International Journal of scientific research and management (IJSRM)
||Volume||3||Issue||1||Pages|| 1914-1921 ||2015||
Website: www.ijsrm.in ISSN (e): 2321-3418

Investigation of PML Decay Function Performance and its Application in Radiation Boundary of Dam Reservoir

Gh. Saeedifar[1], S. Pirboudaghi[2]*, M.H. Ahmadi[1], A. Naseri[2]
[1]Department of Civil Engineering, Beyza Branch, Islamic Azad University, Beyza, Iran
[2] Faculty of Civil Engineering, University of Tabriz, Tabriz, East Azerbaijan Province, Iran
S.pirboudaghi@gmail.com

Abstract: The estimation of precise hydrodynamic forces on dam faces due to earthquakes is an important aspect of the analysis and design of dams. A key issue in the earthquake analysis of this kind of coupled systems is the accurate and inexpensive dynamic modeling of the unbounded reservoir. A suitable boundary condition should be applied along the truncation surface. Sommerfeld boundary condition is the most common traditionally used approach with enough large distance from the dam to obtain accurate results. Other approach to such modeling uses a bounded domain surrounded by an absorbing boundary or layer that absorbs waves propagating outwards from the bounded domain. A perfectly matched layer (PML) is an absorbing layer model for linear wave equations that absorbs, almost perfectly, propagating wave of all non-tangential angels of incidence and of all non-zero frequencies. In this research a finite element program is developed in order to do the seismic analysis of dam-reservoir systems, and the PML boundary condition is adopted as a truncation boundary condition for time-harmonic dam-reservoir interaction. The results obtained from this analysis are compared with the results of the same system but with Hyper elements as exact solution in accuracy. Parametric analysis of the coupled system with different parameters of the PML is done and some useful rules are obtained in order to introduce PML decay function. Results from this problem demonstrate the high accuracy achievable by PML models using small bounded domains and at low computational costs.

Keywords: Perfectly matched layer, absorbing boundary, decay function, Radiation Boundary, Dam – Reservoir interaction.

1. Introduction

A large variety of structures subjected to predominantly fluid loading need to be analyzed precisely for external exciting forces. The estimation of precise hydrodynamic forces on dam faces due to earthquakes is an important aspect of the analysis and design of dams. The hydrodynamic pressure on vertical rigid structure subjected to ground motion was first solved analytically by Westergard (1933). In most of the practical problems, it is difficult to obtain closed form analytical solutions due to complex geometries of the dam–reservoir systems. The finite element method (FEM) is recognized as a powerful numerical tool for solving such practical problems. In the finite element analysis of such problems, difficulties arise mainly because of the large extent of the fluid domain, where the fluid is unbounded. There are a variety of far-boundary conditions reported in the literature. These may be broadly classified as (i) imposition of a boundary condition on the truncation surface (Sommerfeld, 1949; Sharan, 1985; Gogoi, 2010), and (ii) coupling the finite element discretization with other type of discretization such as infinite elements (Saini et al., 1978), or boundary element types (Seghir et al., 2009; Wang et al., 2011). The distance between the structure and truncation surface is not considerably less while adopting the infinite elements (Küçükarslan et al, 2005). On the other hand, the finite element approach has the distinct advantage of being straightforward in implementation. In the finite element analysis of dam–reservoir interaction problems difficulties arise due to unbounded reservoir domain. This difficulty is handled by truncating the unbounded fluid domain at a certain distance away from the dam-reservoir interface. However, for an accurate analysis, the behavior of reservoir fluid at the truncation surface must be truly represented. Therefore, a suitable boundary condition is required along the truncation boundary. The most commonly used truncating boundary condition is the Sommerfeld radiation condition (1949). This boundary condition becomes a rigid stationary boundary for incompressible fluid domains. Another boundary condition is proposed by Sharan (1985). The Sharan boundary condition is obtained by using the exact solution of the reservoir fluid responses for a vertical faced rigid dam to represent the fluid behavior at a sufficiently large distance away from the dam–reservoir interface. Lately Samii and Lotfi (2012) have used H-W boundary condition for dam–reservoir systems.

