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Inventory Model (M, R, T) Constant Lead Times, Quadratic, Exponential And Backorder Costs.

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Abstract
The paper considers the inventory model in which at a review time, when the inventory position is less than or equal to R, some quantity is ordered which is sufficient to bring it to inventory position to M.
The paper follows the method of Hadley and Whitin which however does not specify the costs terms. We develop the equations for linear backorder cost. From which the quadratic and exponential backorder costs are derived. The expected on hand inventory is determined firstly and the probability that there is a stockout is determined. We assume that demand follows a normal distribution and the demand for x units in n periods follows the n-flow convolution. The expected costs per cycle is obtained and also the expected length of a circle. The inventory costs for quadratic and exponential are obtained replacing the linear backorder costs with quadratic and exponential costs respectively.

Introduction

The cost depending upon backorders could depend upon the length of time of a backorder. It could be linear or non-linear cost. Without inventories to meet orders customers would have to wait until their orders were filled from a source or were manufactured. In this paper, we are considering linear backorder costs, quadratic and exponential costs for the periodic review model (M,R,T) in which at a review time, when the inventory position is less than or equal to R, some quantity is ordered which is sufficient to bring the inventory position to M.

Literature Review

The (M, R, T) inventory model was dealt with by Hadley and Whitin (Ref 1) in which backorder costs were not specified.

Uthayakumar and Parrathi (2) investigates a continuous review inventory model to reduce lead time, yield variability and set up costs simultaneously through capital investments. The backorder rate is depending on the lead time through the amount of shortage. Zhang G.U and Dathwo (3) develop a hybrid inventory system with a lead time limit in backorders.

Inventory Model (M, R , T) Constant Lead Times, Quadratic And Exponential Backorder Costs.

Model (M, R, T) is the model in which at a review time when the inventory position is less than or equal to R, some quantity is ordered which is sufficient to bring the inventory position to M.

We develop the model inventory costs when the backorder cost is linear following the method of Hardley in Ref 1. Than we derive the quadratic and exponential backorder cost from it.

We assume that demand follows a normal distribution N(μ, σ^2. In time t demand is normal with mean Dt and variance $\sigma_t^2, N(Dt, \sigma_t^2)$. the probability density function g(x) is given as

$\frac{1}{\sqrt{\sigma^2 t}} \exp - \frac{1}{2}\left(\frac{x-Dt}{\sigma^2 t}\right) - \infty < x < \infty.$

Let R + x be in inventory position immediately after a review at time t and let $G_1(R+x,T)$ be the expected number of backorder incurred from t + L to t + L+T. if the inventory position is R + x at time t.

Hence

$G_1(M+ x,T) = \int_{R+x}^{\infty}(V-(R+x)\,g(v,D(T+L-gv,DLdv$ …………………1

Let $G_2(R+ x,T)$ *be the expected number of unit of years of shortage incurred from* $t+L$ *to* $t+L+T$.At time t + L+ε, $t+L<t+L+\varepsilon<t+L+T$ the expected number of unit years of shortage.

$G_2(R+x,T) = \int_{R+x}^{\infty}(v-R-x)g(v,I)(L+\varepsilon))dv$

Integrating over the states of ε

We obtain $G_2(R+x,T)$

$G_2(R+x,T) = \int_{L}^{T+L}\int_{R+x}^{\infty}(v-R-xg(v,D(L+\varepsilon))dvd\varepsilon$ …………………… 2

Hence the expected number of backorders per year at any point in time.

