

An Inventory Model for Perishable Items with Time Varying Stock Dependent Demand and Trade Credit under Fuzzy type Inflation

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Abstract

In this paper we presented an inventory model for perishable items with time varying stock dependent demand under fuzzy type inflation. It is assumed that the supplier offers a credit period to the retailer and the length of credit period is dependent on the order quantity. The purpose of our study is to minimize the present value of retailer's total cost. A numerical example is also given to demonstrate the presented model.

Keywords

Inventory, Deterioration, Perishable, Credit period, Inflation and Time varying stock dependent demand

Introduction

In the classical inventory models payment for the items paid by the supplier depends on the payment paid by the retailer and in such cases the supplier offers a fixed credit period to the retailer. During which no interest will be charged by the supplier so there is no need to pay the purchasing cost by the retailer. After this credit period up to the end of a period interest charged and paid by the retailer. In such situations the retailer starts to accumulate revenue on his sale and earn interest on his revenue. If the revenue earned by the retailer up to the end of credit period is enough to pay the purchasing cost or there is a budget, the balance is settled and the supplier does not charge any interest, otherwise the supplier charges interest for unpaid balance after the credit period. The interest and the remaining payment are made at the end of replenishment cycle.

In traditional EOQ models the payment time does not affect the profit and replenishment policy. If we consider the inflation then order quantity and payment time can influence both the supplier's and retailer's decisions. A large pile of perishable foods such as fruits, vegetables, milk, bread, chocklet etc. attracts the consumers to buy more. Buzacott [1] considered an EOQ model with different type of pricing policies under inflation. Baker and Urban [2] proposed a deterministic inventory model for deteriorating items with stock level dependent demand rate. Mandal and Phaujdar [3] presented an inventory model for deteriorating items with stock level dependent consumption rate. Vrat and Padmanabhan developed two inventory models [4] and [5]. The model [4] is an inventory model with stock dependent consumption rate under inflation and the model [5] is an EOQ model for perishable products with stock dependent selling rate. Bose et al. [6]

presented an EOQ model for deteriorating items with linear time dependent demand and shortages. They also considered the concept of inflation and time discounting in their inventory model. Mandal and Maiti [7] proposed an inventory model for damageable products with stock dependent demand and variable replenishment rate. Chung and Lin [8] determined an optimal replenishment policy for an inventory model of deteriorating items with time discounting. Chang [9] proposed an EOQ model for deteriorating items under inflation and time discounting. He assumed that the supplier offers a trade credit policy to the retailer, when the retailer's order size is larger than a certain level. Dye and Ouyang [10] developed an EOQ model for perishable items with stock dependent selling rate by allowing shortages. Hou [11] presented an inventory model for deteriorating items with stock dependent consumption rate and shortages. He also considered the effect of inflation and time discounting in his inventory model. Jaggi et al. [12] determined an optimal ordering policy for an inventory model of deteriorating items with time dependent demand. They also introduced the concept of inflation in their inventory model. Sana and Chaudhuri [13] developed a deterministic EOQ model for deteriorating items with stock dependent demand and permissible delay in payments. Valliaththal and Uthayakumar [14] presented an EOQ model for perishable products with stock dependent selling rate and shortages. Roy et al. [15] developed an inventory model for deteriorating items with stock dependent demand. They also considered the fuzzy inflation rate and time discounting over a random planning horizon. Sana developed two inventory models [16] and [21]. The model [16] is a lot size inventory model with time varying deterioration rate and stock dependent demand by allowing shortages. And in the model [21] she considered a control policy for a production system with stock dependent demand. Chang et al. [17] determined an optimal replenishment policy for an inventory model of non-instantaneous deteriorating items with stock dependent demand. Sarkar et al. [18] presented an EMQ (economic manufacturing quantity) model for imperfect production process. They also considered the time dependent demand and time value of money under inflation. Yan [19] considered an EOQ model for perishable items with freshness dependent demand and partial backlogging. Nagrare and Dutta [20] developed a continuous review inventory model for perishable products with inventory level dependent demand. Yang et al. [22] presented an inventory model for perishable products with stock dependent demand and trade credit under inflation. Jana et al. [23] proposed a partial backlogging inventory model for deteriorating items under fuzzy inflation and discounting over random planning horizon. Shabani et al. [24] presents an inventory model with fuzzy deterioration and fully backlogged shortage under inflation. Naserabadi et al. [25] developed a new mathematical model with stochastic and fuzzy deterioration rate under inflation.

In the present paper we presented an inventory model for perishable items with time varying stock dependent demand and trade credit under inflation.