A newly discovered perfectly matched layer (PML) is an absorbing layer model for linear wave equations that absorbs, almost perfectly, propagating waves of all non-tangential angles-of-incidence and of all non-zero frequencies. This method is growing rapidly and has been used in many fields, ranging from eddy-current problems to wave propagation in elastic media. The concept of a PML was first introduced by Berenger (1994) in the context of electromagnetic waves. More significantly, Chew and Weedon (1994) showed almost immediately that the Berenger PML equations arise from a complex-valued coordinate stretching in the electromagnetic wave equations. Since the introduction of these seminal ideas, extensive research has been conducted on various aspects of PMLs for electromagnetic waves. PMLs have been formulated for other linear wave equations too: the scalar wave equation or the Helmholtz equation (Turkel and Yefet, 1998; Harari et al., 2000), the linearized Euler equations (Hu, 1996), the wave equation for poroelastic media (Zeng et al., 2001), and to the time-harmonic elastodynamic wave equation (Basu and Chopra, 2003). The idea that PMLs could be formulated for the elastodynamic wave equation was first introduced by Chew and Liu (1996): they used complex-valued coordinate stretching to obtain the equations governing the PML and presented a proof of the absorptive property of the PML. Furthermore, they presented a finite difference time domain (FDTD) formulation obtained through field splitting or an unphysical additive decomposition of the velocity and stress fields. Contemporaneously, Hastings et al. (1996) applied Beranger's original split-field formulation of the electromagnetics PML directly to the P and S wave potentials and obtained a two-dimensional FDTD scheme for implementing the resultant formulation. Liu (1999) later applied the coordinate stretching idea to the velocity–stress formulation of the elastodynamic equation to obtain split-field PMLs for time-dependent elastic waves in cylindrical and spherical coordinates. Zhang and Ballmann (1997) and Collino and Tsogka (2001) have also obtained split-field, time-domain PMLs for the velocity–stress formulation and presented FDTD implementations. Mehdizadeh and Paraschivoiu (2003) investigated a two-dimensional statistical Helmholtz's equation by SEM and PML as absorbing boundary. Harari and Albocher (2006) have Studied of FE/PML for exterior problems of time-harmonic elastic waves. Kucukcoban and Kallivokas (2011) studied mixed perfectly-matched-layers for direct transient analysis in 2D elastic heterogeneous media. They report on numerical simulations demonstrating the stability and efficacy of the approach.

The focus of the present study is on the application of PML for most practical case; time-harmonic dam-reservoir interaction, as an efficient truncation boundary condition to model the infinite reservoir. It includes the radiation effects properly and can be adopted in the finite element formulation in a simple form. Parametric analysis is done to introduce PML decay function. The dam-foundation interaction is not investigated in this paper and the foundation soil is assumed as rigid.

2. Materials and methods

2.1 The coupled dam-reservoir problem

The dam-reservoir system can be classified as a coupled field problem in which two physical domains of fluid and structure interact at the interface plane. Displacement was chosen as response variable for the structure while pressure may be chosen as a response variable for the fluid (Lagrangian-Eulearian approach). In this case, the equation of motion of the coupled dam-reservoir system is unsymmetrical. Let us now consider harmonic ground excitations with frequency ω i.e. $U_g(t)=u_g(\omega)e^{i\omega t}$. In this case, displacements and pressures will all be have harmonic ($U(t)=u(\omega)e^{i\omega t}$ or $P(t)=p(\omega)e^{i\omega t}$).

2.2 FE formulation of the concrete gravity dam

This problem can be totally discretized by finite elements. Employing the weighted residual method, one obtains the following finite element equations of motion of the dam structure subjected to external excitations in the frequency domain (Bathe, 1996):

$$[-\omega^2 M + (1+2\beta i)K]u(\omega) = -Mu_g(\omega) \tag{1}$$

Where M and K are the mass and stiffness matrices of the dam structure. $u(\omega)$ and $u_g(\omega)$ are the amplitude of displacements and ground excitation respectively. It should be mentioned that hysteretic damping matrix is utilized in the above relation. This means:

$$C = \frac{2\beta}{\omega}K \tag{2}$$

Where β is the constant hysteretic factor of the dam body. Relation (1) is the equation of a dam in frequency domain.

2.3 Governing equations for reservoir and boundary conditions

Neglecting the internal viscosity, and assuming the water to be linearly compressible with a small amplitude irrotational two-dimensional movement, the hydrodynamic pressure distribution in the reservoir system is governed by the wave equation (Bathe, 1996):

$$\frac{1}{c^2}\ddot{P}(x,y,t) - \nabla^2 P(x,y,t) = F(x,y,t) \tag{3}$$

Where $P(x,y,t)$ is the hydrodynamic pressure distribution in excess of the hydrostatic pressure, c is the acoustic wave velocity in water, t is the time variable. By assuming a time-harmonic solution for the hydrodynamic pressure, $P(t)=p(\omega)e^{i\omega t}$ and a time-harmonic source function F, $F(t)=f(\omega)e^{i\omega t}$, the wave Eq. (3) reduces to Helmholtz's equation

$$\nabla^2 p(x,y,\omega) + k^2 p(x,y,\omega) = -f(x,y,\omega) \tag{4}$$

Where k is the wave number defined as $k=\omega/c$. The varioational formulation of this problem with homogeneous boundary conditions and weight function w is:

$$\int_\Omega (\nabla p.\nabla w - k^2 pw)dxdy = \int_\Omega fw\,dxdy \tag{5}$$

Using the finite element discretization of the fluid domain and the Galerkin formulation of Eq. (5), the wave equation can be written in the following matrix form:

$$[-\omega^2 E + H] = R - F \tag{6}$$

Where E and H are the pseudo mass and stiffness matrices of the reservoir and are determined uses the following expressions:

$$E = \frac{1}{c^2}\int_A N^T N dA \tag{7}$$

$$H = \int_A (\nabla N_x \nabla N_y)dA \tag{8}$$

$$R = \int_{s_b} N^T \frac{\partial p}{\partial n}ds = \sum_{b=1}^{4}\int_{s_b} N^T \frac{\partial p}{\partial n}ds \tag{9}$$

Where N is the element shape function, A is the reservoir area and s_b is the prescribed length along the side of boundary elements as shown in Figure 1.

