

# Calculation of Residual Strength of Shot Casing

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## Abstract:

There is a significant spanning difference in scale between the shot-hole borehole and the oil well casing, and the stress singularity at the shot-hole usually occurs in conventional finite element simulations, which cannot be avoided even when elastic-plasticity is brought into the calculation, leading to errors in the simulation results. In addition, a large amount of mesh refinement is required near the shot holes, which also increases the computational volume. In order to solve this problem, this paper adopts the RVE technique, which establishes the RVE model in and around the original shot hole location, and then establishes the casing model after RVE treatment to avoid the occurrence of stress singularity. After RVE treatment, additional mesh refinement is no longer required near the shot hole, which reduces the amount of calculation to a certain extent. The feasibility of the RVE-treated casing in simulation calculations was verified by performing elasto-plastic static finite element simulations of the conventional shot hole casing and the RVE-treated casing in ANSYS finite element simulation software. The results show that the RVE-treated model optimizes the simulation effect in the original shot hole region. The RVE-treated casing model is used to calculate the residual strength coefficients under different shot hole parameters, and the fitting analysis is carried out. The results show that the residual strength of the casing gradually decreases with the increase of the injection hole diameter under the condition of constant casing parameters, and the residual strength of the casing also gradually decreases with the increase of the number of holes. These findings provide an important reference for the design of oil and gas well production programs.

**Keywords:** rve; casing shot hole; stress concentration; stress singularity; residual strength; finite element analysis

## 1 Introduction

In oil well operations, perforating operations can cause some degree of damage to the well wall, and if the residual strength of the wall is insufficient to withstand the downhole pressure or the forces during well operations, it may lead to rupture or collapse of the wall, which in turn affects the integrity of the wellbore and the safety of the operation. By calculating the residual strength, the stability of the well wall can be assessed to determine whether support or reinforcement measures are required to ensure the safe conduct of downhole operations<sup>1</sup>. Therefore, the calculation of the residual strength after the shot hole is very important for assessing the stability of the well wall and determining the subsequent operation plan. However, since the diameter of the shot hole borehole is usually around 10 mm at present, and the complete casing is usually calculated in meters when calculating its residual strength. Since both of them are relatively large in scale span, the stress singularity will appear in the vicinity of the shot hole in the calculation<sup>2</sup>. In order to make its simulation results more in line with the display situation, elastic-plastic calculation is added for optimization, but it still cannot solve the stress singularity phenomenon effectively. In this paper, the shot hole part is treated with RVE so as to optimize the simulation.

In 1991, Xu Daoling, Liu Tieniu, Xing Xianjun<sup>3</sup> and others began to notice the importance of the reduction of the casing strength after the shot hole operation, and established the shot hole casing model and performed the stress field analysis. Yang Bin, Lian Zhanghua<sup>4</sup> established a spatial finite element mechanical model for the strength analysis of shot hole casing with helical perforation using nonlinear elastic-plastic finite element method, which provides some theoretical basis for the reasonable selection of casing and the extension of its service life. Zhang Wenlai<sup>5</sup> studied the relationship between the reduction of anti-extrusion strength of shot hole casing and the stress concentration factor of shot hole casing by shot hole parameters such as eyelet diameter, eyelet distribution and eyelet density. Su Yibao<sup>6</sup> Based on the theory of fluid-solid coupling, combining rock mechanics, elastic-plastic mechanics, damage mechanics and seepage mechanics, the stress coupling model of stratum-cement ring-shooting casing was established, and degraded to a bare casing to compare with the API standard, which verified the feasibility of numerical simulation. Liu Xianbo, Li Jun<sup>7</sup> et al. based on the theory of impact dynamics and tubing column mechanics, combined with the field perforation completion conditions, established a three-dimensional perforation numerical computation model, and calculated the residual strength coefficient of the casing to resist extrusion after perforation. In this way, the effects of casing diameter to thickness ratio, shot hole density, shot hole phase angle, etc. on the stress concentration phenomenon and initial loss near small holes are investigated. Nobuo Morita<sup>8</sup> proposes the directional shot hole technique as an effective method to control the problem of casing caving in high inclination wells. M.A. Rahman<sup>9</sup> proposes a new type of technique called "drilling", and compares it with the "drilling" technique. " and compared its performance with that of conventional perforating.