Assumptions and Notations

We consider the following assumptions and notations corresponding to the developed model

1. The demand rate $R(t)$ is $R(t) = a + bt + kI(t)$, $a \geq 0$, $0 \leq b \leq 1$, $k > 0$.
2. θ is the constant deterioration rate.
3. o_c is the ordering cost per order.
4. h_c is the holding cost per unit.
5. s_c is the shortage cost.
6. M is the credit period.
7. T is the replenish cycle length.
8. r is the inflation rate.
9. \tilde{r} is the fuzzy inflation rate.
10. I_c is the interest charged per \$ per unit time when $T > M$.
11. p_c is the purchasing cost per unit.
12. s_p is the selling price per unit with $s_p > p_c$.
13. Q is the initial inventory level.
14. L is the planning horizon.
15. The supplier sells one single item to the retailer.
16. The items are replenished, when the stock level becomes zero.
17. The supplier provides a credit period, which is dependent on the order quantity.
18. The lead time is zero.
19. Shortages are not allowed.
20. The inventory planning horizon is finite and the numbers of cycles are finite in the planning horizon.
21. $I(t)$ is the inventory level at any time t .

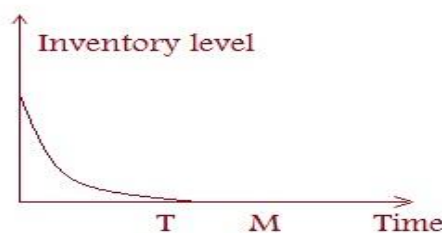


Figure 1 Inventory Model

Mathematical Formulation

Suppose an inventory system contains the maximum inventory level at time $t = 0$. Due to both demand and deterioration the inventory level decreases in the interval $[0, T]$. The replenishment cycle starts with the initial maximum inventory level Q and ends with zero stock level.

The retailer's instantaneous inventory level at any time t in the interval $[0, T]$ is governed by the following differential equation

$$\frac{dI}{dt} + \theta I = -[a + bt + kI(t)], \quad 0 \leq t \leq T \quad (1) \quad \text{with}$$

the boundary condition $I(T) = 0$

The equation (1) can also be written as

$$\frac{dI}{dt} + \alpha I = -[a + bt], \quad 0 \leq t \leq T \quad (2)$$

Here $\alpha = (\theta + k)$

with the boundary condition $I(T) = 0$

By considering only second degree terms in T and t , we get the solution of equation (2)

$$I = a(T - t) + \frac{b}{2}(T^2 - t^2) + \frac{a\alpha}{2}(T^2 + t^2) - a\alpha Tt \quad (3)$$

The initial order quantity Q is obtained by putting $t = 0$ in equation (3)

$$Q = aT + \frac{b}{2}T^2 + \frac{a\alpha}{2}T^2 \quad (4)$$

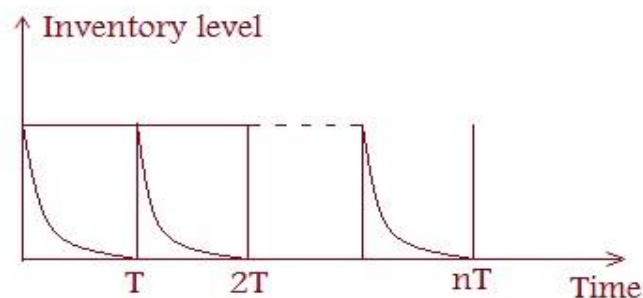


Figure 2 Inventory Model

Now we discuss the case, $M \geq T$

Case

When $M \geq T$ then in this case the retailer can sell all the items before the end of credit period M . Since the credit period M is greater than the replenishment cycle length so no interest will be charged by the retailer. Since the purchasing cost is paid at the end of credit period M .

During the first cycle, the present value of ordering cost is

$$O_c = A \quad (5)$$

During the first cycle, the present value of holding cost is

$$H_C = h_c \left[\int_0^T I(t) e^{-rt} dt \right]$$

$$H_C = h_c \int_0^T e^{-rt} \left[a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{a\alpha}{2}(T^2 + t^2) - a\alpha Tt \right] dt$$

or

$$H_C = h_c \left[\frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{a\alpha}{6}T^3 - \frac{ar}{6}T^3 \right] \quad (6)$$

During the first cycle, the present value of purchasing cost is

$$P_C = p_c Q e^{-rM}$$

Here Q is the initial order quantity and given by the equation (4)

$$P_C = p_c e^{-rM} \left[aT + \frac{b}{2}T^2 + \frac{a\alpha}{2}T^2 \right] \quad (7)$$

During the first cycle, the present value of retailer's total cost is

$$TC_1(T) = [O_C + H_C + P_C]$$

$$TC_1(T) = A + h_c \left[\frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{a\alpha}{6}T^3 - \frac{ar}{6}T^3 \right] + p_c e^{-rM} \left[aT + \frac{b}{2}T^2 + \frac{a\alpha}{2}T^2 \right] \quad (8)$$

