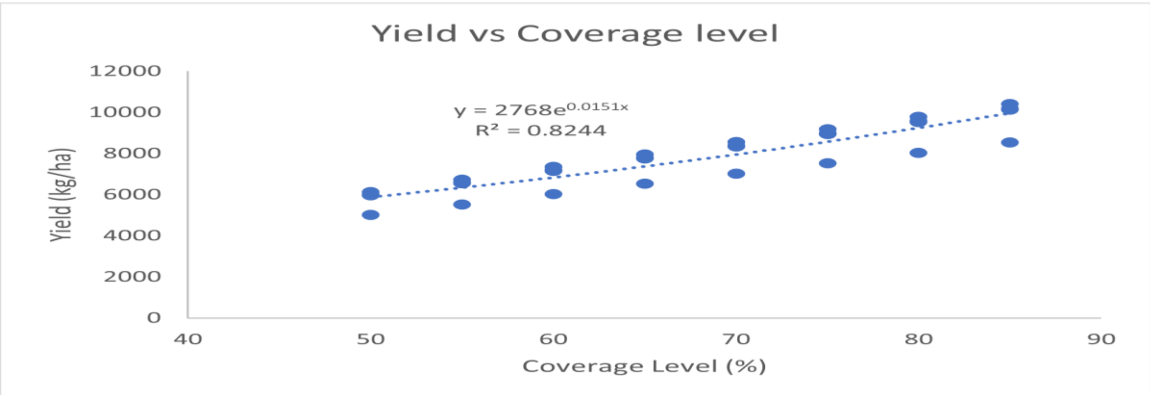

Figure 4: Yield vs. Coverage Level for 25 Counties Randomly Selected in the Area of Pennsylvania, Indiana, Ohio, and Illinois (Crop Insurance Decision Excel Tool, Farmdoc, Illinois, 2022)

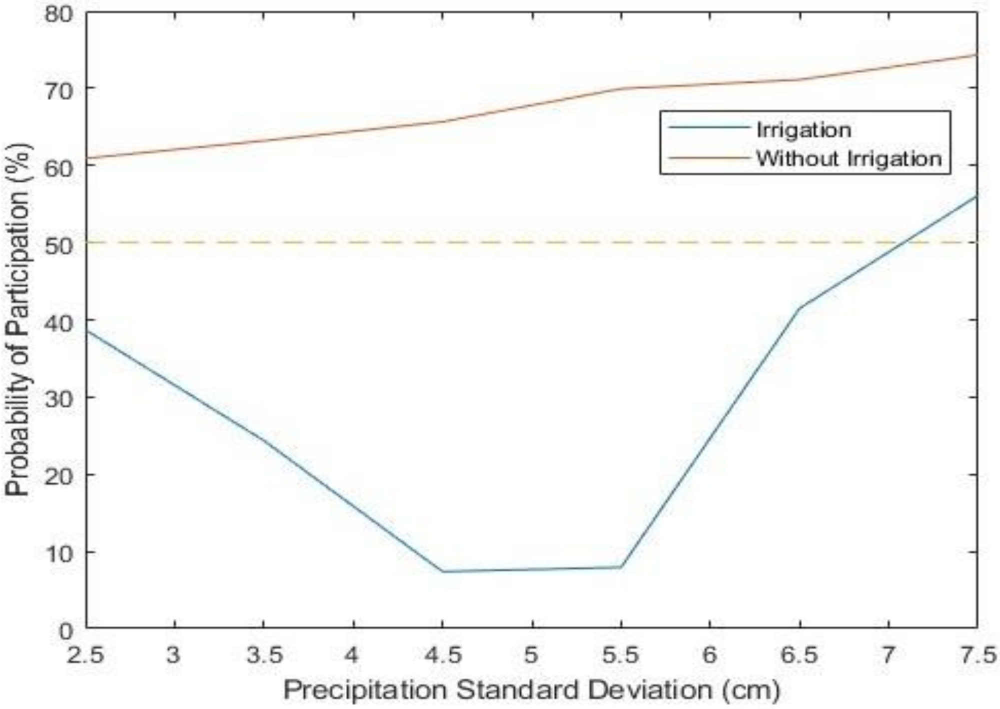

Figure 5: Probability of crop insurance purchase with an increase in the standard deviation of precipitation