$= \frac{1}{T}\int_{L}^{T+L}\int_{R+x}^{\infty}(v-R-x)g\big(v,D(L+\varepsilon dvd\varepsilon$ ………………………….3a

Let $G_3(w,T) = \int_0^L\int_w^{\infty}(x-w)g(x-Dt)dxdt$

$= \left(\frac{\sigma^4 + 2D^4T^2}{4D^3} + \frac{W(\sigma^2-2D^2T)}{2D^2} + \frac{W^2}{2D}\right)F\left(\frac{W-DT}{\sqrt{\sigma^2 T}}\right)$

$+\frac{1}{2}\left(\sigma T^{3/2} - \frac{\sigma^2 T^{1/2}}{D^2} - \frac{T^{1/2}w}{D}\right)g\left(\frac{w-DT}{\sqrt{\sigma^2 T}}\right) - \frac{\sigma^4}{4D^3}esp\left(\frac{2Dw}{\sigma^2}\right)F\left(\frac{w+DT}{\sqrt{\sigma^2 T}}\right)$ ……………….3b

Hence

$G_2(R+x,T) = (G_3(R+x,T+L) - G_3(R+x,L))$ ……………………….4

Let D(R+ x, T) be the expected unit years of storage incurred from t+ L to t+ L+ T.

The expected on hand inventory is equal to the net inventory plus the expected number of backorders.

Hence D (R+ x, T) = $\int_L^{L+T}(R+x-D\varepsilon)d\varepsilon + (G3R+x, T+L- G3(R+x, L))$

$=$ T$\left(R+x-DL-\frac{DT}{2}\right) + G_2(R+x,T)$ ……………………….5

Let $G_4(R+x,T)$ *be the expected cost of carrying inventory and backorders for a lenght of* T time units i.e. form t+L to t+ L+ T.

Hence

$G_4(R+x,T) = hc.T.\left(R+x-DL-\frac{DT}{2}\right) + b_1G_1(R+x,T) + (b_2+hc)G_2(R+x,T)$ …. 6

If the stockout costs is included in $G_4(R+ x_1T)$

Then we need the probability that there is a stockout.

The probability that there is a stockout at anytime t +L+ε, between t+ L and t+ L+ T is the probability that demand exceeds R+ x at t+L+ ε

$= \int_{R+x}^{\infty} g\,(w,D(L+\varepsilon)dvd\varepsilon$

Integrating over the ranges of E, the probability of a stockout is

POUT= $\int_L^{T+L}\int_{R+x}^{\alpha} g(w,D\varepsilon)dwdv\varepsilon$

Integrating

POUT = $\int_{L}^{T+L} F\left(\frac{R+x-D\varepsilon}{\sqrt{\sigma^2\varepsilon}}\right) d\varepsilon$

Let $R_0\,(R,T) = \int_0^T F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) dt$

= $\left(T - \frac{R}{D} - \frac{\sigma^2}{2D^2}\right) F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sqrt{\sigma^2 T}}{D}\, g\, \left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sigma^2}{2D^2} esp\, \left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R\_DT}{\sqrt{\sigma^2 T}}\right)$ …………8

Hence = POUT = $R_0(R+x, T+L) - R_0(R+x, L)$

The cost of stockout

= s$\left(R_0(R+x,T) - R_0(R+x,L)\right)$

= S$G_5(R+x, T+L)\; where\; G_5(R+x,T) = (R_0(R+x,T+L) - R_0(R+x,L))$

Thus $G_4(R+x,T) = hcT\,\left(R + x - DL - \frac{DT}{2}\right) + b_1 G_1(R+x,T)$

+ $(b_2 + hc)G_2(R+x,T)$ ……………………..9

If an order is placed at time T and the next order is placed at time t + nT n ≥ 1, then given that no order has been placed since time t then the probability that the inventory position lies between x and x + dx is the probability that M- R- X units have been demanded in n periods.