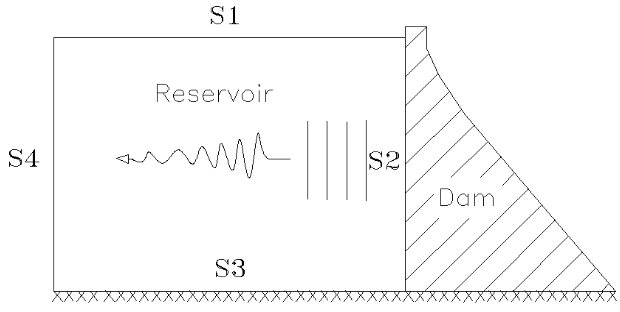

Figure 1. Dam-reservoir interaction system with reservoir domain boundaries

The hydrodynamic pressure distributions within the domain may be obtained by solving equation (1) with the following boundary conditions:

(1) At the free surface:
Neglecting the free surface wave, the only modification at the free surface is $p=0$ that applied with penalty method.

(2) At the dam -reservoir interface:

$$\frac{\partial p}{\partial n} = -\omega^2 \rho a_g(\omega) \tag{10}$$

$$\int_{s4} N_i \frac{\partial p}{\partial n} ds = -\omega^2 [\frac{1}{c} \int_{s4} N_i n N_j ds]\{u_{tot}\} = -\omega^2 \rho [Q^T]\{u \tag{11}$$

Where ρ is the density of water and $a_g(\omega)$ is the component of acceleration amplitude on the boundary along the direction of the inward normal n. u_{tot} is the total acceleration of dam grids and Q is the coupling matrix between dam and reservoir.

(3) At the reservoir bottom:
For simplification of the analytical procedures, the bottom of the reservoir is generally considered to be rigid. So boundary condition used in the current work is as the following expression while reservoir-foundation interaction is neglected:

$$\frac{\partial p}{\partial n} = 0 \tag{12}$$

(4) Truncation boundary condition:
In the finite element modeling of the reservoir a suitable boundary condition should be applied along the truncation surface. Sommerfeld boundary condition is the most common traditionally used approach which is based on the assumption that at a far distance from the dam face, the outgoing waves can be considered as plane waves to represent the fluid behavior at a sufficiently large distance away from the dam–reservoir interface. Consequently in the present analysis, the Sommerfeld radiation boundary condition is used as the following formulas (Sommerfeld, 1949):

$$\frac{\partial p}{\partial n} = -\frac{i\omega}{c} p \tag{13}$$

$$\int_{s4} N_i \frac{\partial p}{\partial n} ds = -i\omega [\frac{1}{c} \int_{s4} N_i N_j ds]\{p\} = -i\omega [A]\{p\} \tag{14}$$

Where A is the radiation damping matrix. The physical meaning of the boundary condition is equivalent to adding dampers and springs to absorb the outgoing waves in the truncation boundary.

2.4 Coupling of dam and reservoir equations

Finally the dam-reservoir interaction is a classic coupled problem, which contains two harmonic differential equations in the frequency domain. The coupled equations of the dam structure and the reservoir can be written in the following form:

$$\begin{bmatrix} -\omega^2 M (1+2\beta i) K & -Q^T \\ -\rho Q \omega^2 & -\omega^2 E + i\omega A + H \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} -M u_g \\ -\rho Q^T u_g \end{Bmatrix} \tag{15}$$

Where $[M]$ and $[K]$ are the mass and stiffness matrices of the dam structure and $[E]$, $[A]$ and $[H]$ are matrices representing the mass, damping and stiffness of the reservoir, respectively. $[Q]$ is the coupling matrix, $[p]$ and $[u]$ are the vectors of hydrodynamic pressures and displacements amplitude. u_g is the ground acceleration amplitude and $u_{tot}=u+u_g$. ρ is the density of the fluid.

