International Journal of Scientific Research and Management (IJSRM)
||Volume||13||Issue||05||Pages||9090-9111||2025||
Website: https://ijsrm.net ISSN (e): 2321-3418
DOI: 10.18535/ijsrm/v13i05.em16

Short-Term Inflation Forecasting In Sierra Leone: A Comparison of Vector Autoregressive VAR(P), Arimax, And Arima Models

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Abstract
This study develops and compares three time series models, ARIMA, ARIMAX, and VAR(p) for short-term inflation forecasting in Sierra Leone to aid evidence-based monetary policy formulation. Using monthly data from January 2018 to June 2023 on the Consumer Price Index (CPI), exchange rate (EXR), and money supply (M2), the study first confirmed that all series are integrated of order one (I(1)) using Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The ARIMA and ARIMAX models were estimated using Box-Jenkins methodology, with the ARIMAX model incorporating the exchange rate as an exogenous variable. A VAR(p) model was also specified using differenced series after cointegration tests showed no long-run relationship. Model performance was evaluated over a 12-month out-of-sample period (July 2023–June 2024) using Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE). Results show that the ARIMAX model significantly outperformed ARIMA and VAR(p), achieving the lowest forecast errors (MAPE = 2.03%, RMSE = 4.76), reflecting a 71.8% and 83.3% improvement in MAPE over ARIMA and VAR(p), respectively. These findings confirm the exchange rate as a critical driver of short-term inflation in Sierra Leone. The superior performance of the ARIMAX model underscores the importance of including exogenous (exchange rate) information in inflation forecasting frameworks. Policymakers are advised to closely monitor exchange rate movements as a leading indicator of inflation, highlighting the centrality of exchange rate pass-through effects in inflation dynamics and to consider exogenous-variable-enriched models like ARIMAX for effective short-term inflation targeting.

Keywords: Inflation Forecasting, ARIMA, ARIMAX, VAR, Exchange Rate, Forecast Accuracy, Monetary Policy

JEL Classification: C32, E31, E37, E52

Introduction

1.1.0 Background

Monetary policy refers to the set of laws and actions adopted by central banks to achieve statutory goals, with the maintenance of low and stable inflation being a primary objective in both developing and advanced economies. In this context, central banks including the Bank of Sierra Leone require credible and data-driven forecasting models to guide policy decisions that promote price stability and support economic growth.

This study is motivated by the practical and theoretical need to identify forecasting models suited to economies with limited data availability and heightened sensitivity to external shocks. Single-equation models, such as the Autoregressive Integrated Moving Average (ARIMA) and its variant with exogenous variables (ARIMAX), are attractive for their simplicity and data efficiency. Conversely, Vector Autoregressive (VAR) models capture interrelationships among multiple macroeconomic indicators, offering a more system-wide view of inflation dynamics.

The inclusion of both model types enables a comparative analysis of forecasting performance, particularly in Sierra Leone's economic environment, where external variables, most notably the exchange rate have played a critical role in shaping inflation trends. This consideration is especially relevant as the Bank of Sierra

Leone prepares to transition from a monetary targeting regime to an inflation targeting (IT) framework, which demands more accurate and forward-looking inflation forecasts for effective policy formulation.
The objective of this study is to develop short-term inflation forecasting models for Sierra Leone using the ARIMA, ARIMAX, and $VAR_{(P)}$ approaches, and to evaluate their predictive performance using standard forecast accuracy tests such as MAPE and RMSE**.**

1.1.1 Overview of Inflation Dynamics in Sierra Leone

Inflation in Sierra Leone has historically been driven by a combination of external shocks and domestic macroeconomic imbalances. During the 1980s and early 1990s, inflation surged in response to excessive monetary expansion, persistent exchange rate depreciation, and adverse external conditions. The 1985 devaluation of the Leone led to an inflation rate of 78.1% and a money supply growth of 66.5%. These pressures were further compounded by chronic foreign exchange shortages and widening fiscal deficits, often financed through the banking system. By 1987, inflation had peaked at 178.7%, alongside a money supply growth of 162.9% (Bank of Sierra Leone Annual Reports, 1980–2017).
As a small, open economy, Sierra Leone has also been highly susceptible to imported inflation, particularly from global oil price shocks dating back to the 1970s. Dependence on imports and administrative price controls have amplified the transmission of these external price movements (Bank of Sierra Leone Annual Reports, 1980–2017).
The civil conflict in the 1990s significantly worsened inflationary pressures. The January 6, 1999, rebel invasion of Freetown severely disrupted economic activity and supply chains, leading to acute shortages. Inflation spiked again between 2006 and 2009 due to rising global and domestic food and fuel prices.
Following the return to democratic governance in 1996, macroeconomic management gradually improved. By 2000, GDP growth had reached 3.8%, and inflation fell to -2.75% in December 2000, down from 36.74% the year before. Between 2007 and April 2014, inflation steadily declined, largely attributed to improve domestic food production particularly rice and other staples.
More recently, Sierra Leone's inflation trajectory has been shaped by major global disruptions. The Ebola outbreak, COVID-19 pandemic, and the Russia-Ukraine war all disrupted supply chains and triggered sharp increases in food and fuel prices. These shocks, combined with significant Leone depreciation, pushed inflation from 10.45% in December 2020 to 17.94% in December 2021, and further to 37.09% by December 2022. Inflation peaked at 52.16% in December 2023, driven by rising costs in both food and non-food components (Bank of Sierra Leone Annual Reports 2018–2024 and Statistics Sierra Leone Monthly Publications).
In response, the Bank of Sierra Leone adopted prudent monetary policy measures, including the tightening of the Monetary Policy Rate (MPR), which helped stabilize the exchange rate throughout 2024. As a result, inflationary pressures began to ease, with headline inflation declining from 47.42% in January to 31.93% by June 2024 (Bank of Sierra Leone Annual Reports 2018–2024 and Statistics Sierra Leone Monthly Publications).

1.1.2 Specific Objective

The objective of this paper is to develop short-term forecasting models for Sierra Leone's inflation rate using the Box-Jenkins ARIMA approach, the ARIMAX method, and the $VAR_{(P)}$ model. Forecast accuracy tests, such as MAPE and RMSE, have been applied to select the model with the highest predictive power.

1.1.3 Specific Contribution to Monetary Policy at the Bank of Sierra Leone

This paper contributes to monetary policy at the Bank of Sierra Leone (BSL) by developing an enhanced inflation forecasting framework, demonstrating that the ARIMAX model incorporating exchange rate dynamics outperforms traditional ARIMA and VAR models in predicting inflation. Given Sierra Leone's high exchange rate pass-through, where the Leone's depreciation has coincided with rising inflation (from 13.09% in 2019 to 54.20% in 2023), the study provides BSL with a data-driven tool to anticipate inflationary pressures more accurately. The findings underscore the critical role of exchange rate stability in inflation control, recommending that BSL integrate exchange rate trends into its forecasting models and adopt policies to mitigate exchange rate volatility. By improving short-term inflation projections, this research supports more proactive and effective monetary policy decisions, ultimately strengthening BSL's ability to achieve price stability
This paper is organized under the following headings: Introduction (provides an overview of inflation

dynamics in Sierra Leone & review of the relevant literature), Materials and Methods (describes the methodology and data), Results (presentation of data analysis), Discussions (interpretation of results), and Conclusion (summary & policy implication).

1.2 Literature Review

1.2.0 Theoretical Literature

Inflation dynamics in Sierra Leone are shaped by a mix of domestic and external factors. The country is highly dependent on imports particularly food and fuel which makes it vulnerable to global price shocks and exchange rate volatility. Domestically, inflation is driven by supply-side constraints such as low agricultural productivity and inadequate infrastructure. In response, the Bank of Sierra Leone has adopted tight monetary policy measures in recent years, including raising the Monetary Policy Rate (MPR) to help contain inflation. However, fiscal pressures and recurrent government borrowing continue to fuel inflationary pressures, underscoring the importance of improved coordination between fiscal and monetary policies.

To understand inflation more broadly, it is helpful to explore key economic theories. Two dominant schools of thought, the monetarist and Keynesian views have long debated the causes of inflation. Monetarists, led by Milton Friedman (Nobel Laureate, 1976), advocate the quantity theory of money, asserting that "inflation is always and everywhere a monetary phenomenon." According to this view, inflation occurs when the money supply grows faster than output.

Classical economists, notably Irving Fisher, also supported the quantity theory. Fisher's equation of exchange ($MV = PT$) posits that the money supply (M) times its velocity (V) equals the price level (P) times the volume of transactions (T). Assuming V and T are stable, increases in M result in proportional increases in P, that is, inflation. Thus, classical and monetarist economists concluded that controlling the money supply is key to controlling inflation.

In contrast, Keynesian economists attribute inflation to three main sources: demand-pull inflation, cost-push inflation, and profit-driven price increases. Keynes (1936) emphasized that output and employment are demand-driven, and inflation arises when aggregate demand exceeds aggregate supply. This excess demand pushes prices upward a phenomenon known as demand-pull inflation.

Keynesians also highlight cost-push inflation, where rising production costs often driven by strong labour unions, monopolistic pricing, or increased import prices force firms to raise prices. For example, global increases in the cost of imported raw materials can lead to domestic inflation, especially in countries reliant on imports like Sierra Leone.

The Phillips Curve adds another dimension to Keynesian theory by illustrating an inverse relationship between unemployment and inflation. Based on UK data from 1861 to 1957, A.W. Phillips suggested that reducing unemployment could result in higher inflation. However, the simultaneous rise in both inflation and unemployment during the 1970s (stagflation) challenged this view.

