The Effect of Object Thickness in Numerical Reflectance and Transmittance using Finite Difference Time Domain (FDTD)

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Abstract

The electromagnetic waves could be simulated using finite difference time domain. This simulation method based on Maxwell equations. Here, the fields were spatially discretized along time discretization. This was the basic concept for updating the fields in every time step. Since the reflectance and transmission of electromagnetic waves were the basic phenomena for optical studies of the waves in materials, we studied the reflectance and transmittance. In FDTD, the reflection and transmission could be calculated using Fourier transformation. We found that the thickness of the object which was inserted to the system affected the reflection and transmission significantly. If the thickness of the object became thicker, then there also increased the appearance frequency of the reflectance with the similar values. It was also happened to the transmittance.

Keyword: 1D-FDTD, object thickness, reflectance and transmittance

Introduction

The most popular method to study electromagnetic waves propagations in numerical way is Finite Difference Time Domain (FDTD) since it was easy and could be used to analyze many problems[1]. The algorithm of FDTD was based on Maxwell's equations[2], especially the rotation operations which were illustrating Faraday $\left(\nabla \times \vec{E} = -\mu \frac{d\vec{H}}{dt}\right)$ and also Ampere $\left(\nabla \times \vec{H} = \varepsilon \frac{d\vec{E}}{dt}\right)$ laws. Hence, the concept of FDTD involved spatial discretization of the fields along the time which was also discretized. The fields were updated in the time domain. This algorithm was firstly proposed by Yee in 1960's[3]. It was already used to study electromagnetic behavior in many applications, such as antennas[4-5], transducers[6] and also interaction electromagnetic waves in organic tissues[7].

The important properties in studying electromagnetic waves propagation were reflectance and transmittance. These phenomena appeared when electromagnetic waves propagated through an objects or obstacles. In FDTD simulation, both reflectance and transmittance could be analyzed using Fourier transformation. Then, by scanning frequency in a certain interval, the reflection and transmission were calculated to obtain the relation of reflection and transmission toward the fields. Here, the geometry of the problems were focused on only one dimension since the purpose of this study was educational. Hence, the numerical region could be limited using only Dirichlet condition.

In this simulation, we set the source of the waves as a Gaussian pulse which propagated along the \hat{z} axis. The electric and magnetic components were directed to the \hat{x} and \hat{y} axis. The object with certain value of permittivity was laid in *x*-*y* plane. In this study, we varied the thickness of the object while we measured the reflectance and transmittance numerically. This was important since the transmission of electromagnetic waves depended on the length of the path of the medium or materials.

The method and formulation

The geometry of this study was given in Figure 1. A non-magnetic object with certain value of permittivity was inserted into free space with source of waves was a Gaussian pulse which propagated along \hat{z} direction. The surface was arranged in *x*-*y* plane, normal to the propagation of the waves. The reflectance was

numerically measured at the left of the object, while transmittance was calculated numerically at the right of the object.



Figure 1. Geometry of the study. The system which contained Gaussian source was located at the free space. The system was represented by big rectangular. Here, the Gaussian pulse which represented in Fig.1 as a star propagated parallel to the \hat{z} direction. The object with certain thickness d was inserted to the system.

Using Maxwell equations, the basic formula to update the fields was given as

$$H_x(k) = H_x(k) + C_m(k) [E_y(k+1) - E_y(k)]$$
(1)

For magnetic field. Here, $C_m(k) = \frac{c_0 \Delta t}{\mu_r \Delta z}$ was a constant. Parameter μ_r was relative permeability, c_0 was the speed of light in free space, Δt and Δz represented the length of temporal and spatial grid. Here, we used normalized magnetic field. Also, for updating the electric field, the basic formulation was

$$E_{y}(k) = E_{y}(k) + C_{e}(k)[H_{x}(k) - H_{x}(k-1)]$$
(2)

where $C_e(k) = \frac{c_0 \Delta t}{\varepsilon_r \Delta z}$ represented an electric constant.

Here, we also limited the numerical region using Dirichlet condition by setting the fields at the edge of the region to zero. For example: for electric component, we had to add the formula in updating the electric fields as

$$E2 = 0; \quad E1 = 0, \tag{3a}$$

$$E_{2} = E_{1}; \quad E_{1} = E_{y}(n_{z}), \tag{3b}$$

$$E_{y}(1) = E_{y}(1) + C_{e}(1)[H_{r}(1) - H_{2}]. \tag{3c}$$

 $E_{y}(1) = E_{y}(1) + C_{e}(1)[H_{x}(1) - H2].$ The similar way also applied for magnetic fields, where updated magnetic fields became

$$H2 = 0; H1 = 0,$$
 (4a)

$$H2 = H1; \quad H1 = H_x(1),$$
 (4b)

$$H_x(n_z) = H_x(n_z) + C_m(n_z) [E2 - E_y(n_z)].$$
(4c)

Here, n_z was the last spatial grid.

