International Journal of Scientific Research and Management (IJSRM)
||Volume||5||Issue||08||Pages||6685-6688||2017||
Website: www.ijsrm.in ISSN (e): 2321-3418
Index Copernicus value (2015): 57.47 DOI: 10.18535/ijsrm/v5i8.15

Some Operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type

K. Rajesh[1], R. Srinivasan[2]

[1] Ph. D Full-Time Research Scholar, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tamilnadu.

[2] Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tamilnadu.

Abstract:
In this paper, we define some operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their relations.

Keywords: Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Sets of Second Type (IFSST), Interval Valued Fuzzy Sets (IVFS), Interval Valued Intuitionistic Fuzzy Sets (IVIFS), Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST)

1. Introduction

An Intuitionistic fuzzy set was proposed by K. T. Atanassov [2], as an extension of fuzzy set introduced by L. A. Zadeh. Intuitionistic fuzzy sets are characterized by two functions expressing the degree of membership and the degree of non - membership respectively. K. T. Atanassov and G. Gargov [3] further introduced the concept of Interval Valued Intuitionistic Fuzzy Sets.

The present authors [4] further introduced the new extension of IVIFS namely Interval Valued Intuitionistic Fuzzy Sets of Second Type and established some of their properties. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we define some operators on IVIFSST and establish some of their relations. This paper is concluded in section 4.

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1[2] Let X be a non - empty set. An intuitionistic fuzzy set (IFS) A in X is defined as an object of the following form.

$$\mathrm{A} = \{< x,\ \mu_{\mathrm{A}}(x),\ \nu_{\mathrm{A}}(x) > \mid x \in \mathrm{X}\}$$

Where the functions $\mu_{\mathrm{A}}: \mathrm{X} \to [0,1]$ and $\nu_{\mathrm{A}}: \mathrm{X} \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in$ X, respectively, and for every $x \in$ X.

$$0 \leq \mu_{\mathrm{A}}(x) + \nu_{\mathrm{A}}(x) \leq 1$$

Definition 2.2[2] Let a set X be fixed. An intuitionistic fuzzy set of second type (IFSST) A in X is defined as an object of the following form.

$$\mathrm{A} = \{< x, \mu_{\mathrm{A}}(x), \nu_{\mathrm{A}}(x) > \mid x \in \mathrm{X}\}$$

Where the functions $\mu_{\mathrm{A}}: \mathrm{X} \to [0,1]$ and $\nu_{\mathrm{A}}: \mathrm{X} \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in$ X, respectively, and for every $x \in$ X.

$$0 \leq {\mu_{\mathrm{A}}}^2(x) + {\nu_{\mathrm{A}}}^2(x) \leq 1$$

Definition 2.3[2] Let X be an universal set with cardinality n. Let [0, 1] be the set of all closed subintervals of the interval [0, 1] and elements of this set are denoted by uppercase letters. If $M \in [0,1]$ then it can be represented as $M = [\mathrm{M_{AL}}, \mathrm{M_{AU}}]$,

where $\mathrm{M_L}$ and $\mathrm{M_U}$ are the lower and upper limits of M. For $M \in [0,1]$, $\bar{M} = 1 - M$ represents the interval $[1 - \mathrm{M_{AL}}, 1 - \mathrm{M_U}]$ and $\mathrm{W_M} = \mathrm{M_U} - \mathrm{M_L}$ is the width of M.

An interval-valued fuzzy set (IVFS) A in X is given by

$$A = \{< x, \mathrm{M_A}(x) > \mid x \in X\}$$

where $\mathrm{M_A}: X \to [0,1]$, $\mathrm{M_A}(x)$ denote the degree of membership of the element x to the set A.