In order to understand the feasibility of RVE processing in the calculation of residual strength of casing after shot hole, and to reduce its calculation amount significantly in the process of calculation, this paper takes the commonly used 5<sup>1</sup>/<sub>2</sub>in×9.17mm P110 casing as an example, adopts the shot hole with 10 holes/m, 90° phase angle, and 10mm hole diameter, establishes the geometrical model of casing, and imports it into Ansys for the mechanical simulation analysis, and analyzes the casing model by using the VonMises yield criterion to assign material properties to the casing model, the perforation part is RVE processed for calculation<sup>10</sup>, and verified the feasibility of the method.

## **2 Theoretical Modeling of Residual Strength of Shot Casing**

### **1.1 Calculation of the area of influence of the shot hole stress**

Since the shot holes will change the stress distribution in the original region, the range of the shot hole stress effect needs to be calculated in order to determine the size of the RVE model. When determining the model size, it should be ensured that the range of shot hole stress action is completely contained within the model, and the shot hole stress action beyond the range is negligible, so the boundary of the stress influence between shot holes is the minimum size of the RVE model.

Therefore, there is a need to investigate the range of inter-hole stress disturbances in the hole-plate theory. For example, a circular hole of radius  $a$  exists in a wide plate and is subjected to a uniform tensile stress  $p$ . The stress distribution in this case is analyzed as shown in Fig. 1<sup>11</sup>.

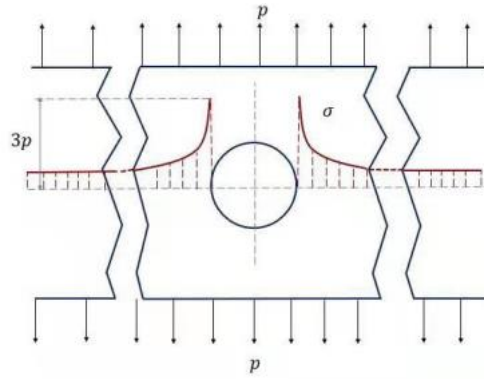


Fig.1 Stresses on an infinite-width plate with holes

Due to the influence of the dug hole in the middle of the infinite width plate, it leads to a significant increase in the stress around the dug hole compared with that when the hole is not dug. Take any point on the infinite width plate, with the increase of the distance  $\rho$  between the point and the center of the dug hole circle, the stress at the point is gradually reduced by the influence of the hole edge stress until it converges smoothly to  $p$ . In order to investigate the range of the influence of the stress at the edge of the hole, the following equations are listed

$$\frac{p}{2} \left( 2 + \frac{a^2}{\rho^2} + \frac{3a^4}{\rho^4} \right) = (1 + \Delta) p \quad (1.1)$$

The solution to Eq. (1-1) is.

$$\rho_{min} = \frac{a\sqrt{6}}{\sqrt{\sqrt{24\Delta+1} - 0.5}} \quad (1.2)$$

Where,  $\Delta$  is the engineering permissible error taken as 2.5%, after substituting (1-1), we get  $\rho_{min} = 2.77a$ . when  $\rho_{min} \leq 2.77a$ , its force will be affected by the common influence of the surrounding holes.

## 1.2 Von-Mises stress

Von-Mises stress is an equivalent stress based on the fourth strength theory and one of the basic theories for calculating equivalent stresses in Ansys simulation software<sup>12</sup>. It is considered that the material reaches the critical value of shape change ratio energy is the main reason for its destruction, which can be used as a basis for plastic materials to reach failure. The Von Mises equivalent stress calculation formula in finite element software is as follows:

$$\sigma_e = \sqrt{\frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right]} \quad (1.3)$$

where  $\sigma_1, \sigma_2, \sigma_3$  are the three principal stresses.

## 2 RVE Modeling

### 2.1 Stress Simulation of Shot Hole Casing Shot Hole Section

The relevant parameters of the physical model are taken from the production well. Take a 5000m deep well with high pumping pressure fracturing as an example to analyze the strength safety of casing under fracturing condition. The shot hole section of this well is 139.7mm×9.17mm P110 casing, which is injected twice with a hole density of 10 holes/m, 90° phase angle and 10mm hole diameter. The establishment of RVE model needs to simulate the original shot hole section to calculate its stress-strain curve, and then import the curve into the RVE model for simulation calculation. From the formula (1-1), it can be seen that the model range of the shot hole part must be larger than 2.77a, and after repeated trial and error, it is more reasonable

to set the side length of the shot hole part as  $3a$  for solving. The model mesh size is  $1/3$  of the wall thickness, and the pressure load is increased by 10MPa per second on the convex surface, fixed constraints on both ends, and the elastic-plastic changes are calculated by using nonlinear materials, and the radial deformation reaches a maximum of 7.88mm when the pressure load reaches the maximum, with a maximum of 14.2% plastic deformation.