Since we needed one certain direction of electromagnetic waves, we had to also apply total field/scattered field (TF/SF). Then, we had to add the updating formula such as

$$E_y(k_{src}) = E_y(k_{src}) - C_e(k_{src})H_{src}(T)$$
(5a)
lectric field and

for electric field, and

$$H_x(k_{src} - 1) = H_x(k_{src} - 1) - C_m(k_{src} - 1)E_{src}(T)$$
 (5b)
For magnetic field. Here, H_{src} and E_{src} were source's fields. The parameter k_{src} was the spatial grid at the source's fields. The temporal grid of time was represented by parameter T .

The reflectance and transmittance were calculated using Fourier transform. The formulation for updating the Fourier transform was given as

$$Ref(n_f) = Ref(n_f) + K(n_f)^T E_y(n_{ref})$$
(6a)

where Ref was reflectance, n_f was frequency's grid, n_{ref} represented the location to measure reflectance and $K = e^{-i2\pi\omega dt}$. For transmittance, the formulation became

$$Tran(n_f) = Tran(n_f) + K(n_f)^T E_y(n_{tran})$$
(6b)

where n_{tran} was location where transmission was measured.

Results and Discussion

In this paper, we used the non-magnetic object with relative permittivity was 10. The surrounding was free space. The simulation of pulse's propagation was presented in Figure 2.



Figure 2. The simulation of Gaussian pulse's propagation. The pulse was represented by solid blue line. The rectangular object was drawn by shaded red line. We captured the propagation in four point time: (a) 0.85 ns, (b) 1.75 ns, (c) 4.2 ns and (d) 10.5 ns.

The source of Gaussian's pulse in this simulation was located at the grid number 20 as it can be seen in Figure 2a. When the waves reached the object, then a part of the incident wave was reflected and propagated in opposite direction to the incident wave while the other part was transmitted (see Figure 2b). The reflected wave was absorbed when it reached numerical boundary. The wavelength inside the object was shorter than wavelength outside the object as it was expected. This was because the density of the object was higher than the density of the surrounding. When the wave left the object and propagated into the surrounding (see Figure 2c), the wavelength became higher since the wave propagate in less dense medium. The waves then absorbed when it reached right numerical boundary at grid number 200 as it was drawn in Figure 2d. Here, we assumed that the incident wave was directed perpendicular to the interface, hence the scattering phenomenon could be excluded.

In calculating reflectance and transmittance, we numerically measured reflection at grid number 10 and transmission at grid number 190. In Fourier transform to obtain reflectance and transmittance, we used frequency 0.25 MHz. The results of numerical calculation were presented in Figure 3. Here, the thickness of the object was varied to analyze the resulted transmittance and reflectance.



Figure 3. The reflection and transmission of Gaussian pulse's propagation. The solid blue lines were represented reflectance. The transmittance was drawn using solid red lines. The addition of reflectance and transmittance were presented using solid black lines. Here, we varied the thickness of the object: (a) 84 mm, (b) 168 mm, (c) 252 mm and (d) 336 mm.

The thickness of the object was varied into four thicknesses: 84 mm, 168 mm, 252 mm and 336 mm as it was presented in Figure 3. Here, the solid red lines were represented reflectance while the solid blue lines were reflectance. The solid black lines were symbolized reflectance plus transmittance which the value was always one since the incident wave was decomposed into reflection and transmission waves at the interface of the object. The Figure 3. Also shown that transmittance was dominant compare to the reflectance. This was probably because the object had only small density since the relative permittivity only 10. The reflectance and transmission was change significantly when the thickness was relatively thick (see Figure 3d).

If we noticed the results in Figure 3, it can be seen that the increased of the thickness' object will increase the appearance frequency of the same values of reflectance or transmittance. This because the increase of the thickness of the object will increase the wave's path inside the object. Hence, for the same interval of frequency, the curvatures for reflectance and transmittance were half 'wavelength' for 84 mm object, one 'wavelength' for 168 mm, 3/2 'wavelength' for 252 mm and around two 'wavelength' for 336 mm.

Conclusion

The increase of object thickness raised the appearance frequency of the values of the both reflectance and transmittance.

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