Definition 2.4[3] An interval-valued intuitionistic fuzzy set (IVIFS) A in X is given by

$$A = \{< x, \mathrm{M_A}(x), \mathrm{N_A}(x) > \mid x \in X\}$$

where $\mathrm{M_A}: X \to [0,1], \mathrm{N_A}: X \to [0,1]$. The intervals $\mathrm{M_A}(x)$ and $\mathrm{N_A}(x)$ denote the degree of membership and the degree of non-membership of the element x to the set A, where $\mathrm{M_A}(x) = [\mathrm{M_{AL}}(x), \mathrm{M_{AU}}(x)]$ and $\mathrm{N_A}(x) = [\mathrm{N_{AL}}(x), \mathrm{N_{AU}}(x)]$ with the condition that

$$\mathrm{M_{AU}}(x) + \mathrm{N_{AU}}(x) \leq 1 \text{ for all } x \in \mathrm{X}.$$

Definition 2.5[4] An Interval-Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) A in X is given by

$$A = \{< x, \mathrm{M_A}(x), \mathrm{N_A}(x) > \mid x \in X\}$$

where $\mathrm{M_A}: X \to [0,1], \mathrm{N_A}: X \to [0,1]$.The intervals $\mathrm{M_A}(x)$ and $\mathrm{N_A}(x)$ denote the degree of membership and the degree of non-membership of the element x to the set A, where $\mathrm{M_A}(x) = [\mathrm{M_{AL}}(x), \mathrm{M_{AU}}(x)]$ and $\mathrm{N_A}(x) = [\mathrm{N_{AL}}(x), \mathrm{N_{AU}}(x)]$ with the condition that

$$M^2{}_{\mathrm{AU}}(x) + N^2{}_{\mathrm{AU}}(x) \leq 1 \text{ for all } x \in \mathrm{X}.$$

Definition 2.6[4] Let $A = \{< x, \mathrm{M_A}(x), \mathrm{N_A}(x) > \mid x \in X\}$ and $B = \{< x, \mathrm{M_B}(x), \mathrm{N_B}(x) > \mid x \in X\}$ be two IVIFSSTs of X, then

i). $A \subset B$ iff $M_{AU}(x) \leq M_{BU}(x)$ & $M_{AL}(x) \leq M_{BL}(x)$ & $N_{AU}(x) \geq N_{BU}(x)$ & $N_{AL}(x) \geq N_{BL}(x)$

ii). $A = B$ iff $A \subset B$ & $B \subset A$

iii). $\bar{A} = \{< x, [\mathrm{N_{AL}}(x), \mathrm{N_{AU}}(x)], [\mathrm{M_{AL}}(x), \mathrm{M_{AU}}(x)] > \mid x \in \mathrm{X}\}$

iv). $A \cup B = \{< x, [\max(\mathrm{M_{AL}}(x),\ \mathrm{M_{BL}}(x)), \max(\mathrm{M_{AU}}(x),\ \mathrm{M_{BU}}(x))], [\min(\mathrm{N_{AL}}(x),\ \mathrm{N_{BL}}(x)), \min(\mathrm{N_{AU}}(x),\ \mathrm{N_{BU}}(x))] > \mid x \in \mathrm{X}\}$

v). $A \cap B = \{< x, [\min(\mathrm{M_{AL}}(x),\ \mathrm{M_{BL}}(x)), \min(\mathrm{M_{AU}}(x),\ \mathrm{M_{BU}}(x))], [\max(\mathrm{N_{AL}}(x),\ \mathrm{N_{BL}}(x)), \max(\mathrm{N_{AU}}(x),\ \mathrm{N_{BU}}(x))] > \mid x \in \mathrm{X}\}$

3. Some operators on IVIFSST

In this section, we define some new operators on IVIFSST and establish some of their relations.

Definition 3.1 Let $A = \{< x, \mathrm{M_A}(x), \mathrm{N_A}(x) > \mid x \in X\}$ and $B = \{< x, \mathrm{M_B}(x), \mathrm{N_B}(x) > \mid x \in X\}$ be two IVIFSSTs of X, then

i). $A + B = \{< x, [\ M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x),$

$$M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)],$$
$$[N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$

ii). $A.B = \{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$

iii). $A \$ B = \{< x, \left[\sqrt{M_{AL}(x)M_{BL}(x)}, \sqrt{M_{AU}(x)M_{BU}(x)}\right], \left[\sqrt{N_{AL}(x)N_{BL}(x)}, \sqrt{N_{AU}(x)N_{BU}(x)}\right] > \mid x \in X\}$

iv). $A \# B$
$$= \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > \mid x \in X\}$$

v). $A @ B$
$$= \{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > \mid x \in X\}$$

Proposition: 3.1 Let X be a non-empty set. For every IVIFSST A and B in X, we have

$(A \# B) \$ (A \# B) = (A \# B)$.