Monetarist Milton Friedman explained stagflation through the expectations-augmented Phillips Curve. He argued that inflation expectations influence wage and price setting. When people anticipate rising inflation, workers demand higher wages, and firms raise prices, shifting the Phillips Curve to the right. This mechanism explains how inflation and unemployment can increase simultaneously.

Ultimately, the key distinction between Keynesians and monetarists lies in their view of central banks. Keynesians see them as inflation fighters, using tools like interest rate changes to manage demand and maintain stability. Monetarists, however, caution that central banks, by mismanaging money supply growth, are often the source of inflation. They argue that inflation control hinges on aligning money supply growth with the real demand for money.

1.2.1 Empirical Literature

Alnaa and Ahiakpor (2011) conducted a study on short-term inflation forecasting in Ghana using the Box-Jenkins ARIMA framework. After applying unit root tests and evaluating model performance with Root Mean Squared Error (RMSE), they concluded that the ARIMA (6,1,6) model was most appropriate for forecasting inflation in Ghana.

Similarly, Doguwa and Alade (2013) developed short-term inflation forecasting models for Nigeria. Their study compared four models using SARIMA and SARIMAX procedures. The results showed that the SARIMAX model outperformed others for headline inflation forecasting using all-items CPI, while the SARIMA model consistently outperformed for core inflation forecasting.

Iqbal and Naveed (2016) compared the forecasting performance of various ARIMA models for Pakistan's inflation. Using quarterly CPI data and the Box-Jenkins methodology, they found that the ARIMA (2,1,1) model was most suitable for short-term inflation forecasting.

Ekpenyong and Udoudo (2016) used seasonal ARIMA models to develop short-term inflation forecasts for Nigeria. Based on monthly inflation data from 2000 to 2015, they identified SARIMA (0,1,0)(0,1,1)[12] as the best-performing model.

Similarly, Okafor and Shaibu (2013) applied a univariate ARIMA model to Nigerian inflation data from 1981 to 2010 and found ARIMA (2,2,3) to be optimal, based on model identification, parameter estimation, diagnostic checks, and forecasting accuracy.

Olajide et al. (2012) applied the Box-Jenkins ARIMA approach to forecast Nigeria's inflation using annual CPI data from 1961 to 2010. Their analysis revealed that the CPI series was integrated of order one, and ARIMA (1,1,1) was selected as the appropriate model.

Faisal (2012) employed several ARIMA models to forecast Bangladesh's inflation using monthly CPI data. The study emphasized the importance of reliable inflation forecasts in achieving macroeconomic targets, such as economic growth, exchange rate stability, and inflation control. The selected ARIMA model outperformed other univariate models in forecasting accuracy.

Empirical work on short-term inflation forecasting is not confined to ARIMA models. Adjepong and Oduro (2013) compared the forecasting performance of the Holt-Winters exponential smoothing method with a Seasonal ARIMA model for Ghana. Based on accuracy metrics (MAE, RMSE, MAPE, and MASE), the Seasonal ARIMA model outperformed the Holt-Winters method.

Jere and Siyanga (2016) forecasted Zambia's inflation using monthly CPI data from 2010 to 2014. They estimated both Holt's exponential smoothing and ARIMA models. Although both models performed well, Holt's method was preferred for its simplicity and ease of implementation compared to the ARIMA model, which requires more sophisticated tools.

Salam et al. (2007) modelled Pakistan's inflation using monthly CPI data and ARIMA models. Their study highlighted the value of reliable inflation forecasts for policymaking and business decision-making. Their selected ARIMA model demonstrated strong predictive capabilities in both in-sample and out-of-sample tests.

Meyler, Kenny, and Quinn (1998) applied ARIMA models to forecast inflation in Ireland using the Harmonized Index of Consumer Prices (HICP). Their study found that ARIMA models outperformed alternative approaches in forecasting accuracy.

In the case of Sierra Leone, empirical studies are limited. Pearce, Alpha, and Pingfeng (2015) examined inflation determinants and found that in the short run, money supply and GDP had a significant positive influence on inflation. However, they reported no significant relationship between inflation and either the exchange rate or imports.

In contrast, Gottschalk, Kalonji, and Miyajima (2008) found that inflation in Sierra Leone is largely driven by nominal exchange rate depreciation, higher oil prices, and money supply growth. Using a VAR model, they suggested that this approach is suitable for inflation forecasting in data-constrained environments like Sierra Leone.

Kallon (1994), in his study *"An Econometric Analysis of Inflation in Sierra Leone,"* adopted a reduced-form inflation equation derived from an IS-LM framework. His results rejected the monetarist proposition that changes in money supply led to proportional changes in inflation in the short run, though he found support for this view in the long run.

Kabundi (2012) investigated inflation dynamics in Uganda using a single-equation Error Correction Model (ECM) grounded in the quantity theory of money. The study identified both domestic and external factors such as monetary aggregates, global food prices, and agricultural supply and demand as long-term determinants of inflation, though it did not produce inflation forecasts.

Kelikume and Salami (2014) compared the forecasting performance of VAR and ARIMA models for Nigeria. Their findings indicated that the VAR model yielded more accurate forecasts by minimizing squared errors.

Mokoti et al. (2009) evaluated the forecasting performance of the Bank of Botswana's short-term inflation models, including the Near-Term Forecasting (NTF) model, ARIMA, and VAR models. Based on forecast evaluation metrics (MAE, RMSE, and Theil's U statistic), the NTF model generally outperformed the others.

Pufnik and Kunovac (2006) compared structural and statistical models for forecasting Croatia's inflation. They noted that while structural models offer deeper economic insights, statistical models like ARIMA provide simpler, behaviour-based forecasts.
Valley (2002) from the Central Bank of Guatemala compared the performance of VAR and ARIMA models in forecasting CPI. The ARIMA model produced more accurate and stable forecasts, particularly during periods of structural change.
Finally, Robinson (1998) from the Central Bank of Jamaica employed a VAR model to forecast inflation, incorporating macroeconomic variables such as CPI, exchange rate, and interest rates. His results showed that the VAR model provided superior predictive accuracy compared to univariate models.

3.0 Materials and Methods

3.1 Background to Modelling Time Series Data

To investigate the most suitable short-term forecasting model for inflation in Sierra Leone, this study utilizes data on the national Consumer Price Index (CPI), money supply (M2), and nominal exchange rate (EXR). In time series analysis, the first critical step is to assess the stationarity of the data. This is essential because modelling non-stationary series can lead to *spurious regression results*, where standard diagnostic statistics such as the t-statistic, R^2, and Durbin-Watson statistic become unreliable and misleading.
A visual inspection of time series plots typically serves as the initial diagnostic step. Such plots can help identify fundamental characteristics of the data, including **trends** (upward or downward shifts in the mean over time), **seasonal patterns**, **outliers** (extreme or anomalous values), and *structural breaks* or discontinuities. Recognizing these features is important because they affect the validity of statistical inference in time series models.
To ensure robust and meaningful analysis, researchers must formally test for stationarity, most commonly using *unit root tests* such as the Augmented Dickey-Fuller (ADF) or Phillips-Perron (PP) test. If the time series is found to be non-stationary, it must be transformed typically through differencing before model estimation can proceed. This step helps to stabilize the mean and variance of the series, making it suitable for econometric modelling and reliable forecasting.

3.2 Estimation Technique

This study applies three existing models:
(i) an Autoregressive Integrated Moving Average (ARIMA) model for the Consumer Price Index (CPI),
(ii) an ARIMAX model incorporating EXR as an exogenous variable, and
(iii) a Vector Autoregressive (VAR) model for CPI, EXR, and M2.
The ARIMA and ARIMAX modelling techniques follow these basic steps:
Model Identification, Model Estimation, Model Diagnostics and Forecasting or Predicting Future Values
Two distinct methodologies are applied in this study. The first is based on the Box-Jenkins (1976) Autoregressive Integrated Moving Average (ARIMA) approach, which is an a-theoretical framework that relies solely on the statistical properties of the data without imposing theoretical economic relationships, used for both ARIMA and ARIMAX models.
The second methodology involves estimating a Vector Autoregressive VAR(p) model for CPI, EXR, and M2 which is a system-based approach where all variables enter the model as endogenous variables. Prior to estimating the VAR model, the study conducted *cointegration tests* using the Johansen procedure to determine the existence of any long-run equilibrium relationships among the variables.
If the *Trace test or Maximum Eigenvalue test* indicates cointegration, a *Vector Error Correction Model (VECM)* is estimated. This allows for the modelling of both short-run adjustments and long-run equilibrium dynamics.
If no cointegration is found, *as is the case in this study*, the variables are differenced, and a **VAR(p)** model is estimated using the first differences of the series.
It is important to note that two non-stationary variables are said to be cointegrated if a linear combination of them is stationary. In this study, since the Johansen cointegration test indicated *no cointegration*, a VAR(p) model was estimated using the first-differenced data.

3.3 Data Source and Description

The dataset used in this study consists of three-monthly variables: the Consumer Price Index (CPI) series compiled by Statistics Sierra Leone, the nominal exchange rate (EXR) and money supply (M2), both compiled by the Bank of Sierra Leone. The study period spans *January 2018 to June 2023* for model estimation. Data from *July 2023 to June 2024* is reserved for *out-of-sample forecast accuracy evaluation*.

The choice of January 2018 as the starting point is based on several considerations: It aligns with the **most recent rebasing** of the CPI by Statistics Sierra Leone, ensuring methodological consistency, it reflects *contemporary economic conditions*, avoiding distortions from out-dated policy regimes and structural dynamics and it improves *computational efficiency* and forecasting relevance by focusing on recent and reliable data. By restricting the sample to this period, the study ensures robustness and avoids the risk of structural breaks that may compromise model accuracy.

