

Mathematical Model for Human Resource Planning through Stochastic Processes

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Abstract

Two category organization subjected to random depart of personnel due to policy decisions taken by the organization is considered. we have found mathematical model and analytical results for the mean and variance of the time for employment at any organization.

Key words: man hours, depart, breakdown point, employment, Mean, Variance.

Introduction

Maintenance activities are the backbone of a successful and profitable organization. Assuming that, organization has two categories of personnel and that the loss of man hours. If maximum loss of man hours due to the depart of personnel crosses a particular level, known as break down point for the organization, the organization reaches an uneconomic status which otherwise be called breakdown point and employment is to be done at this point. In this journal we construct the time for employment in a two category organization. The breakdown point for the organization is the sum of the constant breakdown point for loss of man hours.

Model description

Consider an organization having two categories A_1 and A_2 taking policy decisions at random epochs in the interval $[0, \infty)$. At every decision making epoch a random number of persons depart

the organization. There is a associated loss of man hours if a person depart. The loss of man hours is linear and additive and it forms a sequence of independent and identically distributed random variables. The loss of man hours, the inter decision time process and breakdown points are statically independent. The employment is done whenever the maximum loss of man hours in category 1 and the maximum loss of man hours in category 2 due to policy decisions crosses a combined constant breakdown point C , $C > 0$.

Main result

In this main result, we have derived mean time for employment and variance time for employment.

The survival function of the time for recruitment is

$$P[T > t] = \sum_{k=0}^{\infty} V_k(t) P[\max(R_i^{A_1}, R_i^{A_2}) < C]$$

T – Continuous random variable denoting the time employment in the organization.

$V_k(t)$ - Probability that there are exactly k – decision epochs in $(0, t]$

$R_i^{A_1}$ - Shortage of man hour in category A_1 to the i^{th} decision, $i = 1, 2, \dots$

$R_i^{A_2}$ - Shortage of man hour in category A_2 to the i^{th} decision, $i = 1, 2, \dots$

C - Constant combined breakdown point for the two categories, $C > 0$

$M_k^{A_1}$ - $\max_{1 \leq i \leq k}$, the maximum shortage of man hours in category A_1 in the k decisions

$M_k^{A_2}$ - $\max_{1 \leq i \leq k}$, the maximum shortage of man hours in category A_2 in the k decisions

$$P[T > t] = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] (\psi_{A_1}(C) \cdot \psi_{A_2}(C))^k$$

$\{R_i^{A_1}\}$ - Sequence of independent and identically distributed random variables for category A_1 with distribution $\Psi_{A_1}(\cdot)$

$\{R_i^{A_2}\}$ - Sequence of independent and identically distributed random variables for category A_2 with distribution $\Psi_{A_2}(\cdot)$.

$$P[T > t] = 1 - [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)] \sum_{k=1}^{\infty} F_k(t) (\psi_{A_1}(C) \cdot \psi_{A_2}(C))^{k-1}$$

(1)

From (1)

$$L(t) = [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)] \sum_{k=1}^{\infty} F_k(t) (\psi_{A_1}(C) \cdot \psi_{A_2}(C))^{k-1}$$

(2)

$L(\cdot)$ – Cumulative distribution functions of T .

$F(\cdot)$ - Distribution of the inter decision times with density function $f(\cdot)$; $f(x) = \lambda e^{-\lambda x}$, $\lambda > 0$

$F_k(\cdot)$ - k fold convolution of $F(\cdot)$

Differentiating (2) with respect to t

$$l(t) = [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)] \sum_{k=1}^{\infty} f_k(t) (\psi_{A_1}(C) \cdot \psi_{A_2}(C))^{k-1}$$

$l(\cdot)$ - Probability density function of T .

Taking Laplace transform on both sides of (3) and apply convolution theorem on Laplace transform we get

$$\bar{l}(s) = [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)] \sum_{k=1}^{\infty} (\bar{f}(s))^k (\psi_{A_1}(C) \cdot \psi_{A_2}(C))^{k-1}$$

$\bar{f}(\cdot)$ - Laplace transform of $f(\cdot)$

$$\bar{f}(s) = \frac{\lambda}{\lambda + s}$$

$$\bar{l}(s) = \frac{\lambda [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)]}{\lambda + s - \lambda \psi_{A_1}(C) \cdot \psi_{A_2}(C)} \tag{4}$$

From (4) the expected time of employment is

$$E(T) = \frac{1}{\lambda [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)]} \tag{5}$$

$E(T)$ – Mean time for employment.

$$E(T^2) = 2(E(T))^2$$

$$V(T) = \frac{E(T^2) - (E(T))^2}{V(T)} = \frac{1}{\lambda^2 [1 - \psi_{A_1}(C) \cdot \psi_{A_2}(C)]^2} \tag{6}$$

$V(T)$ – Variance of the time for employment.

(5) and (6) give the mean and variance of the time for employment for the present model.

Sub Result: 1

If $R_i^{A_1}$ and $R_i^{A_2}$ follow Poisson distribution with parameters λ_1 and λ_2 respectively

then mean and variance of the time for employment is

$$E(T) = \frac{1}{\lambda \left[1 - \sum_{k=1}^c \frac{e^{-\lambda_1} (\lambda_1)^k}{k} - \sum_{k=1}^c \frac{e^{-\lambda_2} (\lambda_2)^k}{k} \right]}$$

$$V(T) = \frac{1}{\lambda^2 \left[1 - \sum_{k=1}^c \frac{e^{-\lambda_1} (\lambda_1)^k}{k} - \sum_{k=1}^c \frac{e^{-\lambda_2} (\lambda_2)^k}{k} \right]^2}$$

Sub Result: 2

if $R_i^{A_1}$ and $R_i^{A_2}$ follow exponential distribution with parameters λ_3 and λ_4 respectively

Then mean and variance of the time for employment is

$$E(T) = \frac{1}{\lambda \left[1 - e^{-\lambda_3 c} + e^{-\lambda_4 c} - e^{-(\lambda_3 + \lambda_4)c} \right]}$$

$$V(T) = \frac{1}{\lambda^2 \left[1 - e^{-\lambda_3 c} + e^{-\lambda_4 c} - e^{-(\lambda_3 + \lambda_4)c} \right]^2}$$

Sub Result: 3

if $R_i^{A_1}$ and $R_i^{A_2}$ follow Geometric distribution with parameters λ_5 and λ_6 respectively

Then mean and variance of the time for employment is

$$E(T) = \frac{1}{\lambda \left[1 - \sum_{k=1}^c \lambda_5 (1 - \lambda_5)^k - \sum_{k=1}^c \lambda_6 (1 - \lambda_6)^k \right]}$$

$$V(T) = \frac{1}{\lambda^2 \left[1 - \sum_{k=1}^c \lambda_5 (1 - \lambda_5)^k - \sum_{k=1}^c \lambda_6 (1 - \lambda_6)^k \right]^2}$$

Conclusion

For two category man hour system, the mean and variance for the time for employment are derived.

In this journal we conclude that sub result (2) is preferable than sub result (1) and (3) for average time for employment at any organization.

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