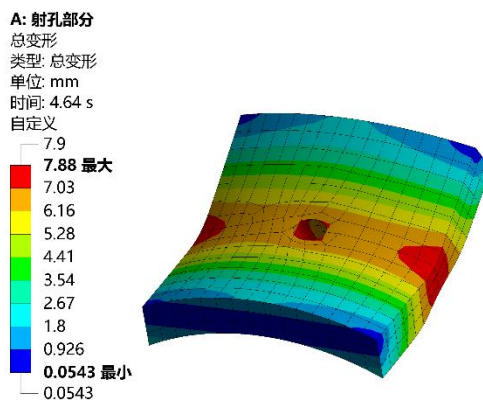


Fig.2 Simulation of the shot hole section

According to the simulation results in Fig. 2, there is an obvious stress concentration phenomenon in the shot hole part, and even a stress singularity, i.e., the stress calculation results in some regions tend to be infinitely large. In practice, stress singularity often leads to inaccurate calculation results and large deviation. In casing completion, the large span between shot hole and casing size easily leads to the appearance of stress singularity. To solve this problem, this paper adopts the RVE treatment to ensure that the mesh division size of the shot hole part and the whole casing does not span too much, so as to avoid the occurrence of stress concentration and stress singularity<sup>13</sup>. RVE processing is an effective method in finite element analysis. By adding appropriate constraints or performing necessary machining near the shot hole portion, a more uniform stress distribution in the region is realized to avoid stress concentration and singularity. By choosing the appropriate size and location of the RVE to replace the shot hole section, the phenomenon of stress concentration and the generation of stress singularity are effectively circumvented while ensuring the accuracy of the calculation results.

## 2.2 RVE Modeling

The stress-strain curves after the simulation of the shot hole part are imported into the same material model without shot holes, which has the same shape and size except the shot holes in order to control the irrelevant variables. The same constraints and loads are set as those of the shot hole part, and then the simulation solution is performed. When the pressure load reaches the maximum, the total deformation reaches 5.64 mm and the maximum plastic deformation reaches 15.9%.

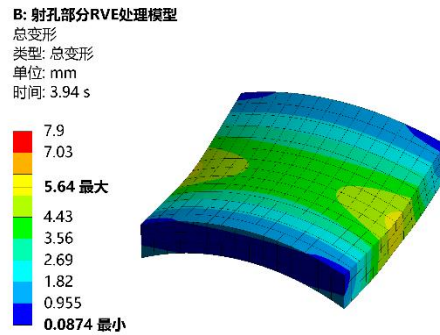


Fig.3 Simulation of the RVE model for the shot hole section

From the simulation results, it can be concluded that the stress concentration phenomenon in the original shot hole region is well avoided, and the stress singularity can also be well solved, so that the simulation results are more consistent with the actual situation and more accurate compared with the results without RVE processing.

### 3 RVE Casing Model Accuracy Verification

#### 3.1 Comparison of Casing Simulation Under External Extrusion Pressure

The model of the shot hole part of the RVE treatment is imported into the model of the complete casing and simulated in ANSYS. A uniform pressure load is added to its outer wall, fixed constraints are applied at both ends, the mesh size is divided with a dimension of 15 mm, the shot hole casing is locally encrypted in the vicinity of the shot holes, and the mesh size of the casing wall is 1/2 of the wall thickness.

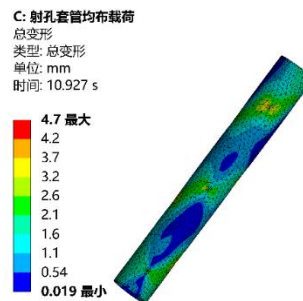


Fig.4 Simulation of the shot casing model

After simulation calculation, the casing has obvious stress concentration near the shot hole, and at the same time, localized stress singular phenomenon occurs in the area of the shot hole, which makes the simulation results near the shot hole inaccurate. The extrusion at the shot hole causes the bulging outward around the shot hole, which is not consistent with the deformation generated by the extrusion of the shot hole in the actual situation, so the simulation results near the shot hole will be different from the expected results. Although the mesh near the shot hole is refined enough for calculation, the occurrence of stress singularity cannot be avoided, which shows that it is difficult to avoid the occurrence of stress singularity in the area of shot hole in the simulation of ordinary shot hole casing.

