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Homomorphism and Anti- Homomorphism of Interval Valued Intuitionistic Fuzzy Ternary Subhemiring of a Hemiring

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Abstract: ***In this paper, we define the concept of Interval-Valued Intuitionistic Fuzzy Ternary Subhemiring (IVIFTSHR) of a Hemiring and also in this note we discussed, some properties of image and pre-image of a level ternary subhemiring of IVIFTSHR with respect the Homomorphism and Anti-Homomorphism.***

Keywords: Fuzzy set, level set,ternary hemiring, interval-valued Intuitionistic fuzzy set, homomorphism.

1. Introduction

Fuzzy set theory has been developed in many fields by many researchers and they concentrated more interesting to working different areas of mathematics such as Groups, Rings, Analysis, Topology, Graphs and so many. Atanassov.K.T. and Gargov.G.[4] are introduced Interval-Valued Intuitionistic Fuzzy Sets(IVIFS), which is a generalization of the IFS. Since 2015 Ezzatallah Baloui Jamkhaneh[6] present generalized interval valued Intuitionistic fuzzy set as an extension of interval valued Intuitionistic fuzzy set. Bhowmik.M and Pal.M.[5],[6] are introduced notions of Partition of Generalized IVIFS and some properties of them. Said Broumi and Florentin Smarandache[13] are discussed interval valued Intuitionistic Hesitant Fuzzy Set by using some new operators.Recently, a few researchers trying to find new ideas of fuzzy algebraic structures as Intuitionistic fuzzy algebraic structures.In this paper, we introduced the concept of Interval-Valued Intuitionistic Fuzzy Ternary Subhemiring under homomorphism and anti-homomorphism.

2. Preliminaries

In this section, we are using some basic definitions and we consider as X is a Universal set.
A Hemiring R is the generalization of semiring and also generalization of Ring defined by Lister.W.G[11].

In briefly, A hemiring is a non-empty set R on which operations of addition and multiplication have been defined such that the following conditions are satisfied: i) (R, +) is a commutative monoid with identity element 0. ii) (R, .) is a semigroup. iii) Multiplication distributes over addition from either side (a.(b+c)=(a.b)+(a.c) and (a+b).c=(a.c)+(b.c)). iv) Multiplication by 0 annihilates R. v) 1 not equal to 0.

A ternary hemiring is also the generalization of ternary Semiring. A non-empty subset T of R is called ternary subhemiring if $a+b \in T$ and $abc \in T$ for all a, b and c are in T.

2.1.Definition: An IFS A in X is defined as an object of the form $A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle : x \in X\right\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and non-membership functions of A respectively and $0 \le \mu_A(x)+\nu_A(x) \le 1$ for each $x \in X$.

2.2.Definition: Interval valued Intuitionistic fuzzy set (IVIFS) A in X is defined as an object of the form $A=\left\{\left\langle x,M_A(x),N_A(x)\right\rangle | x \in X\right\}$,where the functions $M_A(x): X \to [I]$ and $N_A(x): X \to [I]$, denote the degree of membership and degree of non-membership of A respectively, where $M_A(x)=\left[M_{AL}(x),M_{AU}(x)\right]$, $N_A(x)=\left[N_{AL}(x),N_{AU}(x)\right]$, $0 \le M_{AU}(x)+N_{AU}(x) \le 1$ for each $x \in X$.

2.3.Definition: Let [I] be the set of all closed subintervals of the interval [0,1] and $M_A(x)=\left[M_{AL}(x),M_{AU}(x)\right] \in [I]$ and $N_A(x)=\left[N_{AL}(x),N_{AU}(x)\right] \in [I]$ then $N_A(x) \le M_A(x)$ iff $N_{AL}(x) \le M_{AL}(x)$ and $N_{AU}(x) \le M_{AU}(x)$ for each $x \in X$.