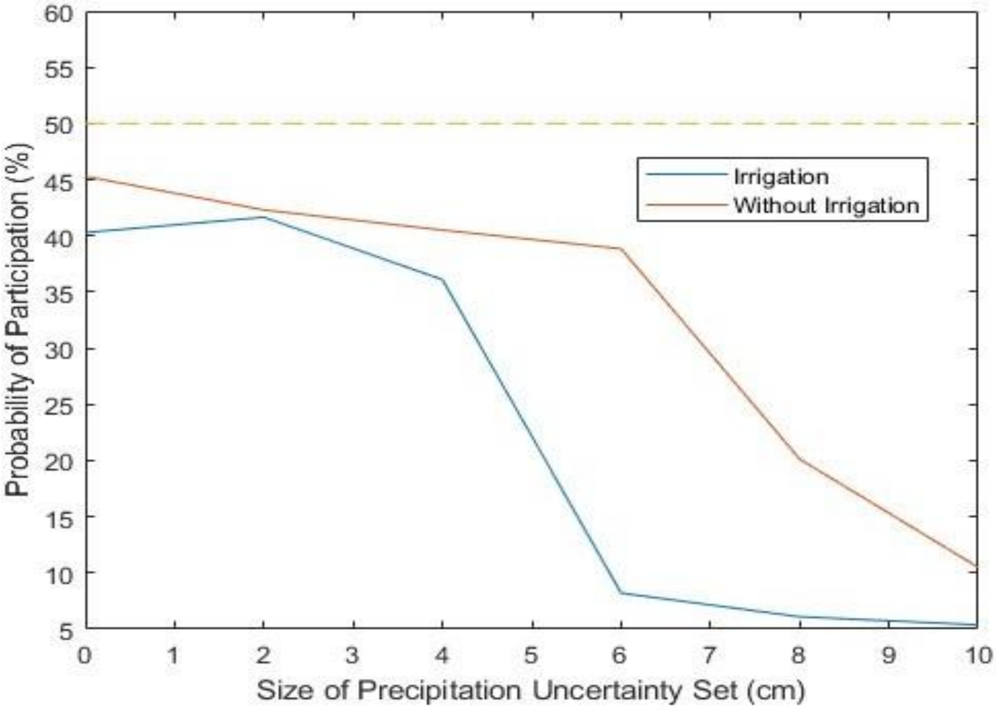

Figure 6: Probability of crop insurance purchase by a myopic farmer with an increase in the size of the precipitation uncertainty set

Table 1: Summary Statistics of the Empirical Variables used in the Estimation of the

Yield Function

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
N (kg/ha)	207	88.9	32.1	159.95	235.38	1,294
$T_g(^\circ C)$	22.4	1.65	15.80	21.37	23.57	27.5
$W_g(cm)$	9.97	3.07	1.9	7.74	11.89	26.9
$T_p(^\circ C)$	13.54	2.02	6.4	12.15	14.95	19.2
$W_p(cm)$	10.6	4.11	1.94	7.71	12.59	35.24
$T_h(^\circ C)$	15.05	1.65	8.95	13.95	16.15	20.5
$W_h(cm)$	8.48	3.84	0.982	5.6	10.75	33.1

Table 2: Yield Function Estimation

	Dependent variable
	yield
Precip. Rate Growing (W_g)	422.343***
	(82.138)
Nitrogen Rate (N)	16.614***
	(2.895)
W_g^2	−21.545***
	(1.146)
N^2	−0.004***
	(0.001)
$W_g * N$	−0.193***
	(0.058)
T_g	3,421.894***
	(250.659)
T_g^2	−64.975***
	(5.447)
$T_g * N$	−0.606***
	(0.119)
$T_g * W_g$	9.495***
	(3.193)
T_p	187.639*
	(100.118)
T_p^2	−6.135*
	(3.627)
T_h	237.291*
	(142.777)
T_h^2	−18.386***
	(4.540)
W_p	35.701**
	(15.549)
W_p^2	−2.613***
	(0.573)
W_h	−13.135
	(14.807)
W_h^2	−1.871***
	(0.660)

Intercept	_42,002.550***
	(2,448.982)
Observations	8,516
R^2	0.579
Adjusted R^2	0.577
Residual Std. Error	1,241.250 (df = 8475)
Note	*p<0.1; **p<0.05; ***p<0.01

The regressions contain fixed effects at the Year and County Level, and the standard deviations are clustered at the county level

Table 3: Simulation Parameters for Corn Case

	Parameters and Variables	Value
	(A) Farming Parameters	
1	Area (A)	180 ha
2	Expected Price Corn (p)	0.23$/kg
3	Nitrogen Price (r_N)	1.44 $/kg
4	Irrigation Water Price (r_W)	1.1 $/ha
5	risk aversion coefficient (ψ_i)	0.005
6	Mean Precipitation Growing Time ($\tilde{\mu}_{W_g}$)	15 cm
7	Range of W_g ($[W_g^{min}, W_g^{max}]$)	[10,27] cm
8	Std. Deviation precipitation ($\sigma_{g,ref}$)	3.07 cm
9	Stock of Carbon Emissions (V_t)	[100,2000] gCO2/kg
10	Time Horizon (T)	5 cm
	(B) Insurance Parameters	
1	Deductible (d)	0.19
2	Coverage Level (c)	50-90 %
	(C) Yield Function Parameters	
1	$\hat{\beta}$	see table 2
	(B) self-protection Parameters	

1	Efficiency of the irrigation technology $\vartheta_{v=1}$	60 %

Table 4: Difference in the Probability of Crop Insurance Purchase for a Myopic-Farmer using irrigation vs. a Myopic-Farmer without irrigation under Risk

Standard Deviation (cm)	Mean probability (%) No Irrigation	Mean probability (%) Irrigation	Difference in Probability	t-stat
$\sigma_{W_g} = 2.5$	60.9	32.3	28.6	3.66 ***
$\sigma_{W_g} = 3.5$	64.1	21.4	42.7	6.50 ***
$\sigma_{W_g} = 4.5$	65.7	19.9	45.8	7.21 ***
$\sigma_{W_g} = 5.5$	69.2	7.9	61.3	14.24

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: Difference in the Probability of Crop Insurance Purchase for a Myopic Farmer without irrigation vs. a Myopic-Hedonic Farmer with irrigation under Uncertainty

Uncertainty set size (cm)	Mean probability (%) No irrigation	Mean probability (%) Irrigation	Difference in Probability	t-stat
$\phi = 0$	45.3	40.2	5.1	4.6 ***
$\phi = 2$	42.3	41.6	0.7	0.63
$\phi = 4$	40.5	36.1	4.4	3.96 ***
$\phi = 6$	38.9	8.2	30.7	27.7 ***

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$