2.5 PML methodology

Two boundary conditions are common in acoustics. Dirichlet boundary conditions are associated with known pressure amplitude on a boundary, which occurs on vibrating boundaries. Homogeneous Neumann boundary conditions are associated with zero velocity on a boundary, which occurs on rigid walls. These two boundary conditions are straightforward to implement, and do not need additional explanations. Problems on unbounded domains, appearing in many applications, require special boundary conditions. For domain-based numerical methods, such as finite element methods, it is obviously impractical to solve the problem on the unbounded domain. An artificial boundary is usually introduced by truncating the unbounded domain. This artificial boundary must be designed in such a way that it does not introduce reflecting waves, which do not exist in the original unbounded problem. The appropriate radiation condition, Sommerfeld radiation condition, must be satisfied (r is the radial direction):

$$\lim_{r\to\infty} r^{1/2} (\frac{\partial \varphi}{\partial r} - ik\varphi) = 0 \tag{16}$$

The method, however, chosen here to create a non-reflecting boundary condition on the artificial boundary is called PML. This method has gained popularity recently, because it is easy to implement. The concept is designed to introducing an absorbing layer that absorbs thoroughly any incident wave without reflection, for any incident angle and at any frequency before discretization. The PML technique can be viewed as a complex change of variable in the frequency domain.

2.6 PML formulation

The main concept is to surround the computation domain R at the infinite media boundary with a highly absorbing boundary layer, as shown in Fig. 2. Generally, the boundary layer is made of the same elements as the computational domain. The formulations of the matrices are the same method for both the computational domain and the boundary layer; however, the boundary layer as slightly different properties.

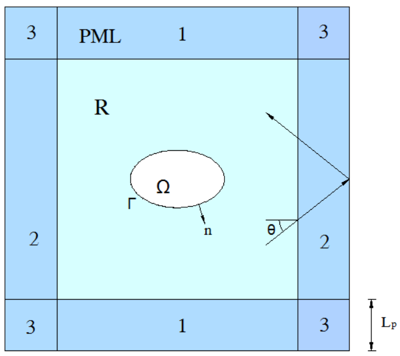

Figure 2. PML model

The PML was first developed for the two-dimensional time dependent Maxwell equations by employing auxiliary fields [12]. It was later shown that the layer can be obtained by performing a complex coordinate stretch [13]. Using this method, the stretched coordinates inside the PML are defined as:

$$\tilde{x}_j = \int_0^{x_j} \psi_j(\xi)\, d\xi \quad (17)$$

The integrand is a complex-valued stretch function of the form:

$$\psi_j = 1 + is_j \quad (18)$$

Here $i = \sqrt{-1}$ is the imaginary unit and s_j is a user specified, non-dimensional, positive attenuation function responsible for wave attenuation inside the layer. The perfectly matched layer surrounds a rectangular computational domain (in Cartesian coordinates) and attenuation is provided through the complex-valued function s_j. For reflection-free transmission this function must depend only on the coordinate axis x_j (Chew and Weedon, 1994). This provides highest attenuation for waves traveling along this axis, and no attenuation for those traveling perpendicular to it.

2.7 Finite element implementation

In this section, a two dimensional finite element formulation is proposed for the PML. The PML method is based on introducing an absorbing layer, after the truncated boundary, to absorb outgoing waves and prevent reflection from the artificial boundary. Based on this method the variational form of Eq. (4) is changed to

$$\int_\Omega (\nabla p . D . \nabla w - pK_x K_y w)\, dxdy = \int_\Omega fwdxdy$$
$$D = \begin{bmatrix} \frac{Ky}{K_x} & 0 \\ 0 & \frac{K_x}{K_y} \end{bmatrix} \quad (19)$$

Where, D is the material property matrix. The mass matrix inside the PML is obtained by multiplying the integrand of the mass matrix of the elastic medium by the stretch functions. These yields:

$$E = \int_A N^T K_x K_y N dA \quad (20)$$

Here N is a shape function and N^T is shape function transpose. The stiffness nodal matrix in a conventional elastic medium takes the form:

$$H = \int_A \left(\frac{K_y}{K_x} . \frac{\partial N^T}{\partial x} . \frac{\partial N}{\partial x} + \frac{K_x}{K_y} . \frac{\partial N^T}{\partial y} . \frac{\partial N}{\partial y}\right) dA \quad (21)$$

$$K_x = k - i\,\sigma_x(x) \,\&\, K_y = k - i\,\sigma_y(y) \quad (22)$$

Eq. (19) reduces to the original Eq. (5) when the coefficients $\sigma_x(x)$ and $\sigma_y(y)$ are zero, which is true in the physical domain. The coefficients are defined to vary from zero at the interface (for the ‘‘perfect match’’) to a maximum value at the truncation of the layer. In layers on the right and left of the main domain $\sigma_y(y)$ is zero (region 2) and similarly, for top and bottom layers, $\sigma_x(x)$ is zero (region 1). In corner absorbing layer regions, both $\sigma_x(x)$ and $\sigma_y(y)$ have non-zero values (region 3).