2.4.Definition: Let (R, +, .) be a ternary hemiring. An interval valued Intuitionistic fuzzy set T is said to be an interval valued Intuitionistic fuzzy ternary subhemiring (IVIFTSHR) of R if the following conditions are satisfied:

$i) M_T(x+y) \ge \min\{M_T(x), M_T(y)\}$
$ii) M_T(xyz) \ge \min\{M_T(x), M_T(y), M_T(z)\}$
$iii) N_T(x+y) \le \max\{N_T(x), N_T(y)\}$
$iv) N_T(xyz) \le \max\{N_T(x), N_T(y), N_T(z)\}$ for each x, y and z in R.

2.5.Definition: Let A be an IVIFS of X. For α,β in [0, 1], the level subset of A is the set $A_{(\alpha,\beta)} = \{x \in X, M_A(x) \geq \alpha, N_A(x) \leq \beta\}$ for each $x \in X$.

2.6.Definition: Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then $f : R \to R^1$ is called a homomorphism if it satisfies the following axioms:

$i)\ f(x+y) = f(x) + f(y)$

$ii)\ f(xyz) = f(x)f(y)f(z)$ for each x, y and z in R.

2.7.Definition: Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then $f : R \to R^1$ is called an anti-homomorphism if it satisfies the following axioms:

$i)\ f(x+y) = f(x) + f(y)$

$ii)\ f(xyz) = f(z)f(y)f(x)$ for each x, y and z in R

3. Some Properties of IVIFSHR

Theorem:3.1 Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then the homomorphic image of an IVIFTSHR of R is an IVIFTSHR of R^1.

Proof: Let $f : R \to R^1$ be a homomorphism. Let $M_T = f(M_U)$, where M_U is an IVIFTSHR of R. Now, $M_T(f(x)+f(y)) = M_U(x+y) \geq \min\{M_U(x), M_U(y)\}$ $\Rightarrow M_T(f(x)+f(y)) \geq \min\{M_T(f(x)), M_T(f(y))\}$ for all $f(x), f(y)$ in R^1 and $M_T(f(x)f(y)f(z)) = M_U(xyz)$ $\geq \min\{M_U(x), M_U(y), M_U(z)\}$ which implies that $M_T(f(x)f(y)f(z))$ $\geq \min\{M_T(f(x)), M_T(f(y)), M_T(f(z))\}$ for all $f(x), f(y)$ *and* $f(z)$ in R^1. On the other hand, let $N_T = f(N_U)$, where N_U is an IVIFTSHR of R. Now, $N_T(f(x)+f(y)) = N_U(x+y) \leq \max\{N_U(x), N_U(y)\}$ $\Rightarrow N_T(f(x)+f(y)) \leq \max\{N_T(f(x)), N_T(f(y))\}$ for all $f(x), f(y)$ in R^1 and $N_T(f(x)f(y)f(z)) = N_U(xyz)$ $\leq \max\{N_U(x), N_U(y), N_U(z)\}$ which implies that $N_T(f(x)f(y)f(z))$ $\leq \max\{N_T(f(x)), N_T(f(y)), N_T(f(z))\}$ for all $f(x), f(y)$ and $f(z)$ in R^1. From all the above, we conclude that T is an IVIFTSHR of R^1. Hence, the homomorphic image of an IVIFTSHR of R is an IVIFTSHR of R^1.

Theorem:3.2 Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then the anti-homomorphic image of an IVIFTSHR of R is an IVIFTSHR of R^1.

Proof: It is trivial.

Theorem:3.3 Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then the homomorphic pre-image of an IVIFTSHR of R^1 is an IVIFTSHR of R.