The probability that M – R- x unit is demanded in n periods is g^n(M-R-x, DT)where g^n (y, T) is the n-fold convolution of g^n (y, T). Using the principle of moment generating function where S= $X_1 + X_2 + \dots\dots + X_n, X_i i = 1\; to\; n\; follows\; a\; normal\; distribution\; N(\mu, \sigma^2)$

M.G.F of S = (MGF of X_i)(M.G.F. of X_2) ………..(MGF of X_n)

= exp $\left(nDT + \frac{n\sigma^2 t^2}{2}\right)$

Hence S is a normal variate with the mean nD and variate n σ^2

Hence

g^n(M- R- x, DT)= $\frac{1}{\sqrt{\sigma^2 \pi n \sigma^2 t}} \exp - \frac{1}{2}\left(\frac{M-R-x, nDT}{\sqrt{n\,\sigma^2 T}}\right)$

the expected cost of carrying inventory for the period is

$G_4(M,T) since\; the\; inventory\; position\; is\; M$

$at\; time\; t\; immediately\; after\; a\; review.$

At other review times nT

n> $1, the\; inventory\; position\; is\; between$

$M\; and\; R\; say\; i.e.$

$R + y, 0 < Y < M - R\; and\; this\; occurs\; with\; probability$

g^n(M-R-Y, DT)

Thus the expected cost per cycle

$\sum_{n=1}^{\infty} \int_0^{M-R} G_4\,(R+Y,T) g"\,(M - R - Y, DT) dy + G_4(M,T)$ …………………….10

The cycle will be precisely one period if the demand is greater than M-R before the last review. The probability of this occurring is F $\left(\frac{M-R-DT}{\sqrt{\sigma^2 T}}\right)$

The probability that a cycle will be n (n ≥ 2) periods is that M- R- Y units have been demanded in n-1 periods and more than y units are demanded in the nth period.

Thus the probability that a cycle contains precisely n periods.

= $\int_0^{M-R} g^{(n-1)}\,(M - R - Y, DTgy{-}DT\sigma 2Tdy$ $n{\geq}2$ …………… 11

Thus the expected length of a cycle is T times the expected number of period per cycle.

$= \quad T\left[f\left(\frac{M-R-Y,DT}{\sqrt{\sigma^2 T}}\right) + \sum_{n=2}^{\infty}\int_0^{M-R} ng^{(n-1)}(M-R-Y,DTFy-DT\sigma 2Tdy\right.$ ………..12

The review cost Rc is incurred in every period but the ordering cost S is incurred only when an order occurs.

Thus the inventory cost from (10) to (12)

$C \quad = \frac{Rc}{T} + S + \sum_{n=1}^{\infty}\int_0^{M-R} G_4(R+Y,T).g^n(M-R-Y,DTd+G4M,T$

$T\left[F\left(\frac{M-R-DY}{\sqrt{\sigma^2 T}}\right) + \sum_{n=2}^{\infty}\int_0^{M-R} ng^{(n-1)}(M-R-Y,DTFY-DT\sigma 2Tdy\right.$ …………… 13

MODEL (M, R, T) QUADRATIC COST TERM

We have expressed $G_4(R+x,T)$ as the expected cost of carrying inventory and backorders for length of T from t+L to t+T+L where R+x is the inventory position at time t. In the quadratic case the cost of a backorder expressed as

$$C_\beta(t) = b_1 + b_2 + b_3\, t^2$$

We have derived the inventory cost when the backorder cost is linear

From equation 9

$$G_4(R+x,T) = T\left(R + x - DL - \frac{DT}{2}\right)$$

$$+ b_1 G_1(R+x,T) + (b_2 + hc)G_2(R+x,T)$$

If a backorder is incurred at time z, z<L then L-Z is the time for which the backorder lasts.