2.8 PML coefficients

The absorbing properties of the PML are determined by a number of parameters. When using the PML as an absorbing layer, an understanding of these parameters is essential for optimizing its performance. Consider a wave propagating in the direction θ with respect to the x axis having a wave number k (Fig. 2). The layer width is L and it is terminated by homogenous Dirichlet boundary conditions. The ratio between the magnitudes of the incident wave and the wave reflected back from the layer was obtained and is referred to as the reflection coefficient (Harari and Albocher, 2006):

$$R = \exp(-2k\, \cos(\theta)) \int_0^L \sigma(x)\, dx) \quad (23)$$

Reducing this value as much as possible seems attractive since it would suggest no pollution of the computational region from the reflected wave. It turns out however, that the perfect matching property of the PML, which ensures no reflection of wave s as they pass through the layer, only holds for a continuous layer, and after discretization it ceases to exist (Singer and Turkel, 2004). One of the parameters that affect the quality of the performance of the PML is the decay function σ(x). Although there are no restrictions on its selection (besides being positive and real which are necessary for attenuation), it is common practice to select it in the form of a polynomial. Assuming a profile of the form

$$\sigma(x) = \sigma^* \left(\frac{x}{L}\right)^n \quad (24)$$

Where σ, σ^* and L_p are attenuation function, positive constant attenuation coefficient (referred to as the attenuation coefficient yet to be specified) and the length of PML layer respectively and x is measured from the interface. In the above relation, n is an unknown parameters that should be defined.

3. Results and discussion

3.1 Basic parameters

The introduced absorbing boundary is employed to analyze a typical dam–reservoir system. Since the exact solution of our problem has been calculated in the frequency domain, the analysis results will be presented in the frequency domain, as well; however, the whole analysis procedure may be carried out in the time domain. The mentioned exact solution, proposed by Hall and Chopra (1982), is based on the hyper-element method. This method treats the infinite dimension of the reservoir analytically and uses the finite element discretization for the cross section of the reservoir.

The general setup of the considered dam–reservoir system is illustrated in Fig. 2. The height of the dam and reservoir is taken as 200 m in all of the analysis cases and the effect of reservoir's length is studied on the behavior of the analysis method. Dam and reservoir are assumed to be placed on a rigid foundation, and the system is excited with horizontal ground motion of frequency ω. The concrete is assumed to be homogeneous and isotropic, while water is considered as a compressible and inviscid fluid. In dam structure, Hysteric damping method is used and its relevant coefficient is 5%. The modulus of elasticity, unit weight and Poisson's ratio of concrete were taken as 27.5 GPa, 24 KN/m^3 and 0.2, respectively. The dam is assumed to be in the case of plain stress. The velocity of pressure waves in water was taken as 1440 m/s.

One of the important aspects of the analysis procedure is its reliability in the frequency range, which is applied to the real structure. The nature of the dynamic loading applied to our problem is seismic base excitation; the dominant frequencies of the common earthquake strong motions are located between 1 and 10 Hz. In this study, the response of the system will be calculated for excitation frequencies below 12 Hz.

The employed finite element mesh for dam and reservoir consists of 2D quadratic isoparametric elements. The size of the elements should be able to simulate the shape of waves, which are propagating inside dam and reservoir. In this study, the accuracy of higher order NRBCs is going to be compared with the semi-analytical exact solutions, with the same mesh and material properties. Therefore, satisfying the minimums for the mesh size should make this study perfectly reliable. As a result, the size of the fluid and solid elements is taken smaller than 40 m. This will result in a mesh with five layers of elements along the height of the dam; the number of elements in x-direction is variable with the length of the reservoir. Nevertheless, In order to evaluate the sensitivity of the response to the mesh size, one of the experiments has been carried out for a model with 20 m mesh size. As will be shown in this example, the 40 m mesh size is totally satisfactory, especially for our comparison purposes.

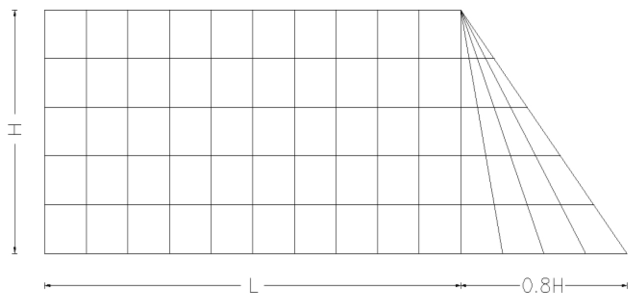

Figure 3. The finite element model of Dam-reservoir

3.2 Results verification

A special program was developed for dynamic analysis of the system by using two-dimensional elements in MATLAB. In order to verify the results obtained from the developed code, the results are compared with Sommerfeld boundary condition. In order to examine this and find an accurate length of the reservoir, different lengths can be tried. Sommerfeld boundary condition is known to converge to the exact solution of the problem, when the truncation boundary is located at an infinitely large distance from the wave source (L/H≥2). Fig. 4 shows the horizontal acceleration at dam crest with various length of the reservoir when Sommerfeld boundary condition is used. As shown in Fig. 4, the results are converging to the exact solution; however, even for L/H=3, Sommerfeld BC exhibits some major instabilities, in our desired frequency range.