Proof: Let $f : R \to R^1$ be a homomorphism. Let $M_T = f(M_U)$, where M_T is an IVIFTSHR of R^1. Now, $M_U(x+y) = M_T(f(x)+f(y))$ $\geq \min\{M_T(f(x)), M_T(f(y))\}$ $\Rightarrow M_U(x+y)$ $\geq \min\{M_U(x), M_U(y)\}$ for all x and y in R and $M_U(xyz) = M_T(f(x)f(y)f(z))$ $\geq \min\{M_T(f(x)), M_T(f(y)), M_T(f(z))\}$ which implies that $M_U(xyz) \geq \min\{M_U(x), M_U(y), M_U(z)\}$ for all x, y and z in R. On the other hand, let $N_T = f(N_U)$, where N_T is an IVIFTSHR of R. Now, $N_U(x+y) = N_T(f(x)+f(y))$ $\leq \max\{N_T(f(x)), N_T(f(y))\}$ $\Rightarrow N_U(x+y)$ $\leq \max\{N_U(x), N_U(y)\}$ for all x and y in R and $N_U(xyz) = N_T(f(x)f(y)f(z))$ $\leq \max\{N_T(f(x)), N_T(f(y)), N_T(f(z))\}$ which implies that $N_U(xyz) \leq \max\{N_U(x), N_U(y), N_U(z)\}$ for all x, y and z in R. From all the above, we conclude that U is an IVIFTSHR of R. Hence, the homomorphic pre-image of an IVITFSHR of R^1 is an IVIFTSHR of R.

Theorem:3.4 Let $(R,+,.)$ and $(R^1,+,.)$ be any two ternary hemirings. Then the anti-homomorphic pre-image of an IVIFTSHR of R^1 is an IVIFTSHR of R.

Proof: It is trivial.

Theorem:3.5 Let A be an IVIFTSHR of R. Then for α, β in [0, 1], $A_{(\alpha,\beta)}$ is an IVIFTSHR of R.

Proof: Let x,y and z in $A_{(\alpha,\beta)}$. Now, $M_A(x+y) \geq \min\{M_A(x), M_A(y)\} \geq \min\{\alpha,\alpha\} \Rightarrow M_A(x+y) \geq \alpha$ and $M_A(xyz) \geq \min\{M_A(x), M_A(y), M_A(z)\} \geq \min\{\alpha,\alpha,\alpha\} \Rightarrow M_A(xyz) \geq \alpha$. On the other hand $N_A(x+y) \leq \max\{N_A(x), N_A(y)\} \leq \max\{\beta,\beta\} \Rightarrow N_A(x+y) \leq \beta$ for all x and y in $A_{(\alpha,\beta)}$ and $N_A(xyz) \leq \max\{N_A(x), N_A(y), N_A(z)\} \leq \max\{\beta,\beta,\beta\} \Rightarrow N_A(xyz) \leq \beta$. From all the above, we get x+y and xyz in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a IVIFSHR of R.

Theorem:3.6 Let (R, +, .) and $(R^1,+,.)$ be any two ternary hemirings. If $f: R \to R^1$ is a homomorphism, then the homomorphic image of a level ternary subhemiring of an IVIFTSHR of R is a level ternary subhemiring of an IVIFTSHR of R^1.