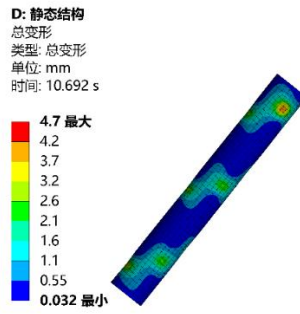


Figure5 RVE casing simulation diagram

The simulation analysis of the casing after RVE treatment shows that there is no obvious stress singularity phenomenon near the original shot hole, which indicates that the RVE treatment makes the casing effectively avoid the occurrence of stress singularity in the shot hole part. The calculation time of the shot hole casing reaches 1h, while the calculation time of the casing after RVE treatment is only 49min, which can be seen that the casing after RVE treatment reduces the calculation time to a certain extent and saves time and cost.

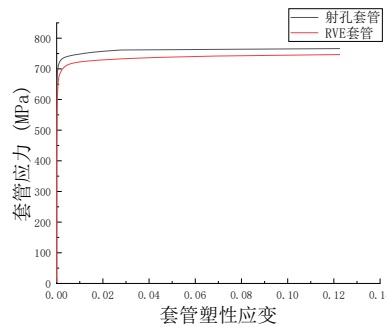


Fig.6 Plastic strain stress curve of RVE casing and shot hole casing

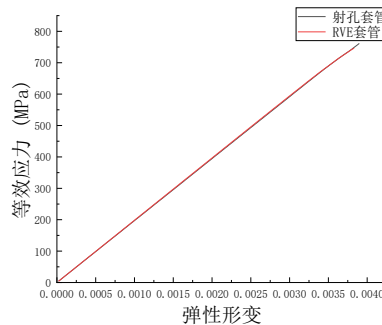


Fig.7 Elastic strain stress curve of RVE casing vs. shot hole casing

Comparing the elastic-plastic stress-strain curves of the two types of casing, it can be seen that the overall mechanical characteristic curves of the two types of casing are relatively close to each other, which indicates that the mechanical characteristics of the casing after RVE treatment are similar to those of the original shot hole casing. This indicates that the RVE treatment of the casing does not change the normal mechanical properties of the overall casing, but only optimizes the stress singularity phenomenon generated by the stress concentration in the shot hole part, which leads to more accurate results when the casing is subjected to mechanical simulation.

### 3.2 Comparison of Single Path Simulation for RVE Model

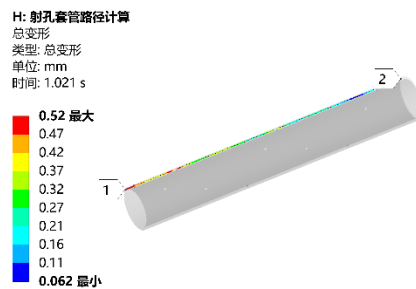


Figure8 Shot hole casing

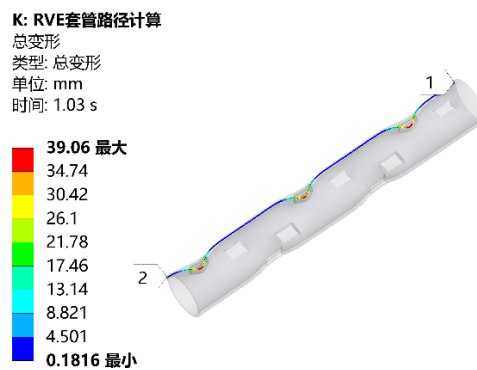


Figure9 RVE Casing

A point is selected at one end of the shot casing and another point is selected at the other end to connect a path perpendicular to the bottom surface and which passes through both shot regions. The same path is set up for the RVE treatment casing, and fixed constraints are applied to both casing ends, and uniformly distributed pressure loads are applied to its outer wall. According to Figure 9 it can be seen that the deformation increases significantly in the RVE section, which is in accordance with the actual situation that the deformation in the shot hole region is the largest when the casing is pressurized. According to the Fig. , it can be seen that the simulation path is discontinuous in the shot hole part, which is because there is no solid unit in the shot hole part, so it is not possible to simulate the shot hole region, while the RVE treated casing has solid units in a region near the shot hole for normal simulation.