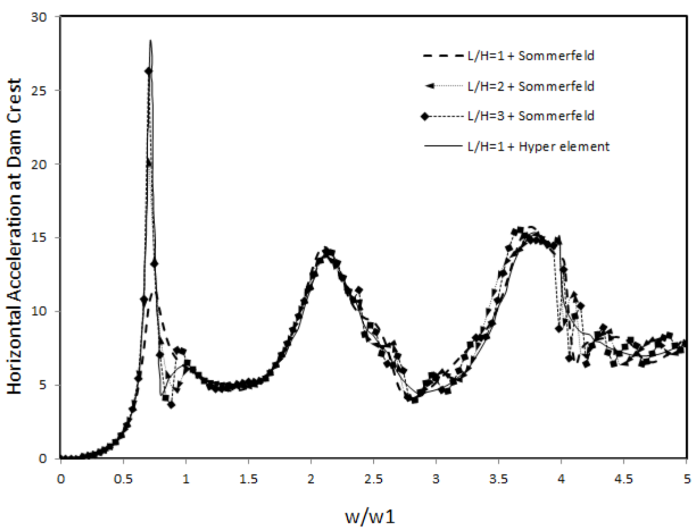

Figure 4. Comparison of horizontal acceleration at dam crest from the present study and Hall and Chopra (1982)

3.3 Dam-reservoir analysis by using PML at far end of reservoir

The geometry of the finite element model of the dam-reservoir system with PML is shown in Fig. 5.

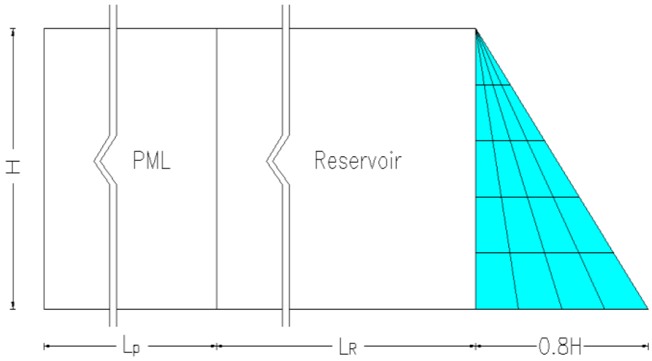

Figure 5. The model of Dam-reservoir-PML

Because of compatibility, the same vertical discretization should be used for the PML region. Obviously if one use smaller mesh rather than the mesh shown in figure 5, PML has better performance because PML is developed for continuous formulation essentially. However using other meshes for dam-reservoir interaction problem in comparison to the proposed mesh is not favor generally and not studied in the present study. In order to proper optimizing PML, the history of dam crest displacement amplitude is compared with exact solution. So an error is defined as below:

$$Error = \sum(\varphi_{Exact} - \varphi_{PML})^2 \qquad (25)$$

PML, while it has revolutionized absorbing boundaries for wave equations, especially (but not limited to) electromagnetism, is not a panacea. Some of the limitations and failure cases of PML are discussed in this section, along with workarounds.

3.4 PML decay function

There are some parameters in the decay function relation that good performance of PML is very sensitive to proper selection of these parameters. So effects of these parameters are analyzed in this research. L_p is the PML length's that should be chosen large enough properly. As noticed, a polynomial profile is assumed to the PML attenuation function. So experience shows that a simple quadratic or cubic turn-on of the PML absorption usually produces negligible reflections for a PML layer of only half a wavelength or thinner. (Increasing the resolution also increases the effectiveness of the PML, because it approaches the exact wave equation.) Guidelines for the selection of the polynomial were established by Singer and Turkel [27], which in general suggest it to be of second order at least in the discrete case (in contrast to the continuous formulation in which a low order profile is recommended). If a lower order polynomial is used, PML absorption rate changes more gradually and it can adopt own to the system easily. So Assume a profile of second order (n=2). In this profile σ^* as the attenuation coefficient should be specified. Good performance of the PML depends on proper selection of this value. Taking it too small would result in pollution of the computational domain due to insufficient attenuation. Taking it too large causes spurious reflection from the interface due to inadequate representation of the PML by the discretized layer. Proper selection of σ^* rests on defining a reasonable reflection coefficient which should be derived from the accuracy of the discretization and reservoir and PML region lengths.

3.5 Discretization and numerical reflections

First, and most famously, PML is only reflectionless while solving the exact wave equations. As soon as discretizing the problem (whether for finite difference or finite elements), it is only solving an approximate wave equation, and the analytical perfection of PML is no longer valid. PML is still an absorbing material: waves that propagate within it are still attenuated, even discrete waves. The boundary between the PML and the regular medium is no longer reflectionless, but the reflections are small because the discretization is (presumably) a good approximation for the exact wave equation. The key fact is that, even without a PML, reflections can be made arbitrarily small as long as the medium is slowly varying. That is, in the limit as "turning on" the absorption more and more slowly, reflections go to zero due to an adiabatic theorem. With a non-PML absorber, there is a need to go slowly (i.e. a very thick absorbing layer) to get acceptable reflections (Oskooi et al., 2008). However, with PML the constant factor is very good to start with, so a constant magnitude is tried for this coefficient. In Fig. 6 and 7, comparison between reservoir lengths, L2 error of horizontal dam crest displacement amplitude, PML layer thickness and constant damping ratio of PML are shown.