Proof: Let $f: R \to R^1$ be a homomorphism. Let $M_T = f(M_U)$, where M_U is an IVIFTSHR of R. Clearly, M_T is an IVIFTSHR of R^1(If x, y and z in R, then f(x),f(y) and f(z) in R^1). Let $A_{(\alpha,\beta)}$ be a level ternary subhemiring of M_U .Suppose x,y and z in $A_{(\alpha,\beta)}$, then x+y and xyz in $A_{(\alpha,\beta)}$. Now $M_T(f(x)) = M_U(x) \geq \alpha \Rightarrow M_T(f(x)) \geq \alpha$, similarly $M_T(f(y)) \geq \alpha$ and $M_T(f(z)) \geq \alpha$, for all $f(x), f(y) and f(z)$ in R^1. Now $M_T(f(x)+f(y)) = M_U(x+y) \geq \min\{M_U(x), M_U(y)\}$ $\geq \min\{M_T(f(x)), M_T(f(y))\} \geq \min\{\alpha,\alpha\} \Rightarrow$ $M_T(f(x)+f(y)) \geq \alpha$ for all $f(x) and f(y)$ in R^1. And $M_T(f(x)f(y)f(z)) = M_U(xyz)$ $\geq \min\{M_U(x), M_U(y), M_U(z)\}$ $\geq \min\{M_T(f(x)), M_T(f(y)), M_T(f(z))\} \geq \min\{\alpha,\alpha,\alpha\}$ $\Rightarrow M_T(f(x)f(y)f(z)) \geq \alpha$ for all $f(x), f(y)$ and $f(z)$ in R^1. On the other hand, let $N_T = f(N_U)$, where N_U is an IVIFTSHR of R. Clearly, N_T is an IVIFTSHR of R^1. Let $A_{(\alpha,\beta)}$ be a level ternary subhemiring of N_U .Suppose x,y and z in $A_{(\alpha,\beta)}$, then x+y and xyz in $A_{(\alpha,\beta)}$. Now $N_T(f(x)) = N_U(x) \leq \beta \Rightarrow N_T(f(x)) \leq \beta$, similarly $N_T(f(y)) \leq \beta$ and $N_T(f(z)) \leq \beta$, for all $f(x), f(y) and f(z)$ in R^1. Now $N_T(f(x)+f(y)) = N_U(x+y) \leq \max\{N_U(x), N_U(y)\}$ $\leq \max\{N_T(f(x)), N_T(f(y))\} \leq \max\{\beta,\beta\} \Rightarrow$ $N_T(f(x)+f(y)) \leq \beta$ for all $f(x) and f(y)$ in R^1. And $N_T(f(x)f(y)f(z)) = N_U(xyz)$ $\leq \max\{N_U(x), N_U(y), N_U(z)\}$ $\leq \max\{N_T(f(x)), N_T(f(y)), N_T(f(z))\} \leq \max\{\beta,\beta,\beta\}$ $\Rightarrow N_T(f(x)f(y)f(z)) \leq \beta$ for all $f(x), f(y)$ and $f(z)$ in R^1. Hence the homomorphic image of a level ternary subhemiring of an IVIFTSH of R is a level ternary subhemiring of an IVIFTSH of R^1.

Theorem:3.7 Let (R, +, .) and $(R^1,+,.)$ be any two ternary hemirings. If $f: R \to R^1$ is an anti-homomorphism, then the anti-homomorphic image of a level ternary subhemiring of an IVIFTSHR of R is a level ternary subhemiring of an IVIFTSHR of R^1.

Proof: It is trivial.

Theorem:3.8 Let (R, +, .) and $(R^1,+,.)$ be any two ternary hemirings. If $f: R \to R^1$ is a homomorphism, then the homomorphic pre-image of a level ternary subhemiring of an IVIFTSHR of R^1 is a level ternary subhemiring of an IVIFTSHR of R.

Proof: Let $f: R \to R^1$ be a homomorphism. Let $M_T = f(M_U)$, where M_T is an IVIFTSHR of R^1 . Clearly, M_U is an IVIFTSHR of R. Suppose f(x), f(y) and f(z) in $A_{(\alpha,\beta)}$, then f(x)+f(y) and f(x)f(y)f(z) in $A_{(\alpha,\beta)}$. Now $M_U(x) = M_T(f(x)) \geq \alpha \Rightarrow M_U(x) \geq \alpha$, similarly $M_U(y) \geq \alpha$ and $M_U(z) \geq \alpha$, for all x, y and z in R. Now $M_U(x+y) = M_T(f(x)+f(y))$ $\geq \min\{M_T(f(x)), M_T(f(y))\} \geq \min\{M_U(x), M_U(y)\}$ $\geq \min\{\alpha,\alpha\} \Rightarrow M_U(x+y) \geq \alpha$ for all x and y in R. And $M_U(xyz) = M_T(f(x)f(y)f(z))$