3.4 Model Specification And Estimation Technique of Arima And Arimax Models

1. ARIMA Model

The general equation of an ARIMA(p,d,q) model is given as follows:

$Y_t = c + \phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$ ------------------------------ (1)

Where:

$\phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p}$ is the Autoregressive (AR) part,

$\theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$ is the Moving Average (MA) part,

ε_t is the error term.

As noted above, ARIMA models do not accommodate additional predictors within the model. In contrast, dynamic regression models allow the incorporation of other predictor variables (covariates), which can provide relevant information for the analysis. A general ARIMAX model could be specified as follows:

2. ARIMAX Model - General Form

A general ARIMAX model can be expressed as:

$\hat{y}_t = f(Y_{t-1}, Y_{t-2}, \ldots, X_{t-1}, X_{t-2}, \ldots)$ --(2)

3. ARIMAX Model - Linear Form

$Y_t = c + \phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t + \beta X_t$ ------------------ (3)

Here, β is the coefficient of the exogenous variable X_t.

4. ARIMAX Model - Reparametrized

$Y_t = \beta X_t + \eta_t$ --- (4a)

$\eta_t = \phi_1 \eta_{t-1} + \phi_2 \eta_{t-2} + \ldots + \phi_p \eta_{t-p} - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} + \varepsilon_t$ ------------------------(4b)

Where:

η_t represents the ARMA component of the residuals,

βX_t captures the effect of the exogenous variable, making β interpretable in its usual form.

3.5 Model Specification And Estimation Technique of the VAR(p)

This study employs three existing models, two of which have been thoroughly explained in the previous sections. The third model, the Vector Autoregressive VAR(p) model, is estimated to capture additional information that could help explain inflation dynamics, moving beyond just replying on the past values of the CPI series to predict current and future inflation. Specifically, a 3-variable VAR(p) model is estimated, incorporating the following variables:

CPI (Consumer Price Index), representing inflation, EXR (Nominal Exchange Rate) and M2 (Money Supply)

Equation (5): The equation of CPI is given by:

$$\ln CPI_t = \alpha_1 + \sum_{i=1}^{p} \beta_{1i} \ln CPI_{t-i} + \sum_{j=1}^{p} \phi_{1j} \ln EXR_{t-j} + \sum_{m=1}^{p} \sigma_{1m} \ln M2_{t-m} + \in_{1t}$$ ------------ (5)

Where α_1 is the constant term, β_{1i}, ϕ_{1i}, σ_{1i} are the coefficients for the lagged values of CPI, EXR, and M2 respectively, and ε_{1t} is the residual.

Equation (6): The equation for EXR (Nominal Exchange Rate) is given by:

$$\ln M2_t = \alpha_2 + \sum_{i=1}^{p} \beta_{2i} \ln CPI_{t-i} + \sum_{j=1}^{p} \phi_{2j} \ln EXR_{t-j} + \sum_{m=1}^{p} \sigma_{2m} \ln M2_{t-m} + \in_{2t}$$ ------------ (6)

Where α_2 is the constant term, β_{2i}, ϕ_{2i}, σ_{2i} are the coefficients for the lagged values of CPI, EXR, and M2

respectively, and ε_{2t} is the residual.
Equation (7): The equation for M2 (Money Supply) is given by:

$$\ln EXR_t = \alpha_3 + \sum_{i=1}^{p} \beta_{3i} \ln CPI_{t-i} + \sum_{j=1}^{p} \phi_{3j} \ln EXR_{t-j} + \sum_{m=1}^{p} \sigma_{3m} \ln M2_{t-m} + \in_{3t} \quad \text{----------- (7)}$$

Where α_3 is the constant term, β_{3i}, ϕ_{3i}, σ_{3i} are the coefficients for the lagged values of CPI, EXR, and M2 respectively, and ε_{3t} is the residual.
In these equations, each variable is a function of its own past values as well as the past values of the other variables in the system. The lag length, p=2, was selected based on *Information Criteria*, ensuring that the model adequately captures the dynamics between the variables without over fitting. The variables in the VAR model were re-estimated using the optimal lag-length of 2 (two). A key characteristics of a VAR(p) model is its stability. Stability ensures that the system generates stationary time series, with time-invariant means, variances and covariance structures, provided that the initial conditions are appropriate. Stability in the VAR model can be assessed by examining the *moduli of the eigenvalues* of the companion matrix. For stability to hold, the moduli of all eigenvalues must be *less than one* (i.e., the system must be stable). In this research work, the stability condition was satisfied, confirming that the VAR system is stable.
Diagnostic Tests
Serial correlation: The VAR model was tested for serial correlation in the residuals. This is important because, an absence of serial correlation in the residuals will suggest that the model adequately captures the relationships among the variables.
Heteroskedasticity: The residuals were further examined for constant variance (homoscedasticity), thereby ensuring that the variances of the residuals do not change over time.
Normality: The residuals of the model were tested for normality. This is crucial for the validity of the statistical inference.
Forecasting: Lastly, the VAR(p) model was used to generate *a 12-month ahead* forecast of CPI, based on the estimated relationships among CPI, EXR, and M2. The forecast from the VAR(p) could provide value insights into the future inflation trends.

4.0 Results
4.1 Presentation and Discussion of Results
This paper focuses on forecasting the *Consumer Price Index (CPI)* in its level form rather than the inflation rate to avoid potential distortions caused by data transformations. Using CPI in levels preserves crucial information about overall price trends and seasonality, which could be lost if transformed into inflation. This approach also ensures consistency across variables, as key factors like *exchange rate (EXR)* and *money supply (M2)* are typically modelled in levels. Additionally, ARIMA, ARIMAX, and VAR models are well-suited to handle CPI in levels, provided stationarity is addressed. The use of CPI in levels enhances the interpretation of relationships within these models, offering more meaningful insights. The methodology aligns with prior studies, including Robinson (1998), Valley (2002), and others, who used CPI for forecasting rather than inflation. More importantly, at the Central Bank of Sierra Leone, CPI is forecast before its associated inflation is calculated. Therefore, I want my work to be in conformity with the Bank and hence the justification for using CPI rather than Inflation rate in this study.
Table1 displays the descriptive statistics of the national CPI, M2 (money supply) and the EXR (nominal exchange rate) series in Sierra Leone. The maximum values of the series are 171.11, 22.70 and 23005.61 for CPI, Exchange Rate and Money Supply respectively. Also, the minimum values are 59.99, 7.54, and 6515.769 for CPI, Exchange Rate and Money Supply respectively. The CPI, Exchange Rates and Money Supply series display upward trends.

Table 1: summary statistics for CPI, EXR and M2

STATISTIC	CPI	EXR	M2
Mean	**93.089**	**11.215**	**11806.00**
Median	**85.045**	**9.942**	**10802.03**
maximum	**171.11**	**22.70**	**23005.61**

minimum	**59.988**	**7.54**	**6515.769**
Std.Dev	**27.83**	**3.82**	**4703.244**
Sum	**6143.877**	**740.175**	**779196.3**
Number of observations	**66**	**66**	**66**

Source: author's computation

4.2 Time plot of the CPI, EXR and M2 Series

Clearly from figures 1, 2 & 3, plots of the national consumer price index, nominal exchange rate and money supply depict exponentially increasing trends with unstable means as they keep increasing and decreasing at certain points.

Figures 1, 2, & 3 are time plots of the CPI, NER, & M2 Series

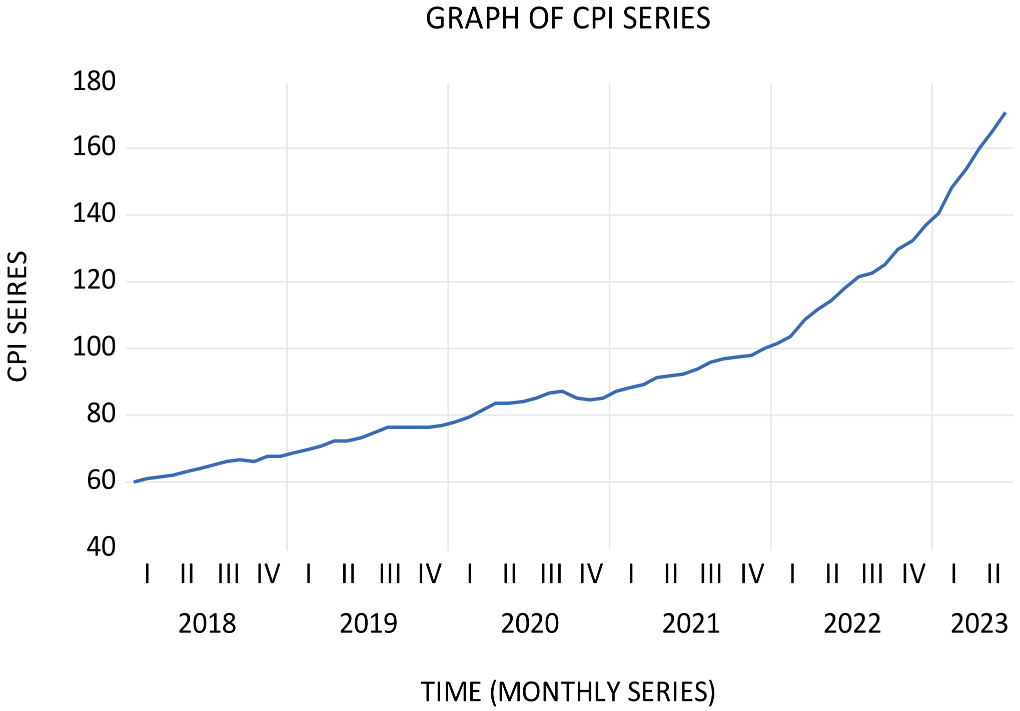


Fig. 1 Graph of the CPI series at levels depicts upward trend and non-constant mean indicating non-stationarity