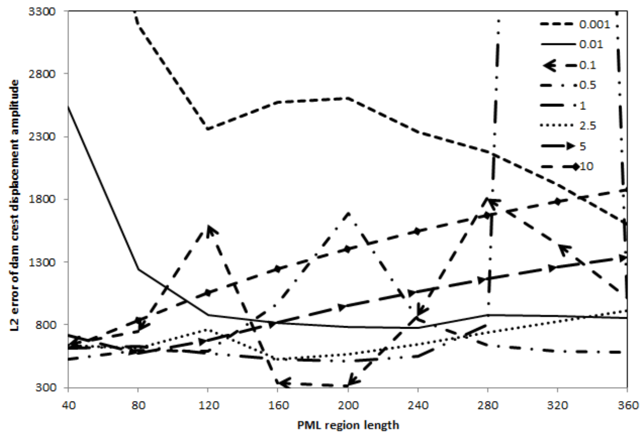

Figure 6. Comparison between layer thickness and constant damping ratio of PML (L_R=40m)

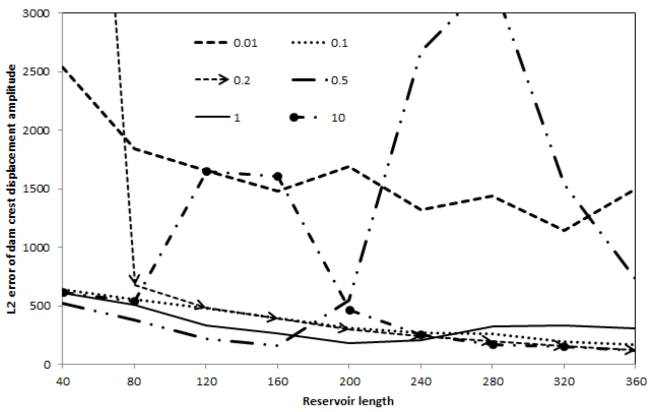

Figure 7. Comparison of reservoir length and PML constant damping ratio (L_p=40m)

The large amount of error is because of some shift or delay in the results or differences in first mode magnitude. From the Figs. 6 and 7, it is obvious that the range of selection for damping ratio of PML decay function σ^* is restricted. One should care for choose. If one chooses a large or small σ^*, there is some oscillations in the results. This is because of different wave numbers in the system and actually problem is not held for one wave number or statistical. By considering increasing PML domain length with constant reservoir length (L_R=40m) in Fig. 6 or vice versa in Fig. 7, large amount of reservoir length has better results than large amount of PML length. Also it is obvious that for every matter only one value converges to the exact solution. Results indicate that a minimum length of the reservoir or PML region is needed to have acceptable error near to the exact solution. So reservoir length must be more than 160m. If a minimum PML region like L_p=40m is selected, it has acceptable results. From the figure 6, taking σ^* any larger would not improve the results, and in the case of a coarse mesh is likely to impair them. As noticed, for most matters at the beginning there is same error magnitude. In Fig. 8 comparisons of exact solution and PML with σ^*=0.1, L_P=160m, L_R=40m and in Fig. 9 PML with σ^*=0.5, L_P=40m, L_R=160m are shown. The PML is truncated with homogenous Dirichlet boundary conditions. Fig 10 shows the real part of hydrodynamic pressure at middle of PML domain end for σ^*=0.5, L_P=40m, L_R=160m.