If the inventory position of the system is R+Y immediately after a review at time t, then the expected backorder costs at time t + L is

$D\int_0^L \int_0^t \frac{1}{\sqrt{\sigma^2 t}} C_\beta(t-z) g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$…………..15

Similarly the expected backorder costs at time t+ L+T

$D\int_0^{L+T} D\int_0^t \frac{1}{\sqrt{\sigma^2 t}} C_\beta(t-z) g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$ ………16

Defining $G_1(R+Y,T)$ as expected cost of a backorder from t+L to t+L+T

For quadratic case

$G_6(R+Y,T) = D\int_0^{T+L} D\int_0^t \frac{C_\beta(t-z)}{\sqrt{\sigma^2 t}} g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$

$- D\int_0^L \int_0^t \frac{C_\beta(t-z)}{\sqrt{\sigma^2 t}} g\left(\frac{R+y+Dz}{\sqrt{\sigma^2 t}}\right) dzd$…………………..17

$= D\int_0^{T+L}\int_0^t \frac{b_1 + b_2(t-z) + b_3(t-z)^2}{\sqrt{\sigma^2 t}} g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$

$- D\int_0^L \int_0^t \frac{(b_1 + b_2 t + b_3 t^2)}{\sqrt{\sigma^2 t}} g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$………..….. 18

From equation 3b

$G_3(R,T) = \left(\frac{\sigma_4 + 2D^4T^2}{4D^3} + R\frac{(\sigma^2 - 2D^2T)}{2D^2} + R22DFR-DT\sigma 2T\right.$

$+ \quad \frac{1}{2}\left(\sigma T^{3/2} - \frac{\sigma^3 T^{1/2}}{D^2} - \frac{T^{1/2}R}{D}\right) g\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sigma^4}{4D^3} esp\left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R+DT}{\sqrt{\sigma^2 T}}\right)$

$G_5(R,T) = \sqrt{\sigma^2}Tg\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) - (R-DT)F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)$

$G_7(R,T) = D\left(\frac{R^3}{3D^3} + \frac{\sigma^2 R^2}{2D^4} + \frac{\sigma^4 R}{2D^5} + \frac{\sigma^6}{4D^6} - \frac{\sigma^2 T^2}{2D^2} + T33-T2RD-R2TD2R-DT\sigma 2T\right.$

$+ \frac{D}{\sqrt{\sigma^2 T}} g\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)\left(-\frac{2}{3}\frac{\sigma^2 R^2}{D^2} + \frac{\sigma^3 T^3}{3D} + \frac{\sigma^2 R^2 T}{3D^3} - \sigma 2RT2D4 + \sigma 4T26D3 + 8\sigma 6TD5 + \right.$ $esp\left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\frac{\sigma^6}{4D^5}$

Integrating 17, we have $G_6(R+Y,T)$

$= \quad b_1\big(G_9(R+Y,T+L) - G_9(R+Y,L)\big) + b_2\big(G_3(R+Y,T+L) - G_3(R,L)\big) + b_3(G_7(R+Y,T+L) - G_7(R+Y,L))$ ……………………………21

Let $G_8(R+Y,T)$ be the expected cost of carrying inventory and backorder including the cost of a stockout dependent on the number of stockouts only.

The difference between $G_8(R+Y,T)$ and $G_4(R+Y,T)$, equation 9 is the $b_3$factor

Hence $G_8(R+Y,T) = hcT\Big(R+Y-DL-$ $DT2+b1($ $G9R+Y,T+L-$ $G9R+Y,L)+$ $b2+hc$ $(G_3(R+Y,T+L) - G_3(R+Y,L)) +$ $b_3(\,G_7(R+Y,T+L) - G_7(R+Y,L) +$ $sG_5(R+Y,T)$ …………………………………………………… ………..22

Replacing $G_4(R+Y,T) in\ equation$ 13 $by\ G_8(R+Y,T)$ we obtain the inventory cost for quadratic backorder cost terms to be

$$C = \frac{\frac{Rc}{T}+S+\sum_{n=1}^{\infty}\int_0^{M-R} G_8(R+Y,T)g^n(M-R-Y,DT)dY + G_8(M,T)}{T\left[F\left(\frac{M-R-DT}{\sqrt{\sigma^2 T}}\right)+\sum_{n=z}^{\infty}\int_{n=z}^{M-R} ng^{n-1}(M-R-Y,DT)F\left(\frac{Y-DT}{\sqrt{\sigma^2 T}}\right)dY\right]}$$

………………………………………..23

SECTION B EXPONENTIAL BACKORDER COST TERMS

In section 3 we developed $G_4(R+x,T)$ where $G_4(R+x,T)$was the expected cost of carrying inventory and backorders for a length of T from t+L to t+L+T, where R+x is the inventory position at time t, for linear and backorder cost. In this section we develop the corresponding cost when the cost per backorder is an exponential function of length of time of backorder.