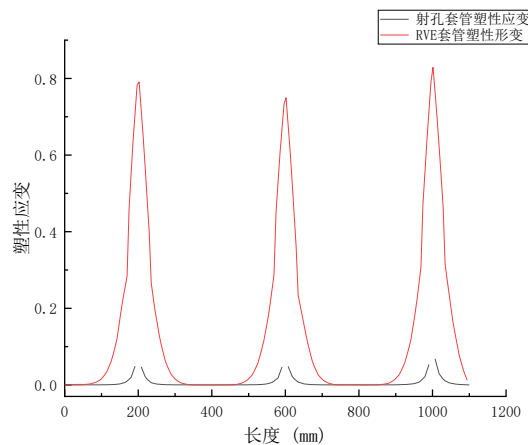


Fig.10 Plastic strain on two casing paths

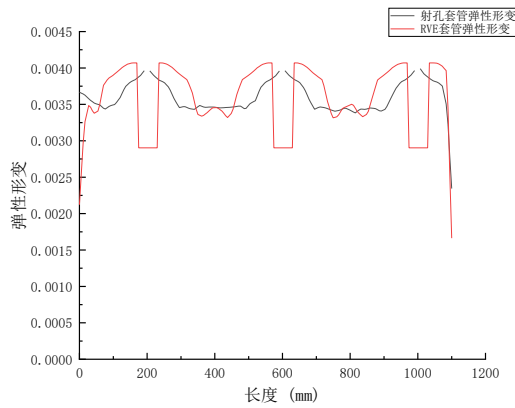


Fig.11 Elastic strain on two casing paths

From Fig.10 and Fig.11, it can be seen that the stress-strain curves of the two types of casing are relatively close to each other at the same locations except near the shot hole, which also indicates that the calculation of the overall strength of the casing after the RVE treatment does not cause a great error, and the results are accurate. In the vicinity of the shot hole, the strain of RVE-treated casing increases continuously and reaches a peak at the center of the shot hole. The shot hole casing, on the other hand, shows no data at the shot hole because there is no solid unit at the shot hole. Comparing the two, it can be seen that the results of the overall casing simulation after RVE treatment are accurate and have different degrees of optimization for the shot hole part.

In addition to the RVE treatment, parameters such as the location, shape or size of the shot holes can be adjusted in order to reduce the span between the shot holes and the casing dimensions, so as to reduce the occurrence of the stress concentration phenomenon. In addition, reasonable design of shot hole parameters such as density, angle and size can also effectively reduce the effect of stress concentration phenomenon<sup>14</sup>. Here in this paper, we only discuss the hole diameter and number of holes of the shot holes, and calculate their effects on the strength of the casing.

## 4 Analysis of Residual Strength Factor of Shot Casing

### 4.1 Residual Strength Factor

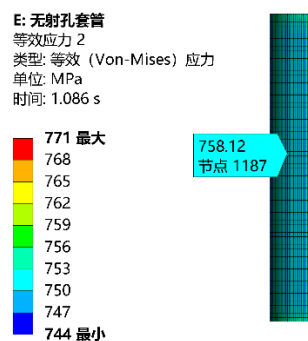


Figure12 Unshot Casing



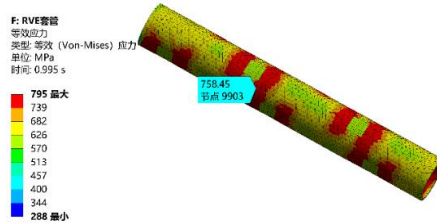


Figure13 RVE Casing

Through the signal mutation in the casing extrusion experiment, the sound of casing extrusion can be used as the basis of whether the casing is destroyed or not. For the damage and instability of the casing in the simulation calculation, the strain value of the casing is adopted as the basis for judging whether the casing is destroyed or not. For the casing simulation calculation contains different shot hole parameters, through the fourth strength theory Mises stress as the casing whether the destruction of the judgment basis is more appropriate.