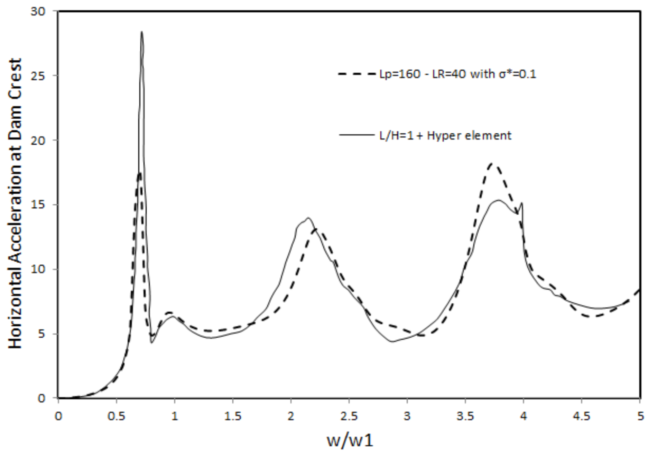

Figure 8. Comparison of exact solution and PML with σ^*=0.1, L_p=160m, L_R=40m

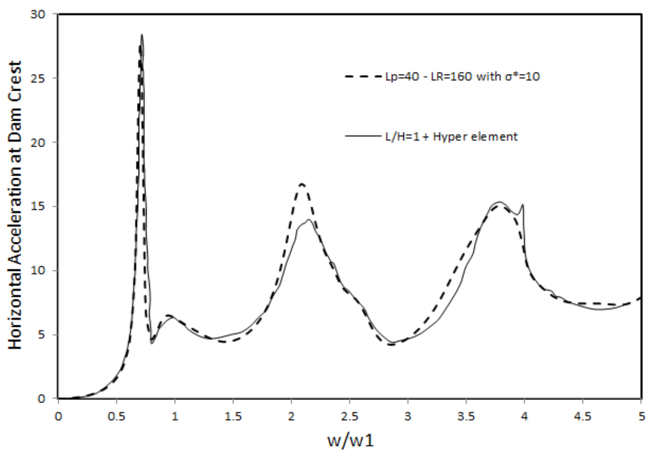

Figure 9. Comparison of exact solution and PML with σ^*=0.5, L_P=40m, L_R=160m

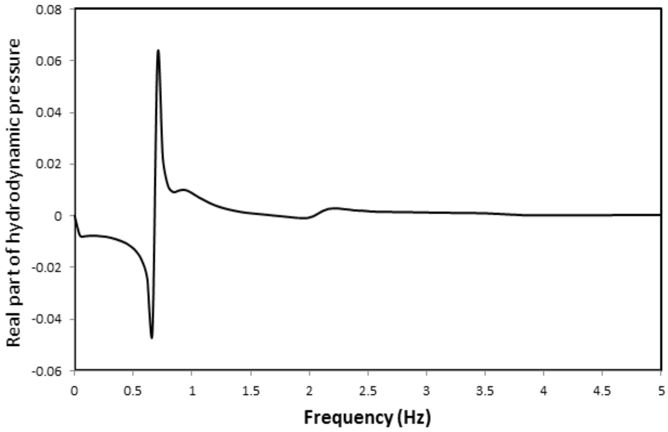

Figure 10. Real part of hydrodynamic pressure at middle of PML domain end for $\sigma^*=0.5$, $L_P=40m$, $L_R=160m$

The results obtained with the PML in Fig. 9 show good agreement with the exact solution, validating our formulation and verifying our implementation. As noticed in Fig. 10, waves damping truly because it is around zero. This can be advantage of the PML that it is necessary only to model half of the infinite reservoir in comparison to traditionally Sommerfeld boundary condition. Results from this problem demonstrate the high accuracy achievable by PML models using lower bounded domains and at low computational costs.

4. CONCLUSION

The dam–reservoir-PML interaction problem was studied by using the finite element method. A finite element program is developed in order to do the seismic analysis of system with the PML boundary condition is adopted as a truncation boundary for time-harmonic dam-reservoir interaction. The program results verified with the exact solution of the same system, but with applied large length Sommerfeld boundary condition in accuracy. Polynomial profile of second order has been assumed to the PML decay function and tried a constant magnitude for damping ratio of the function. The range of selection for damping ratio of PML decay function σ^* is restricted. Taking σ^* any larger or smaller would not improve the result, and in the case of a coarse mesh is likely to impair them. On the other hand, some instability can be seen in the results because the system is frequency dependent. Constant values for σ^* may be use sufficiently when there is a one wave number in the problem or in fact there is no dispersion and problem is statistical. Results indicate that a minimum length of the reservoir or PML region is needed to have acceptable error near to the exact solution. According to this study, if one choose a dam-reservoir-PML system with $\sigma^*=0.5$, $L_P=40m$, $L_R=160m$, the obtained results show good agreement with the exact solution, validating our formulation and verifying our implementation. This can be advantage of the PML that it is necessary only to model half of the infinite reservoir in comparison to traditionally Sommerfeld boundary condition. The proposed truncation boundary condition may be located even at a relatively small distance away from the structure, resulting in great computational advantages.

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Author Profile

Gholamreza Saeedifar received the B.S. and M.S. degrees in civil Engineering from Shiraz University, Iran respectively. He is now Faculty member at Islamic Azad University, Beiza Branch, Fars, Iran.

Sajjad Pirboudaghi received the B.S. and M.S. degrees in civil Engineering from Urmia and Tabriz University, Iran respectively. He is now PhD Candidates in civil Engineering at Tabriz University, Iran.

Mohamad Hossein Ahmadi received the B.S. and M.S. degrees in civil Engineering from Shahid Bahonar and Tabriz University, Iran respectively. He is now PhD Candidates in civil Engineering at Tabriz University and also Faculty member at Islamic Azad University, Beiza Branch, Fars, Iran.

Akbar Naseri received the B.S. and M.S. degrees in civil Engineering from Khaje nasir and Tabriz University, Iran respectively.