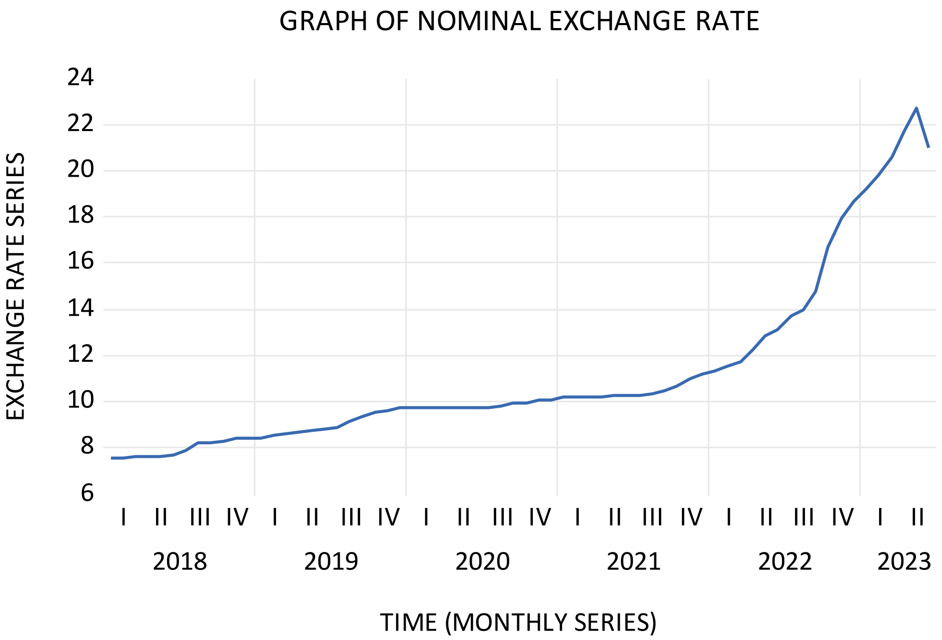


Fig. 2 Graph of the nominal exchange (EXR) rate at levels depicts upward trend and non-constant mean indicating non-stationarity.

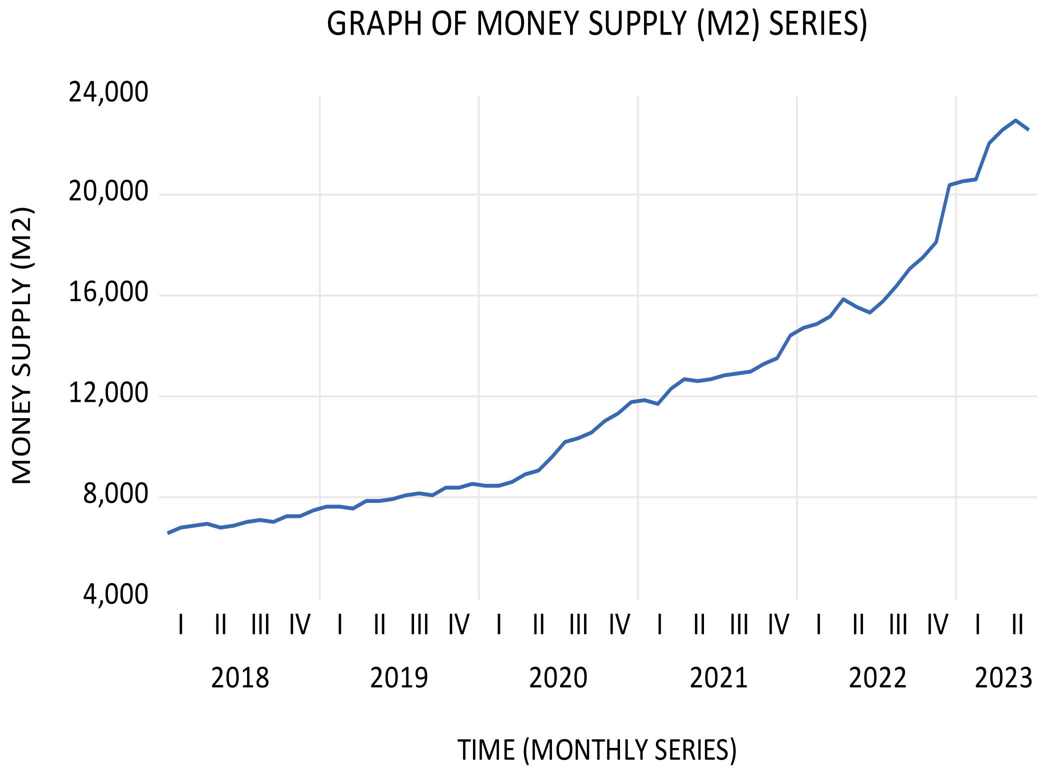


Fig. 3 Graph of money supply (M2) at levels depicts upward trend non-constant mean indicating non-stationarity

4.3 Time plot of the log-difference of CPI, EXR and, M2 series

The plots of the log-differenced CPI, EXR and M2 series show evidence of mean-reversion. Indicating that, graphically, the differenced log-transformed series are stationary. These plots are displayed below.

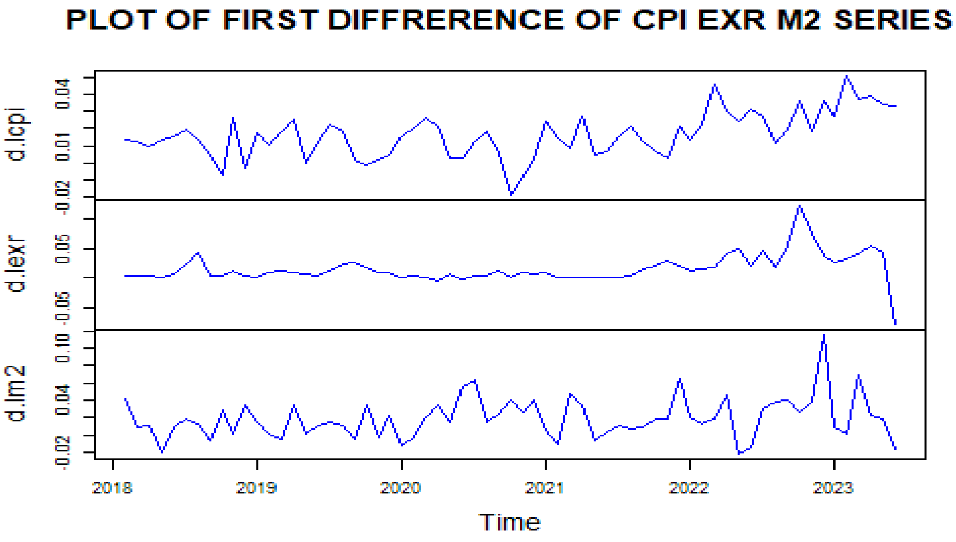


Figure.5 graph of the first differenced of the log-transformed series (CPI, EXR & M2) show mean reversion

4.4 Unit Root Test

In this study, the Augmented Dickey-fuller and Phillips-Perron unit root tests have been applied to check for the stationarity of the log-transformed series of CPI, EXR and M2 and their differenced series.

Table.2a ADF ***Unit Root test for logCPI, logEXR, logM2 and first differences***

Test_at_Level	t-statistic	P_value
Augmented Dickey-Fuller, [LCPI]	0.859056	0.9998
Augmented Dickey-Fuller, [LEXR]	-2.960826	0.1525
Augmented Dickey-Fuller, [LM2]	-1.564005	0.7964

Test_at_first_difference	t-statistic	P_value
Augmented Dickey-Fuller, [DLCPI]	-5.518721	0.0001
Augmented Dickey-Fuller, [DLEXR]	-3.622296	0.0367
Augmented Dickey-Fuller, [DLM2]	-7.603337	0.0000

Table.2b PP Unit Root test results ***for logCPI, logEXR, logM2 and first differences***

Test	t-statistics	P-value
PP.test (LCPI)	2.129030	1.0000
PP.test (DLCPI))	-5.518721	0.0001
PP.test (LEXR)	-0.113180	0.9936
PP.test (DLEXR)	-3.071499	0.0338
PP.test (LM2)	-1.545377	0.8034
PP.test (DLM2)	-7.603698	0.0000

4.4 Unit Root Test Results

Tables 2a & 2b present the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root test results for the CPI, EXR, and M2 series in both levels and first differences. Key findings are summarized below:

Tests in Levels: For all three series (CPI, EXR, M2), both the ADF and PP unit root tests fail to reject the null hypothesis of a unit root ($p > 0.05$), indicating non-stationarity at levels.

Tests in First Differences: After applying first-order differencing (denoted as ΔLCPI, ΔLEXR, ΔLM2), the results change decisively: Both ADF and PP unit root tests reject the null hypothesis ($p < 0.05$), confirming stationarity in the differenced series and the lag structure (selected via AIC/SIC) and bandwidth (for PP) support robust inference.