$\geq \min\{M_T(f(x)), M_T(f(y)), M_T(f(z))\}$
$\geq \min\{M_U(x), M_U(y), M_U(z)\} \geq \min\{\alpha, \alpha, \alpha\} \Rightarrow$
$M_U(xyz) \geq \alpha$ for all x, y and z in R. On the other hand, let $N_U = f(N_T)$, where N_T is an IVIFTSHR of R^1. Clearly, N_U is an IVIFTSHR of R. Now $N_U(x) = N_T(f(x)) \leq \beta \Rightarrow$ $N_U(x) \leq \beta$, similarly $N_U(y) \leq \beta$ and $N_U(z) \leq \beta$, for all x, y and z in R. Now $N_U(x+y) = N_T(f(x)+f(y))$
$\leq \max\{N_T(f(x)), N_T(f(y))\} \leq \max\{N_U(x), N_U(y)\}$
$\leq \max\{\beta, \beta\} \Rightarrow N_U(x+y) \leq \beta$ for all x and y in R. And $N_U(xyz) = N_T(f(x)f(y)f(z))$
$\leq \max\{N_T(f(x)), N_T(f(y)), N_T(f(z))\}$
$\leq \max\{N_U(x), N_U(y), N_U(z)\} \leq \max\{\beta, \beta, \beta\} \Rightarrow$
$N_U(xyz) \leq \beta$ for all x, y and z in R. Hence the homomorphic pre-image of a level ternary subhemiring of an IVIFTSH of R^1 is a level ternary subhemiring of an IVIFTSH of R.

Theorem:3.9 Let (R, +, .) and $(R^1,+,.)$ be any two ternary hemirings. If $f: R \to R^1$ is an anti-homomorphism, then the anti-homomorphic pre-image of a level ternary subhemiring of an IVIFTSHR of R^1 is a level ternary subhemiring of an IVIFTSHR of R.

Proof: It is trivial.

In the following Theorem is the Composition of Two Functions:

Theorem:3.10 Let T be an IVIFTSHR of R^1 and f is a homomorphism from a hemiring R into R^1. Then $T \circ f$ is an IVIFTSHR of R.

Proof: Let T be an IVIFTSHR of R^1. Then we have, $(M_T \circ f)(x+y) = M_T(f(x+y)) = M_T(f(x)+f(y))$
$\geq \min\{M_T(f(x)), M_T(f(y))\}$ which implies that $(M_T \circ f)(x+y) \geq \min\{(M_T \circ f)(x), (M_T \circ f)(y)\}$ for all x and y in R. And $(M_T \circ f)(xyz) = M_T(f(xyz))$
$= M_T(f(x)f(y)f(z))$
$\geq \min\{M_T(f(x)), M_T(f(y)), M_T(f(z))\} \Rightarrow$
$(M_T \circ f)(xyz)$
$\geq \min\{(M_T \circ f)(x), (M_T \circ f)(y), (M_T \circ f)(z)\}$ for all x, y and z in R. On the other hand $(N_T \circ f)(x+y)$
$= N_T(f(x+y)) = N_T(f(x)+f(y))$
$\leq \max\{N_T(f(x)), N_T(f(y))\} \Rightarrow$
$(N_T \circ f)(x+y) \leq \max\{(N_T \circ f)(x), (N_T \circ f)(y)\}$ for all x and y in R. And $(N_T \circ f)(xyz) = N_T(f(xyz))$
$= N_T(f(x)f(y)f(z))$
$\leq \max\{N_T(f(x)), N_T(f(y)), N_T(f(z))\} \Rightarrow$
$(N_T \circ f)(xyz)$
$\leq \max\{(N_T \circ f)(x), (N_T \circ f)(y), (N_T \circ f)(z)\}$ for all x, y and z in R. Hence $T \circ f$ is an IVIFTSHR of R.

Theorem:3.11 Let T be an IVIFTSHR of R^1 and f is an anti-homomorphism from a hemiring R into R^1. Then $T \circ f$ is an IVIFTSHR of R.

Proof: It is trivial.

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