There are m cycles in the planning horizon L . Therefore, the present value of retailer's total cost over L is

$$TC(T) = \sum_{n=0}^{m-1} TC_1(T) e^{-rnT}$$

$$TC(T) = \left(\frac{1 - e^{-rnT}}{1 - e^{-rT}} \right) TC_1(T)$$

or

$$TC(T) = \left(\frac{L}{T} \right) \left[A + h_c \left\{ \frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{a\alpha}{6}T^3 - \frac{ar}{6}T^3 \right\} + p_c \left\{ aT + \frac{b}{2}T^2 + \frac{a\alpha}{2}T^2 - arMT - \frac{brM}{2}T^2 - \frac{a\alpha rM}{2}T^2 \right\} \right]$$

or

$$TC(T) = \frac{L}{T} \left[A + ap_c(1 - rM)T + \frac{1}{2}(ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c)T^2 + \frac{1}{6}(2bh_c + a\alpha h_c - arh_c)T^3 \right] \quad (9)$$

The necessary condition for $TC(T)$ to be minimum is

$$\frac{dTC(T)}{dT} = 0, \text{ and solving this equation, we get the optimum values of } T \text{ say } T^*.$$

The sufficient condition for $TC(T)$ to be minimum is

$$\frac{d^2TC(T)}{dT^2} > 0, \quad \forall T = T^*$$

From the equation (9), we get we obtain

$$\begin{aligned} \frac{dTC(T)}{dT} = \frac{L}{T} [ap_c(1-rM) + (ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c)T + \frac{1}{2}(2bh_c + a\alpha h_c - arh_c)T^2] \\ - \frac{L}{T^2} [A + ap_c(1-rM)T + \frac{1}{2}(ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c)T^2 + \frac{1}{6}(2bh_c + a\alpha h_c \\ - arh_c)T^3] \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d^2TC(T)}{dT^2} = \frac{L}{T} [(ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c) + (2ah_c + a\alpha h_c - arh_c)T - \frac{L}{T^2} [2ap_c(1-rM) \\ + 2(ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c)T + (2bh_c + a\alpha h_c - arh_c)T^2] + \frac{2L}{T^3} [A \\ + ap_c(1-rM)T + \frac{1}{2}(ah_c + bp_c + a\alpha p_c - brMp_c - ar\alpha Mp_c)T^2 + \frac{1}{6}(2bh_c + a\alpha h_c - arh_c)T^3] \end{aligned} \quad (11)$$

Fuzzy Model

Let us consider an inventory model in fuzzy environment, due to the uncertainty in the parameter r .

Let $r = (r_1, r_2, r_3)$ be a triangular fuzzy number. The present value of retailer's total cost over L is

$$\begin{aligned} \tilde{TC}(T) = \frac{L}{4T} [4A + 4ap_cT - ap_cM(r_1 + 2r_2 + r_3)T + 2\{a(h_c + \alpha p_c) + bp_c\}T^2 - \frac{a\alpha Mp_c(r_1 + 2r_2 + r_3)}{2}T^2 \\ - \frac{bMp_c(r_1 + 2r_2 + r_3)}{2}T^2 + \frac{2(2bp_c + a\alpha h_c)}{3}T^3 - \frac{ah_c(r_1 + 2r_2 + r_3)}{6}T^3] \end{aligned} \quad (12)$$

The necessary condition for $\tilde{TC}(T)$ to be minimum is

$$\frac{d\tilde{TC}(T)}{dT} = 0 \text{ and solving this equation, we get the optimum values of } T \text{ say } T^*.$$