4.5 Model Identification

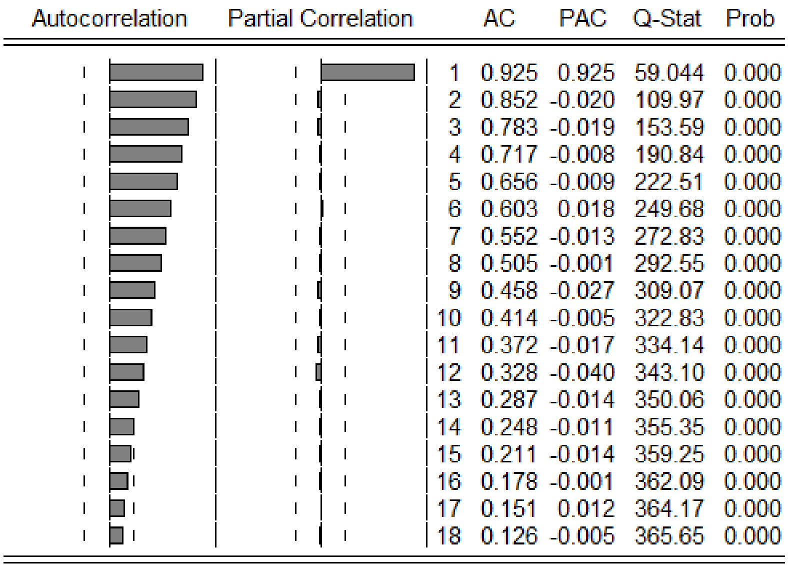
Initial Assessment (Levels):

The ACF (autocorrelation function) and PACF (partial autocorrelation function) plots of the CPI series in levels (Figure 6) suggest non-stationarity, as evidenced by; ACF plot has Large initial spikes that persist without rapid decay toward zero, and indicating a slow-moving or trending process, while the PACF plot has a statistically significant spike at lag 1, with subsequent lags diminishing abruptly. This pattern is characteristic of a unit root process.

Transformation to Stationarity;

After applying first-order non-seasonal differencing, the differenced CPI series (Figure 7) exhibits behaviour consistent with stationarity; The ACF and PACF plots now show rapid decay after a few lags, with no significant autocorrelations beyond the 95% confidence intervals. This confirms that the integrated (I) component of the ARIMA model should be set to $d = 1$.

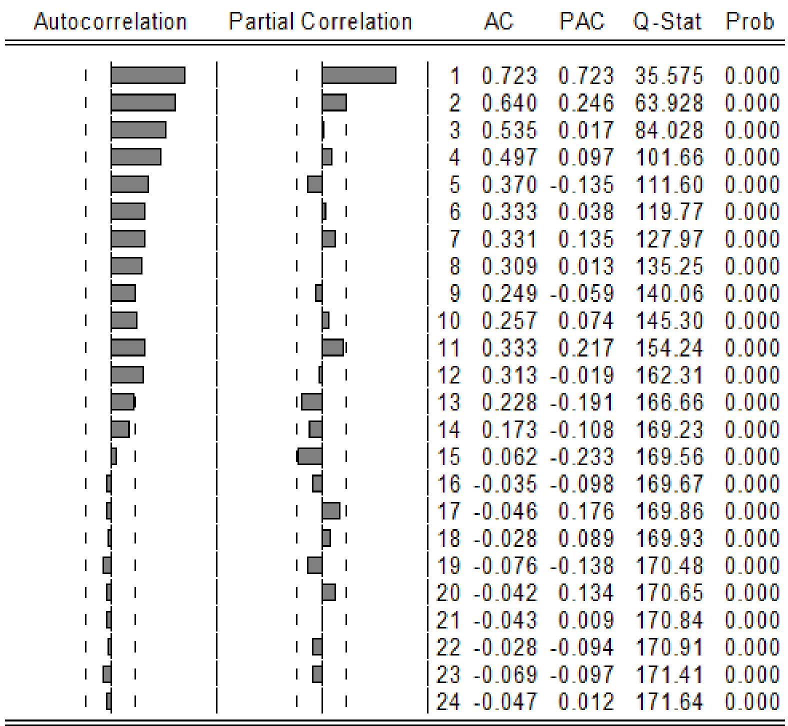
Fig. 6 ACF and PACF plots of CPI SERIES at levels exhibiting large spikes without decaying rapidly to zero



Lag	AC	PAC	Q-Stat	Prob
1	0.925	0.925	59.044	0.000
2	0.852	-0.020	109.97	0.000
3	0.783	-0.019	153.59	0.000
4	0.717	-0.008	190.84	0.000
5	0.656	-0.009	222.51	0.000
6	0.603	0.018	249.68	0.000
7	0.552	-0.013	272.83	0.000
8	0.505	-0.001	292.55	0.000
9	0.458	-0.027	309.07	0.000
10	0.414	-0.005	322.83	0.000
11	0.372	-0.017	334.14	0.000
12	0.328	-0.040	343.10	0.000
13	0.287	-0.014	350.06	0.000
14	0.248	-0.011	355.35	0.000
15	0.211	-0.014	359.25	0.000
16	0.178	-0.001	362.09	0.000
17	0.151	0.012	364.17	0.000
18	0.126	-0.005	365.65	0.000

Fig. 6 ACF and PACF of the level series

Fig. 7 ACF and PAC plots of the first difference of CPI series showing rapid decay after a few lags, with no significant autocorrelations beyond the 95% confidence intervals



Lag	AC	PAC	Q-Stat	Prob
1	0.723	0.723	35.575	0.000
2	0.640	0.246	63.928	0.000
3	0.535	0.017	84.028	0.000
4	0.497	0.097	101.66	0.000
5	0.370	-0.135	111.60	0.000
6	0.333	0.038	119.77	0.000
7	0.331	0.135	127.97	0.000
8	0.309	0.013	135.25	0.000
9	0.249	-0.059	140.06	0.000
10	0.257	0.074	145.30	0.000
11	0.333	0.217	154.24	0.000
12	0.313	-0.019	162.31	0.000
13	0.228	-0.191	166.66	0.000
14	0.173	-0.108	169.23	0.000
15	0.062	-0.233	169.56	0.000
16	-0.035	-0.098	169.67	0.000
17	-0.046	0.176	169.86	0.000
18	-0.028	0.089	169.93	0.000
19	-0.076	-0.138	170.48	0.000
20	-0.042	0.134	170.65	0.000
21	-0.043	0.009	170.84	0.000
22	-0.028	-0.094	170.91	0.000
23	-0.069	-0.097	171.41	0.000
24	-0.047	0.012	171.64	0.000

Fig.7 ACF and PACF of the First Difference Series

4.6 Model Estimation

Model selection was based on the BIC and AIC criteria. After multiple iterations, the following models were estimated using the differenced, log-transformed CPI series.

Table 3: Model selection criteria for the ARIMA model

Model	AIC	Hann-Quinn	BIC
ARIMA(0,1,1)	-5.847	-5.947	--5.908

ARIMA(1,1,0)	**-6.104**	-6.064	**-6.064**
ARIMA(1,1,1)	--56.018	-5.966	-5.885

4.6 Model Selection

The optimal ARIMA specification was determined through an iterative process comparing multiple candidate models using the **Akaike Information Criterion (AIC)** and **Bayesian Information Criterion (BIC)**. Three competitive models were evaluated in the final selection stage. As shown in Table [3], the **ARIMA(1,1,0)** specification demonstrated; the **lowest AIC value** (-6.104), indicating superior goodness-of-fit while penalizing complexity and the **minimal BIC value** (-6.064), confirming parsimony and statistical efficiency.

This model was therefore selected as the most appropriate for forecasting the national CPI series, as it optimally balances; **Model fit** (capturing key patterns in the data), **Parsimony** (avoiding over parameterization), and **Theoretical consistency** (aligning with the unit root and ACF/PACF diagnostics)

4.7 Diagnostic Testing for ARIMA (1,1,0) Model

Prior to implementation in the ARIMAX framework, the selected ARIMA(1,1,0) model underwent rigorous diagnostic evaluation to verify its statistical adequacy:

4.8 Stability Test

The stability of the ARIMA (1,1,0) specification was verified through rigorous examination of its characteristic roots:

Root Location Analysis: All inverse roots of the AR polynomial fall strictly within the unit circle and the dominant root modulus of 0.353918 confirms exponential decay in impulse responses.

Stationarity Verification: Absolute value of AR coefficient ($\varphi_1 = 0.353918$ satisfy $|\varphi_1| < 1$ condition and the Companion matrix eigenvalues lie within the complex unit circle

Implications for the model

The stability condition ensures; Finite variance and mean-reverting behaviour, Convergent forecast intervals, and non-explosive long-term projections

These results mathematically validate the stationarity of the differenced series and the suitability of the ARIMA (1,1,0) specification for forecasting. The stable configuration suggests that shocks to the CPI series will dissipate over time at a rate determined by the AR(1) coefficient.

Table4a. Stability test for the ARIMA(1,1,0)

SAMPLE: 2018M01 – 2023M06 INCLUDED OBSERVATIONS :65		
AR ROOTS	MODULUS	CYCLE
0.353918	0.353918	

No root lies outside the unit circle

ARMA model is stationary

Table 4b. ACF Plot of ARIMA(1,1,0)

The Ljung-Box Q-statistic p-values for residual autocorrelation are all statistically insignificant ($p > 0.05$), indicating no remaining serial correlation in the model residuals. This supports the adequacy of the ARIMA(1,1,0) specification.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.070	-0.070	0.3364	0.562
		2	0.159	0.155	2.0794	0.354
		3	-0.046	-0.027	2.2305	0.526
		4	0.113	0.087	3.1498	0.533
		5	0.002	0.025	3.1501	0.677
		6	0.073	0.046	3.5479	0.738
		7	-0.005	0.005	3.5494	0.830
		8	-0.030	-0.057	3.6170	0.890
		9	-0.067	-0.075	3.9673	0.914
		10	-0.034	-0.043	4.0584	0.945
		11	0.185	0.204	6.8065	0.815
		12	-0.023	0.012	6.8503	0.867
		13	0.039	-0.004	6.9793	0.903
		14	-0.095	-0.074	7.7483	0.902
		15	0.001	-0.046	7.7483	0.933
		16	-0.048	-0.031	7.9527	0.950
		17	0.173	0.155	10.664	0.873
		18	-0.017	0.025	10.690	0.907

The probability values of the Q-statistic are all greater than 0.05 ($p>0.05$).