Proof:
Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | x ∈ X}
and B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | x ∈ X}
Then,
$$A \# B = \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > \mid x \in X\}$$
Now
$$(A \# B)\$(A \# B) = \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > \mid x \in X\}\$$$
$$\{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > \mid x \in X\}$$
$$= \{< x, \left[\sqrt{\left(\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}\right)^2}, \sqrt{\left(\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}\right)^2}\right], \left[\sqrt{\left(\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}\right)^2}, \sqrt{\left(\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}\right)^2}\right] > \mid x \in X\}$$
$$= \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > \mid x \in X\}$$
$= A \# B$

Therefore, $(A \# B) \$ (A \# B) = (A \# B)$.

Proposition: 3.2 Let X be a non-empty set. For every IVIFSST A and B in X, we have

$(A + B) \$ (A + B) = (A + B)$.

Proof:
Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | x ∈ X}
and B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | x ∈ X}
Then,
$$A + B = \{< x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
Now
$$(A + B)\$(A + B) = \{< x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}\$$$
$$\{< x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
$$= \{< x, \left[\sqrt{\left(M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x)\right)^2}, \sqrt{\left(M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)\right)^2}\right], \left[\sqrt{(N^2{}_{AL}(x)N^2{}_{BL}(x))^2}, \sqrt{(N^2{}_{AU}(x)N^2{}_{BU}(x))^2}\right] > \mid x \in X\}$$
$$= \{< x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
$= (A + B)$

Therefore, $(A + B)\$(A + B) = (A + B)$

Proposition: 3.3 Let X be a non-empty set. For every IVIFSST A and B in X, we have the following

$(A . B) \$ (A . B) = (A . B)$

Proof:
Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | x ∈ X}
and B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | x ∈ X}
Then,
$$A.B = \{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
Now
$$(A.B)\$(A.B) = \{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}\$$$
$$\{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
$$= \{< x, \left[\sqrt{\left(M^2{}_{AL}(x)M^2{}_{BL}(x)\right)^2}, \sqrt{\left(M^2{}_{AU}(x)M^2{}_{BU}(x)\right)^2}\right], \left[\sqrt{\left(N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x)\right)^2}, \sqrt{(N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x))^2}\right] > \mid x \in X\}$$
$$= \{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$$
$= A . B$

Therefore, (A . B) \$ (A .B) = A . B.

Proposition: 3.4 Let X be a non-empty set. For every IVIFSST A and B in X, we have

$(A @ B) \$ (A @ B) = (A @ B)$

Proof:
Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | x ∈ X}
And B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | x ∈ X}
Then,
$$A @ B = \{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > \mid x \in X\}$$

Now

$(A @ B)\$(A @ B)$

$$= \{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > | \; x \in X \}\$$$

$$\{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > | \; x \in X \}$$

$$= \{< x, \left[\sqrt{\left(\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}\right)^2}, \sqrt{\left(\frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right)^2}\right], \left[\sqrt{\left(\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}\right)^2}, \sqrt{\left(\frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right)^2}\right] > | \; x \in X\}$$

$$= \{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > | \; x \in X \}$$

$= A @ B$

Therefore, $(A @ B)\$(A @ B) = A @ B$.

Proposition: 3.5 Let X be non-empty set. For every IVIFSST A and B in X, we have

$$(A \# B) \$ (A @ B) = (A \$ B)$$

Proof:

Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | $x \in$ X}
and B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | $x \in$ X}
Then,

$$A \# B = \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > |x \in X\}$$

And,

$$A @ B = \{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > | \; x \in X \}$$

Now,

$(A \# B) \$ (A @ B) =$

$$= \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > | \; x \in X\}\$$$

$$\{< x, \left[\frac{M^2{}_{AL}(x) + M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x) + M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x) + N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x) + N^2{}_{BU}(x)}{2}\right] > | \; x \in X \}$$

$$= \{< x, \left[\sqrt{\frac{2M_{AL}(x)M_{BL}(x)}{(M^2{}_{AL}(x) + M^2{}_{BL}(x))} \cdot \frac{(M^2{}_{AL}(x) + M^2{}_{BL}(x))}{2}}, \sqrt{\frac{2M_{AU}(x)M_{BU}(x)}{(M^2{}_{AU}(x) + M^2{}_{BU}(x))} \cdot \frac{(M^2{}_{AU}(x) + M^2{}_{BU}(x))}{2}}\right],$$