The cost per backorder

$C_\beta(t) = b_1 esp\,(b_2 t), b_2 > 0$ where t is the length of time of backorder. ………………………..24

If the inventory position of the system is R+Y immediately after review at time t, then the expected backorder costs at time t+L is

$$D\int_0^L D\int_0^t \frac{1}{\sqrt{\sigma^2 t}} C_\beta(t-z) g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$$

$$= D\int_0^L D\int_0^t \frac{1}{\sqrt{\sigma^2 t}} b_1 esp(b_2 t) g\left(\frac{R+y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$$

Similarly the expected backorder costs at time t+L+T

$$= D\int_0^{L+T} D\int_0^t \frac{1}{\sqrt{\sigma^2 t}} b_1 esp(b_2 t) g\left(\frac{R+y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$$

……………………………………………25

Hence expected backorder cost $G_{11}(R+Y,T)$

$$G_{11}(R+Y,T) = D\int_L^{L+T} D\int_0^t \frac{b_1 esp(b_2 t)}{\sqrt{\sigma^2 t}} g\left(\frac{R+y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$$

$$= D\int_L^{L+T}\int_0^t \frac{b_1 esp(b_2 t)}{\sqrt{\sigma^2 t}} \frac{1}{\sqrt{2\pi\sigma^2 t}} esp\left(\frac{R+y-Dz}{\sqrt{\sigma^2 t}}\right) dzdt$$

………………………………………26

By integrating solving

$G_{11}(R+Y,T) = G_{10}(R+Y,T+L) - G_{10}(R+Y,L)$

Where

$G_{10}(R,L) = \frac{2D^2 b_1}{b_4\left(\sigma^2 b^2{}_2 + 2D^2 b_2\right)} esp\left[L\left(\frac{\sigma^2 b^2{}_2 + 2D^2 b_2}{2D^2}\right) -\right.$ $b2RDFR-L(D+\sigma2b2)\sigma2L-b1b2R-DLesp\sigma2L$ $2\pi12R-DL\sigma2L-b1b2FR-DL\sigma2L+\sigma2b2b1b2\sigma$ $2b22+2D2b2esp(2DR)\sigma2FR+DL\sigma2L$ ………………….……………27

Hence when the cost of a stockout dependant only on the number of stockouts is included and the cost of carrying inventories is included.

$$G_{12}(R+Y,T) = hc\left(R+Y-DL-\frac{DT}{z}\right) + hc\big(G_3(R+Y,T+L) - G_3(R+Y,L)\big) + \big(G_{10}(R+Y,T+L) - G_{10}(R+Y,L)\big) + sG_5(R+Y,T) \quad \ldots\ldots 28$$

Hence the inventory cost for model M,R,T is obtained by replacing $G_8(R+Y,T)$ by $G_{12}(R+$ *Y,Tin equation 23*

C=

$$\frac{\frac{Rc}{T}+S+\sum_{n=1}^{\infty} G_{12}(R+Y,T)g^{n}(M-R-Y,DT)dY+G_{12}(M,T)}{\left[TF\left(\frac{M-R-DT}{\sqrt{\sigma^2 T}}\right)+\sum_{n=2}^{\infty}\int_0^{M-R} ng^{n-1}(M-R-Y,DT)F\left(\frac{Y-DT}{\sqrt{\sigma^2 T}}\right)\right]} \quad \ldots\ldots 29$$