4.9 The exogenous variable in the ARIMAX model

The ARIMAX framework extends our baseline ARIMA(1,1,0) specification by incorporating the nominal exchange rate (EXR) as an exogenous regressor, implemented through the following systematic approach:

Dynamic specification: The ARIMAX model consists of an endogenous component: ARIMA(1,1,0) for CPI (as previously validated) and the exogenous component, EXR series modelled independently as ARIMA(1,1,4) before incorporating it into the ARIMAX model.

Exchange Rate Modelling: The EXR series was differenced once (I=1) to achieve stationarity and optimal ARMA structure (p=1, q=4) determined through AIC/BIC comparison across candidate models.

Forecast Integration of the EXR into the ARIMAX model: EXR point forecasts (Jul 2023 - Jun 2024) generated from validated ARIMA(1,1,4) and the dynamic forecasts incorporated as exogenous inputs in ARIMAX framework with the Covariance structure accounted for in prediction intervals.

4.10 Diagnostic test of the Exchange Rate model

The ARIMA(1,1,4) specification for the exchange rate series was subjected to comprehensive diagnostic testing: *Stability Valuation***,** all inverse roots of the AR polynomial lie within the unit circle (table 5a); MA polynomial, roots demonstrate invertibility (maximum modulus = 0.877503)

Characteristic equation solutions

AR root is 0.692398 (absolute value < 1) and MA roots is 0.000000 ± 0.877503i (modulus = 0.877503)

Table5a. Stability test result of EXR

AR Root(s)	Modulus	cycle
0.692398	0.692398	

No root lies outside the unit circle
ARMA model is invertible

MA Root(s)	Modulus	Cycle
0.000000 ± 0.877503i	0.877503	4.000000
0.877503	0.877503	
-0.877503	0.877503	

No root lies outside the unit circle.
ARMA model is invertible

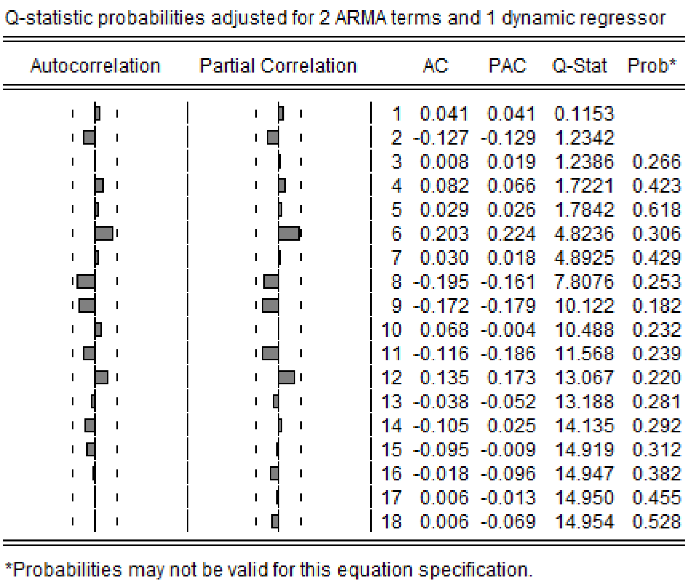
Table 5b presents *visual inspection* for residual autocorrelation in the ARIMA(1,1,4) exchange rate model prior to its inclusion in the ARIMAX framework.

Visual Inspection of the ACF/PACF Plots from table 5b

No spikes exceed the 95% confidence bounds at any lag and the residuals exhibit white noise properties.

This confirms that, the ARIMA(1,1,4) specification adequately captures the EXR series' dynamics and also, Validates the model's use as an exogenous input in the ARIMAX system

Table 5b



Q-statistic probabilities adjusted for 2 ARMA terms and 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.041	0.041	0.1153	
		2	-0.127	-0.129	1.2342	
		3	0.008	0.019	1.2386	0.266
		4	0.082	0.066	1.7221	0.423
		5	0.029	0.026	1.7842	0.618
		6	0.203	0.224	4.8236	0.306
		7	0.030	0.018	4.8925	0.429
		8	-0.195	-0.161	7.8076	0.253
		9	-0.172	-0.179	10.122	0.182
		10	0.068	-0.004	10.488	0.232
		11	-0.116	-0.186	11.568	0.239
		12	0.135	0.173	13.067	0.220
		13	-0.038	-0.052	13.188	0.281
		14	-0.105	0.025	14.135	0.292
		15	-0.095	-0.009	14.919	0.312
		16	-0.018	-0.096	14.947	0.382
		17	0.006	-0.013	14.950	0.455
		18	0.006	-0.069	14.954	0.528

*Probabilities may not be valid for this equation specification.

Table 5b's residual ACF/PACF plots shows that, the Ljung-Box Q-stat results (all $p > 0.05$) confirm the residuals are serially uncorrelated, showing no spikes beyond the 95% confidence intervals.

4.11 ARIMAX Model Estimation output

The final ARIMAX specification combines the following:

Endogenous Component: Differenced log CPI series [Δln(CPI)] from ARIMA(1,1,0); *Exogenous Component*: Differenced log exchange rate [Δln(EXR)]; *Dynamic Structure*: AR(1) term capturing persistence effects

Model Equation:

$\Delta\ln(CPI_t) = \varphi_1\Delta\ln(CPI_{t-1}) + \beta_1\Delta\ln(EXR_{t-1}) + \varepsilon_t$

where:

φ_1 = [0.34865] (AR coefficient with p_value=0.0090 i.e. $p<0.05$)

β_1 = [1.1085] (EXR impact coefficient with p_value=0.0000 i.e. $p<0.05$)

$\varepsilon_t \sim N(0, \sigma^2)$ with σ = [0.010817]

Table6. ESTIMATION OF THE ARIMAX MODEL

Variable	Coefficient	Std.Error	t-Statistic	Prob.
Dlog(exr_f)	1.108523	0.137839	8.04158	0.0000
AR(1)	0.348649	0.129164	2.699263	0.0090
SIGMASQ	0.000117	2.10E-05	5.589610	0.0000
R-squared	0.342909			
Adjusted R-squared	0.321006	Mean dependent var	0.016233	
S.E. of regression	0.011094	S.D. dependent var	0.013463	
Sum squared resid	0.007384	Akaike info criterion	-6.116388	
Log likelihood	195.6662	Schwarz criterion	-6.014334	
Durbin-Watson stat	2.032006	Hannan-Quinn criter.	-6.076250	

Inverted AR Roots	.35			

4.12 Analysis Of The Arimax Model Results

ARIMAX(1,1,0) Model Results: Re-specification of the ARIMAX model after regression $\Delta\log(CPI_t) = 1.11\ \Delta\log(EXR\_F_t) + 0.35\ \Delta\log(CPI_{t-1}) + \varepsilon_t$

Summary of key Estimates from table 6 (ARIMAX regression results)

Parameter	Coefficient	t-stat	p-value	Interpretation
Δlog(EXR_F)	1.11***	8.04	<0.001	1% EXR depreciation → 1.11% CPI inflation
AR(1)	0.35**	2.70	0.009	35% monthly persistence
σ^2	0.0001***	5.59	<0.001	Minimal residual variance

The Fit Statistics for the ARIMAX:

$R^2 = 0.34$: EXR explains 34% of inflation variation

DW = 2.03: No 1st-order autocorrelation

AIC = -6.12: Superior fit to previous specification

4.12 Diagnostic Test Of The Arimax Model Estimated

Stability Test Of The Arimax Model Estimated

In table 7a, The AR(1) coefficient of 0.35 corresponds to a characteristic equation root of z=2.86 (inverse root: 0.35), satisfying the stability condition $|z^{-1}|<1$. This ensures shock persistence decays at 35% per month.

Let us consider the **characteristic equation** derived from the AR(1) term in the ARIMAX model: $\Delta\log(CPI_t)=0.35\Delta\log(CPI_{t-1})+\cdots+\epsilon_t$

Let us rewrite the AR(1) component as a polynomial:

Then we have: $1-0.35L=0$ (where L is the lag operator)

Solving for the Root:

$1-0.35z=0 \implies z=1/0.35\approx 2.861$

The root z=2.86 is **outside the unit circle** (since $|z|>1$).

Inverse root: $1/z\approx 0.351$ (lies *inside* the unit circle, confirming stability).