$$\left[\sqrt{\frac{2N_{AL}(x)N_{BL}(x)}{(N^2{}_{AL}(x) + N^2{}_{BL}(x))} \cdot \frac{(N^2{}_{AL}(x) + N^2{}_{BL}(x))}{2}}, \sqrt{\frac{2N_{AU}(x)N_{BU}(x)}{(N^2{}_{AU}(x) + N^2{}_{BU}(x))} \cdot \frac{(N^2{}_{AU}(x) + N^2{}_{BU}(x))}{2}}\right] > | \; x \in X\}$$

$$= \{< x, \left[\sqrt{M_{AL}(x)M_{BL}(x)}, \sqrt{M_{AU}(x)M_{BU}(x)}\right], \left[\sqrt{N_{AL}(x)N_{BL}(x)}, \sqrt{N_{AU}(x)N_{BU}(x)}\right] > | \; x \in X\}$$

= A \$ B

Hence, (A # B) \$ (A @ B) = A \$ B.

Proposition: 3.6 Let X be non-empty set. For every IVIFSST A and B in X, we have

$$(A \cup B) \# (A \cap B) = (A \# B)$$

Proof:

Let A = {< x, [$M_{AL}(x)$, $M_{AU}(x)$], [$N_{AL}(x)$, $N_{AU}(x)$] > | $x \in$ X}
and B = {< x, [$M_{BL}(x)$, $M_{BU}(x)$], [$N_{BL}(x)$, $N_{BU}(x)$] > | $x \in$ X}
Then,

$$A \cup B = \{< x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))] > | \; x \in X\}$$

$$A \cap B = \{< x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))], [\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))] > | \; x \in X\}$$

Now,

$$(A \cup B)\#(A \cap B) = \{< x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))] > | \; x \in X\} \#$$

$$\{< x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))], [\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))] > | \; x \in X\}$$

$$= \{< x, \left[\frac{2\max(M_{AL}(x), M_{BL}(x))\min(M_{AL}(x), M_{BL}(x))}{(\max(M_{AL}(x), M_{BL}(x)))^2 + (\min(M_{AL}(x), M_{BL}(x)))^2}, \frac{2\max(M_{AU}(x), M_{BU}(x))\min(M_{AU}(x), M_{BU}(x))}{(\max(M_{AU}(x), M_{BU}(x)))^2 + (\min(M_{AU}(x), M_{BU}(x)))^2}\right],$$

$$\left[\frac{2\max(N_{AL}(x), N_{BL}(x))\min(N_{AL}(x), N_{BL}(x))}{(\max(N_{AL}(x), N_{BL}(x)))^2 + (\min(N_{AL}(x), N_{BL}(x)))^2}, \frac{2\max(N_{AU}(x), N_{BU}(x))\min(N_{AU}(x), N_{BU}(x))}{(\max(N_{AU}(x), N_{BU}(x)))^2 + (\min(N_{AU}(x), N_{BU}(x)))^2}\right] > | \; x \in X\}$$

$$= \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x) + M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x) + M^2{}_{BU}(x)}\right] \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x) + N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x) + N^2{}_{BU}(x)}\right] > |x \in X\}$$

$= A \# B$

Therefore, $(A \cup B) \# (A \cap B) = A \# B$.

4. Conclusion

We have introduced some new operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) and established some of their relations. It is still open to define some more operators on IVIFSST.

References

1. K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.
2. K. T. Atanassov, Intuitionistic Fuzzy Sets – Theory and Applications, Springer Verlag, New York, 1999.
3. K. T. Atanassov and G. Gargov, Interval-Valued Fuzzy Sets, Fuzzy Sets and Systems 31 (1989) 343-349.
4. K. Rajesh and R. Srinivasan "Interval Valued Intuitionistic Fuzzy Sets of Second Type", Advances

in Fuzzy Mathematics, Volume 12, Number 4 (2017), pp. 845-853.
5. L. A. Zadeh, Fuzzy Sets, Inform and Control 8 (1965) pp. 338-353.
6. M. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21 (1987) 1-17.