To add uniform pressure load to the outer wall of the casing, the external uniform pressure load is increased uniformly, and the load is treated as external extrusion stress. From Figure 12 Figure 13, it can be seen that the inner fabric of the casing has yielded before the casing is destabilized, and at this time, the load borne by the outer wall of the casing has also reached the maximum, and if the external load continues to be increased, the casing will enter into the state of destabilization. Therefore, the ratio of the maximum external pressure to which the casing is subjected as the maximum bearing limit of the casing to the maximum bearing limit of the casing without shot hole is taken as the residual strength factor of the casing<sup>15</sup>.

The RVE-treated casing is subjected to a constant loading of external extrusion force, and the load of the casing reaches its maximum at 115 MPa, which is the extrusion strength of the casing without injection, i.e. 115 MPa. The load of the RVE-treated casing reaches its maximum at an external extrusion force of 107 MPa, which is the extrusion strength of the casing after injection, i.e. 107 MPa. The residual strength coefficient of RVE treated casing is 93.1%.

#### 4.2 Analysis of Residual Strength Coefficients for Different Shot Hole Parameters

Number of holes	Injection	Residual strength
10	10	93.1
	11	92.3
	12	91.8
	13	91.3
	14	90.8
11	10	90.9
	11	89.9
	12	89.3
	13	89.0
	14	88.5
12	10	90.0
	11	89.1
	12	88.5
	13	87.9

	14	87.5
13	10	89.2
	11	88.6
	12	88.1
	13	87.7
	14	87.3
14	10	88.8
	11	88.3
	12	87.8
	13	87.3
	14	87.0

Fig.14 Residual strength factor of casing with different shot hole parameters

From the calculation results can be obtained in the same conditions of the number of injection holes, with the increase of injection hole diameter, the casing residual strength coefficient gradually decreases; in the same conditions of the injection hole diameter, with the increase of injection hole diameter casing residual strength coefficient gradually decreases.

### 4.3 Casing Residual Strength Fitting

Calculated data is relatively discrete, direct use for field operations is more troublesome, in order to simplify the operation process, the casing residual strength coefficient is fitted to a function of the hole diameter and the number of holes, the function is.

$$S=C_1+C_2x+C_3x^2+C_4y+C_5y^2 \quad (5.1)$$

where S is the residual strength factor, x is the shot hole diameter, y is the number of shot holes, and C1, C2, C3, C4, and C5 are the fitting coefficients.

The fitting function is:

$$S=165.708-9.08771x+0.33857x^2-2.27429y+0.07143y^2 \quad (5.2)$$

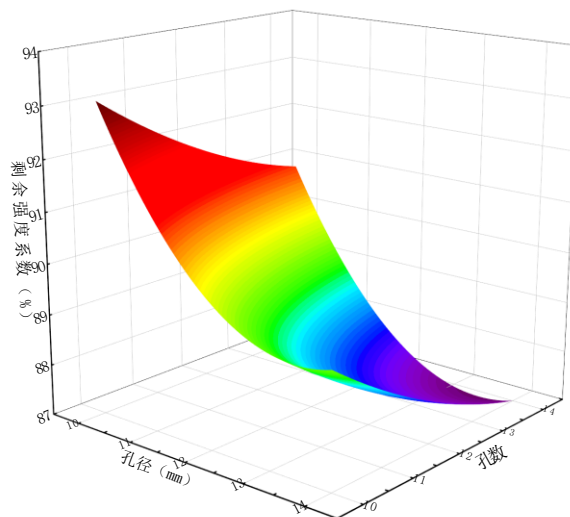


Figure15 Fitted Residual Strength Factor for RVE Casing

After comparative calculations, the maximum error between the calculated results of the fitted function and the original data of the finite element calculation results does not exceed 1%, therefore, the finite

element simulation results can be represented by this function.

## 5 Reach a Verdict

(1)The shot hole section is RVE treated and the RVE treated section is applied to the casing.

(2)RVE treated casing in finite element calculation can well present the stress-strain as well as the total deformation of casing's stress appearing, and partially optimize the shot hole part.

(3)The residual strength coefficients for different combinations of shot hole diameters and number of shot holes were calculated by RVE casing, and the correlation function between the residual strength coefficients and the hole diameters and number of holes was fitted. The maximum error between the raw data and the fitted data does not exceed 1%.

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