Table7a. Stability test of the ARIMAX model

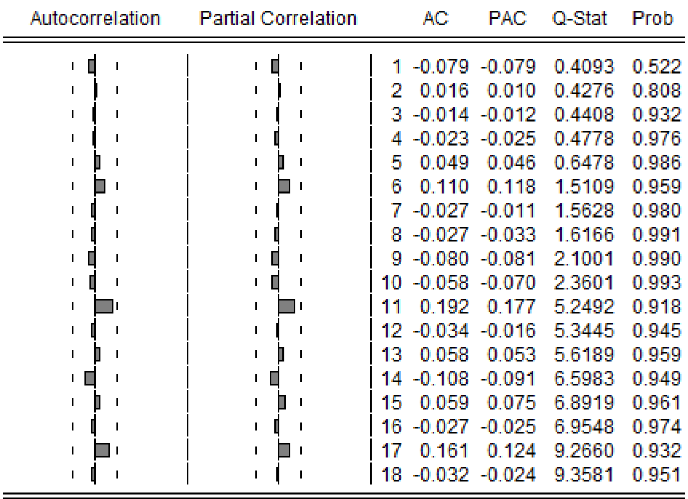
AR ROOT(S)	MODULUS	CYCLE
0.348649	0.348649	

No root lies outside the unit circle

ARMA model is stationary

Table 7b.'s residual ACF/PACF plots shows that, the Ljung-Box Q-stat results (all p > 0.05) confirm the residuals are serially uncorrelated, showing no spikes beyond the 95% confidence intervals

Table 7b. ACF/PACF residual test of the ARIMAX model



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.079	-0.079	0.4093	0.522
		2	0.016	0.010	0.4276	0.808
		3	-0.014	-0.012	0.4408	0.932
		4	-0.023	-0.025	0.4778	0.976
		5	0.049	0.046	0.6478	0.986
		6	0.110	0.118	1.5109	0.959
		7	-0.027	-0.011	1.5628	0.980
		8	-0.027	-0.033	1.6166	0.991
		9	-0.080	-0.081	2.1001	0.990
		10	-0.058	-0.070	2.3601	0.993
		11	0.192	0.177	5.2492	0.918
		12	-0.034	-0.016	5.3445	0.945
		13	0.058	0.053	5.6189	0.959
		14	-0.108	-0.091	6.5983	0.949
		15	0.059	0.075	6.8919	0.961
		16	-0.027	-0.025	6.9548	0.974
		17	0.161	0.124	9.2660	0.932
		18	-0.032	-0.024	9.3581	0.951

4.13 The Vector Autoregressive Model ($VAR_{(P)}$) model

The third model estimated is the Vector Autoregressive (VAR(p)) model, which incorporates three endogenous variables; CPI (Consumer Price Index), EXR (Nominal Exchange Rate), and M2 (Money Supply). All series were confirmed to be integrated of order one I(1) through unit root testing (see section 4.4), satisfying the pre-requisite for VAR modelling in first differences.

4.13.1 Lag-length selection

The optimal lag length for the VAR system comprising CPI, EXR, and M2 was determined through a comprehensive analysis of information criteria and statistical tests. At levels, the variables were tested for optimal lag length using several information criteria: the Schwarz Information Criterion (SIC), Akaike Information Criterion (AIC), Final Prediction Error (FPE), Sequential Modified Likelihood Ratio (LR), and Hannan-Quinn Criterion (HQ).

The lag selection criteria in table 8 present conflicting indicators for the optimal lag length. The Schwarz information Criteria (SIC), which prioritizes parsimony, selected lag-1; the Akaike Information Criteria (AIC), Hannah-Quinn (HQ) and Final Prediction Error (FPE) unanimously favoured lag-2; while the Sequential Modified LR test suggested lag-6. However, given that the VAR model's primary purpose in this research is for short-term forecasting, the AIC was prioritized due to, its demonstrated efficacy in minimizing forecast error variance (Lutkepohl, 2005), its balanced trade-off between model fit and complexity and its consistency with comparable monetary VAR studies (Bernanke et al, 2005). Therefore, lag-2 was chosen for estimating the VAR(p) model. The empirical output in table8 provides the details of the lag-selection criteria.

Table 8 Lag-length selection criteria

lag	logL	LR	FPE	AIC	SC	HQ
0	157.3484	NA	$1.17e^{-06}$	-5.144947	-5.040230	-5.103986
1	470.9743	585.4351	$4.56e^{-11}$	-15.29914	**-14.88028***	-15.13530
2	486.2143	26.92395	**$3.71e^{-11}$***	**-15.50714***	-14.77412	**-15.22042***
3	491.5291	8.857921	$4.22e^{-11}$	-15.38430	-14.33713	-14.97470
4	498.3290	10.65322	$4.59e^{-11}$	-15.31097	-13.94964	-14.77848
5	503.1378	7.052898	$5.38e^{-11}$	-15.17126	-13.49578	-14.51589
6	519.3711	**22.18550***	$4.35e^{-11}$	-15.41237	-13.42274	-14.63412

*indicates lag order selected by the criterion
LR: sequential modified Likelihood Ratio test statistic (each test at 5% level)
FPE: Final Prediction Error
AIC: Akaike Information Criteria
SC Schwarz Information Criteria

4.13.2 Cointegration Analysis

Following the lag length selection, we tested for cointegration among CPI, EXR, and M2 at levels using the *Johanson (1991) procedure.* Two I(1) variables are cointegrated if a linear combination of them yields a stationary process (Engle & Granger, 1987). Results from table 9a indicates that, both the *Trace statistic and Maximum Eigenvalue* test failed to reject the null hypothesis of *no cointegration at the 5% significance level*. That is, the test statistics fell below their critical values across all ranks (r = 0,1,2), confirming the absence of long-run equilibrium relationships. Given these results, we estimated the VAR(p) model in first differences to avoid spurious regression. The lag length was re-estimated in first differences, with AIC again selecting lag2. The empirical output for the cointegration test is presented in Table.9a. The estimated VAR(p) model was subsequently tested for serial correlation, heteroskedasticity, and stability.

Table 9a test for cointegration in VAR(p) model Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob** Critical Value

None	0.256052	26.68595	29.79707	0.1095
At most 1	0.096408	8.051555	15.49471	0.4599
At Most 2	0.027156	1.734456	3.841465	0.1878

Trace test indicates no cointegration at 0.05 level
*denotes rejection of the hypothesis at 0.05 level
**Mackinnon-Haung-Michelis (1999) p-values
Unrestricted Cointegration Rank Test (Max-eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob** Critical Value
None	0.256052	18.63439	21.13162	0.1079
At most 1	0.096408	6.317099	14.26460	0.5729
At Most 2	0.027156	1.734456	3.841465	0.1878

Max-eigenvalue test indicates no cointegration at 0.05 level
*denotes rejection of the hypothesis at 0.05 level
**Mackinnon-Haung-Michelis (1999) p-values
From table 9b, the stability condition of the model is satisfied, as all inverse roots of the AR polynomial lie strictly within the unit circle. Additionally, the absolute values of the AR root moduli are less than one, confirming that the estimated model is stable and stationary.

Table.9b stability test of the VAR(p) model estimated.

Root	Modulus
0.514412-0.084384i	0.521287
0.514412+0.084384i	0.521287
0.046269-0.463789i	0.466091
0.000822-0.112171i	0.112175
0.000822+0.112171i	0.112175

No root lies outside the unit circle.
VAR satisfies the stability condition.
From table11, *The Breusch-Godfrey (BG) serial correlation LM (LRE* and Rao F) for the VAR residuals at lags 1–2 with (p-value > 0.05), fail to reject the null hypothesis of no autocorrelation (Lag 1: LRE* p = 0.208, Rao p = 0.208; Lag 2: LRE* p = 0.471, Rao p = 0.436). This suggests that the VAR(p) specification adequately captures the system's dynamics, with residuals exhibiting no significant temporal dependence.

Table 11 VAR Residual Serial Correlation LM Test Null hypothesis: No serial correlation at lag h

lag	LRE*stat	df	Prob.	Rao F-stat	df	prob
1	12.10067	9	0.2077	1.371613	(9,124.3)	0.2080
2	8.646322	9	0.4705	0.966727	(9,124.3)	0.4361

Null hypothesis: No serial correlation at lag 1 to h

lag	LRE*stat	df	Prob.	Rao F-stat	df	prob
1	12.10067	9	0.2077	1.371613	(9,124.3)	0.2080
2	18.26856	18	0.4381	1.022000	(18,136.2)	0.4395

*Edge worth expansion correlated likelihood ratio statistic.

Heteroskedasticity test

In table 12, the joint chi-square test (joint $\chi^2_{(72)}$ p = 0.2108; all individual p > 0.05), for heteroskedasticity fails to reject the null hypothesis of homoskedasticity indicating constant residual variance across the system. Individual component tests for ARCH effects (e.g., res1*res1: p = 0.8651) and cross-residual correlations (e.g., res3*res2: p = 0.1480) also show no significant evidence of volatility clustering or spill overs. Thus, the VAR model satisfies the homoskedasticity assumption, and standard inference procedures remain valid.

Table 12 VAR(p) residual heteroscedasticity test

JOINT TEST					
Chi-square	Df	Prob.			
81.35937	72	0.2108			
Individual components:					
dependent	R-squared	F(12,50)	Prob.	Chi-sq.(12)	Prob.
res1*res1	0.109279	0.511191	0.8976	6.884563	0.8651
res2*res2	0.224720	1.207737	0.3042	14.15737	0.2908
res3*res3	0.283176	1.646008	0.1089	17.84007	0.1206
res2*res1	0.237047	1.294571	0.2513	14.93397	0.2451
res3*res1	0.240464	1.319138	0.2378	15.14922	0.2334
res3*res2	0.270520	1.545166	0.1396	17.04277	0.1480

The probability of the joint test is greater than 0.05 (Prob.>0.05) implying that the residuals have a constant variance that is; our model does not suffer from heteroscedasticity issues and the standard errors are reliable.

5.0 Discussion and interpretation of forecast results

5.1 Forecast performance measures applied in this research.

The forecasting performance of the ARIMA, ARIMAX, and VAR(2) models was rigorously evaluated using a 12-month holdout sample (July 2023 – June 2024). Two complementary metrics; Mean Absolute Percentage Error (MAPE) and the Root Mean Squared Error (RMSE), were employed in the analysis. These metrics were calculated for out of-sample forecasts to ensure robust model comparison.

The Mean Absolute Percentage Error (MAPE) measures relative forecast accuracy as a percentage.

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right| \times 100$$

Where n = number of forecasted observations

y_t = the actual value at time t

Y-hat = the forecasts value at time t

$|\ |$ = The two vertical bars stand for "absolute value operator"

The MAPE formula above expresses the error as a percentage of the actual observations, thus making the MAPE a scale-independent that is, it enables cross-variable comparisons measure of forecast accuracy test that is easy to interpret. It's ideal value is, when it is closer to zero. However, it is undefined when y_t is equal to zero.