The sufficient condition for $\tilde{TC}(T)$ to be minimum is

$$\frac{d\tilde{TC}(T)}{dT^2} > 0, \quad \forall T = T^*$$

From the equation (12), we get

$$\begin{aligned} \frac{d\tilde{TC}(T)}{dT} = & \frac{L}{4T} [4ap_c - aMp_c(r_1 + 2r_2 + r_3) + 4\{a(h_c + \alpha p_c) + bp_c\}T - a\alpha Mp_c(r_1 + 2r_2 + r_3)T \\ & - bMp_c(r_1 + 2r_2 + r_3)T + 2(2bp_c + a\alpha h_c)T^2 - \frac{ah_c(r_1 + 2r_2 + r_3)}{2}T^2] - \frac{L}{4T^2} [4A + 4ap_cT \\ & - aMp_c(r_1 + 2r_2 + r_3)T + 2\{a(h_c + \alpha p_c) + bp_c\}T^2 - \frac{a\alpha Mp_c(r_1 + 2r_2 + r_3)}{2}T^2 \\ & - \frac{bMp_c(r_1 + 2r_2 + r_3)}{2}T^2 + \frac{2(2bp_c + a\alpha h_c)}{3}T^3 - \frac{ah_c(r_1 + 2r_2 + r_3)}{6}T^3] \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d^2\tilde{TC}(T)}{dT^2} = & \frac{L}{4T} [4\{a(h_c + \alpha p_c) + bp_c\} - a\alpha Mp_c(r_1 + 2r_2 + r_3) - bMp_c(r_1 + 2r_2 + r_3) + 4(2bp_c + a\alpha h_c)T \\ & - ah_c(r_1 + 2r_2 + r_3)T] - \frac{L}{4T^2} [8ap_c - 2aM(r_1 + 2r_2 + r_3) + 8\{a(h_c + \alpha p_c) + bp_c\}T \\ & - 2a\alpha Mp_c(r_1 + 2r_2 + r_3)T - 2bMp_c(r_1 + 2r_2 + r_3)T + 4(2bp_c + a\alpha h_c)T^2 \\ & + \frac{L}{4T^3} [4A + 4ap_cT - aMp_c(r_1 + 2r_2 + r_3)T + 2\{a(h_c + \alpha p_c) + bp_c\}T^2 - \frac{a\alpha Mp_c(r_1 + 2r_2 + r_3)}{2}T^2 \\ & - \frac{bMp_c(r_1 + 2r_2 + r_3)}{2}T^2 + \frac{2(2bp_c + a\alpha h_c)}{3}T^3 - \frac{ah_c(r_1 + 2r_2 + r_3)}{6}T^3] \end{aligned} \quad (14)$$

Numerical example Let us consider the following data for parameters in appropriate units

$$a = 100, b = 3, k = 10, p_c = 6, \alpha = 12, A = 50, h_c = 5, p_c = 15, r = 0.05, M = 4, L = 80$$

Table 1 variation in total cost with r

r	T	TC
0.05	0.066810	196144
0.10	0.062048	164886
0.15	0.058174	134987
0.20	0.054943	106072
0.25	0.052119	77893

Here in table 1, as we increase r , the values of T and TC are decreased.

Table 2 variation in total cost with M

M	T	TC
4	0.066810	196144
8	0.062047	164887
12	0.058172	134991
16	0.054941	106077
20	0.052193	77900.262869

Here in table 2, as we increase M , the values of T and TC are decreased.

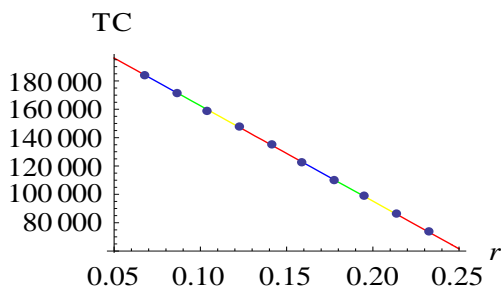


Figure 3 variation of total cost with r

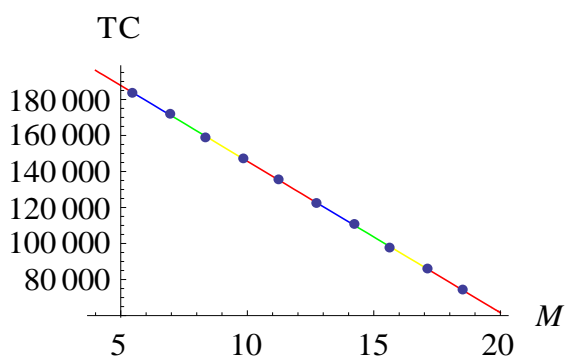


Figure 4 variation of total cost with M

When $M < T$, there are two possibilities,

1- Let $Pe^{rM} \int_0^M R(t)e^{-rt} dt > CQ$, The revenue earned by the retailer is more than the purchasing cost, so in this case no interest will be charged by the supplier although the credit period M is smaller than the replenishment cycle length T . Therefore, the present value of retailer's total will be same as that given equation (9).

2- Let $Pe^{rM} \int_0^M R(t)e^{-rt} dt < CQ$, The revenue earned by the retailer is less than the purchasing cost and the retailer has a budget to pay the remaining short purchasing cost, so in this case there is still no interest charged by the supplier, although the credit period M is smaller than the replenishment cycle length T . Therefore, the present value of retailer's total will be same as that given by the equation (9).

Fuzzy Model

Let $r = (0.05, 0.10, 0.15)$ be a triangular fuzzy number. The solution of the fuzzy inventory model is given by the signed distance method.

Table 3 variation in total cost with fuzzy number r

r	T	TC
$r = (0.05, 0.10, 0.15)$	0.092455	157842
$r = (0.10, 0.15, 0.20)$	0.110689	119292
$r = (0.15, 0.20, 0.25)$	0.145990	77096.239468

Conclusion

In the numerical analysis, we see that when the credit period is short then the retailer wants to order less and decreases the chargeable interest. When the credit period is large enough then the retailer wants to order more and he earns enough revenue on his sell to pay the purchasing cost. Therefore the credit period attracts the retailers to buy more or less.

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