Similarly, the Root Mean Squared Error (RMSE) quantifies absolute forecast error.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}$$

Where n=number forecasted observations

y_t = actual value at time t

Y-hat= the forecast value at time t

While the MAPE provides a percentage-based error metric, making it useful for comparing forecast accuracy across different models, the RMSE emphasizes larger errors, giving insight into the overall forecast performance in terms of absolute differences. Both metrics are widely used for comparing forecast accuracy across models like ARIMA, ARIMAX, and VAR(p). It's ideal value is when it is closer to zero.

Cpi Forecast Comparison For All Three Models Estimated

Table 13: forecast comparison

DATE	CPI ACTUAL	ARIMAX	$VAR_{(P)}$	ARIMA

		FORECAST	FORECAST	FORECAST
July-2023	**176.00**	182	171.62	161.30
Aug-2023	**185.34**	185	173.04	166.12
Sept.-2023	**193.41**	189.55	173.84	171.14
Oct.2023	**200.72**	193.29	174.67	176.39
Nov.2023	**203.91**	197.09	176.01	181.89
Dec.2023	**208.59**	200.94	177.92	187.64
Jan-2024	**207.69**	204.85	180.18	193.66
Feb-2024	**211.37**	208.81	182.68	199.96
Mar-2024	**216.44**	212.82	185.35	206.55
Apr-2024	**220.89**	216.89	188.13	213.46
May-2024	**224.77**	221.01	191.00	220.69
Jun-2024	**225.74**	**225.18**	193.95	228.27

Table 13 compares observed CPI values (July 2023–June 2024) with out-of-sample forecasts generated by ARIMAX, ARIMA, and VAR(*p*) specifications.

5.2 Measures of Out-of Sample Forecast Accuracy

After estimating the ARIMA, ARIMAX, and VAR(*p*) models, their forecasting performance was evaluated using a 12-month out-of-sample holdout period (July 2023–June 2024). Forecast accuracy was assessed using two widely accepted metrics: the **Mean Absolute Percentage Error (MAPE)** and the **Root Mean Squared Error (RMSE)**, computed separately for each model. The models generated 12-month CPI forecasts, which were compared against the actual observed values (Table 13). Based on this comparison, MAPE and RMSE were calculated to determine each model's predictive accuracy over the forecast horizon. Lower values of the two metrics indicate better performance.

5.3 Summary of Forecast Performance of the Three Models Estimated

The forecasting performance of the three estimated models, **ARIMA**, **ARIMAX**, and **VAR(p)** was evaluated using the **Root Mean Squared Error (RMSE)** and **Mean Absolute Percentage Error (MAPE)**, as defined in Section 5.1. These metrics assess the accuracy of Consumer Price Index (CPI) forecasts over a 12-month out-of-sample period (July 2023 to June 2024).

The **RMSE** measures the square root of the average squared differences between actual and forecasted values, giving more weight to larger errors. Lower RMSE values indicate greater forecasting accuracy. In contrast, the **MAPE** expresses forecast error as a percentage of actual values, offering a scale-independent and intuitively interpretable measure of accuracy. For example, a MAPE of 2.03% suggests an average deviation of 2.03% from the observed CPI values.

Table 14 presents the RMSE and MAPE values for all three models, facilitating a direct comparison of their predictive performance. The results highlight substantial differences in model accuracy, with the ARIMAX model demonstrating the most consistent and precise forecasts.

Table14 Forecast Accuracy Metrics

Accuracy measure	ARIMA	ARIMAX	$VAR_{(P)}$
MAPE	7.19	**2.03**	12.15
RMSE	16.09	**4.76**	26.97

Interpretation of Metrics Evaluation-Key Findings

The ARIMAX model outperformed all other models across standard forecast evaluation metrics over the evaluation period (July 2023–June 2024):

Absolute Performance:
The ARIMAX model achieved the lowest **Mean Absolute Percentage Error (MAPE)** of **2.03%** and **Root Mean Squared Error (RMSE)** of **4.76**. A MAPE of 2.03% indicates that, on average, the model's forecasts deviated from actual CPI values by only 2.03%. The low RMSE suggests the model was also effective at minimizing larger forecast errors.
Relative Improvement over ARIMA:
Forecast accuracy improved by **71.8%** relative to ARIMA:
$1-(2.03/7.19) \times 100 = 71.8\%$
Forecast errors (RMSE) reduced by **70.4%**:
$1-(4.76/16.09) \times 100 = 70.4\%$
ARIMA's forecasting errors were approximately **3.5 times** larger (MAPE) and **3.4 times** larger (RMSE).
Relative Improvement Over VAR(p):
Forecast accuracy improved by **83.3%** relative to the VAR(p) model:
$1-(2.03/12.15) \times 100 = 83.3\%$
Forecast errors (RMSE) reduced by **82.4%**:
$1-(4.76/26.97) \times 100 = 82.4\%$
VAR(p)'s forecasting errors were **6.0 times** larger (MAPE) and **5.7 times** larger (RMSE).
Theoretical Interpretation
The ARIMAX model's superior performance underscores the importance of incorporating relevant exogenous information in inflation forecasting. Specifically, the exchange rate appears to carry significant predictive power for inflation dynamics in Sierra Leone. In contrast:
The **ARIMA** model, being purely univariate, fails to account for such external influences.
The **VAR(p)** model likely suffers from:

(1) *Limited sample size* (n = 65), which restricts parameter stability and increases estimation noise.
(2) *Omitted variable bias*, particularly the exclusion of commodity price shocks or other relevant external factors.
(3) *Suboptimal lag selection*, which may have resulted in over fitting without meaningful improvements in predictive performance.

Implications of this result
For central banks and policy analysts, the findings suggest that **ARIMAX** offers the most reliable short-term inflation forecasts among the evaluated models. Exchange rate movements in particular, should be monitored as key leading indicators of inflationary pressure in Sierra Leone. VAR models, while theoretically appealing, require more extensive data and robust specification strategies to be practically useful in this context.

Limitations and Directions for Future Research
1. The model's performance assumes a **stable exchange rate pass-through** mechanism, which may not hold in periods of structural change.
2. The evaluation period (12 months) is relatively short and may not capture **all macroeconomic regimes**, including external shocks or regime shifts.
3. Future research could explore:
(i) The inclusion of **additional exogenous variables** (e.g., commodity prices, international interest rates).
(ii) **Time-varying parameter** models to account for evolving economic relationships.
(iii) Alternative **multivariate frameworks** such as Bayesian VARs, Structural VARs, or Dynamic Factor Models for richer dynamic insights.

5.2 Conclusion
5.2.1 Summary
The primary objective of this study was to develop a suite of inflation forecasting models to support monetary policy operations at the Bank of Sierra Leone. The analysis utilized monthly data on the national Consumer Price Index (CPI), Nominal Exchange Rate (EXR), and Money Supply (M2).

Initial stationarity assessments using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests indicated that all variables were non-stationary at levels but became stationary after first differencing. Consequently, all series were determined to be integrated of order one, I(1).
To model the inflation process, the Box-Jenkins methodology was applied to the differenced CPI series, resulting in the estimation of both ARIMA and ARIMAX models. In the ARIMAX specification, the nominal exchange rate (EXR) was incorporated as an exogenous regressor. The EXR itself was forecasted over a twelve-month horizon and included in the ARIMAX model for generating out-of-sample inflation forecasts.
In parallel, a Vector Autoregressive (VAR) model was estimated using the differenced series of CPI, EXR, and M2. Lag selection criteria identified a lag length of two as optimal. Johansen's cointegration tests both the Trace and Maximum Eigenvalue tests revealed no evidence of cointegration among the variables. As a result, the VAR model was specified in first differences.
Among the models assessed, the ARIMAX model demonstrated superior forecast performance, outperforming both the univariate ARIMA and the multivariate VAR(p) models. This result highlights the significance of the exchange rate as a leading indicator of inflation in Sierra Leone and reinforces its relevance in inflation forecasting frameworks intended to inform monetary policy decisions.

5.22 Policy Implication

The study of inflation remains a central concern for monetary policy authorities, particularly in countries pursuing inflation-targeting regimes. Developing a reliable short-term inflation forecasting model for Sierra Leone provides the Bank's decision-makers with timely insights into the inflation outlook. These forecasts serve as an essential input in the design and execution of effective monetary policies aimed at maintaining price stability.
Given the inherent time lags in the transmission of monetary policy, regular and accurate updates on the inflation trajectory are crucial. Reliable forecasts help policymakers anticipate future inflation trends and act pre-emptively by adjusting policy instruments to mitigate emerging inflationary or deflationary pressures. This proactive approach enhances the effectiveness of monetary interventions and supports broader macroeconomic stability.
The empirical findings from this study underscore the superiority of the ARIMAX model, which includes the exchange rate as an exogenous variable. It consistently outperformed both the ARIMA and VAR(p) models, highlighting the exchange rate's significant influence on inflation in Sierra Leone. Notably, the exchange rate has exhibited a steady upward trend over the past five years, during which the inflation rate surged from 13.09 per cent in November 2019 to 54.20 per cent in November 2023. This co-movement signals a strong exchange rate pass-through effect into domestic prices.
Accordingly, monetary authorities are advised to adopt measures that reduce exchange rate volatility and strengthen the transmission mechanism of monetary policy. Incorporating exchange rate dynamics into inflation forecasting models, as demonstrated by the ARIMAX framework, offers a robust approach to improving inflation targeting in Sierra Leone.

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