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Solving Fuzzy Bottleneck Transportation Problems Using Blocking Zero Point Method

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Abstract

Transportation Problem is a well known topic and is used very often in solving problems of management science. This paper analyzes the Blocking Method for finding an optimal solution to Fuzzy Bottleneck Transportation Problems which is very different from other methods. And also finding the efficient solutions of Fuzzy Bottleneck Transportation Problems by using Blocking Zero Point Method. A numerical example is given to show the efficiency of the method.

Keywords : Fuzzy Transportation Problem, Bottleneck Transportation Problem, Blocking Method, Optimal Solution, Efficient Solution, Blocking Zero Point Method.

1.Introduction

The Transportation Problem is a special type of Linear programming problem which deals with the distribution of the single product from various sources of supply to various destination of demand in such a way the total transportation cost is minimized. The time-minimizing or Bottleneck Transportation Problem (BTP) is a special case of a transportation problem in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destinations. In a BTP, the time of the transporting items from origins to destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations. Many researchers [3,4] developed various algorithms for solving time minimizing transportation problems. The transportation time is relevant in a variety of real transportation problems, too.

Bottleneck-Cost Transportation Problem (BCTP) is a kind of a bicriteria transportation problem. The bicriteria transportation problem is a particular case of multi objective transportation problem which had been proposed and also, solved by Aneja and Nair [2] and until, now many researchers [6,7] also, have great interest in this problem, and some method used their special techniques in finding the solutions for two objective functions approximately approaching to the ideal solution.

2. Mathematical Formulation of Bottleneck transportation problem

Consider the following BTP :

Minimize z = [Maximize t_{ij} / $x_{ij} > 0$]

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1, 2, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1, 2, \ldots, n \qquad (2)$$

$x_{ij} \geq 0$, for all i and j and integers, (3)

where m is the number of supply points; n is the number of demand points; x_{ij} is the number of units shipped from supply point i to demand point j; t_{ij} is the time of transporting goods from supply point i to demand point j; a_i is the supply at supply point i and b_j is the demand at demand point j.

In a BTP, time matrix $[t_{ij}]$ is given where t_{ij} is the time of transporting goods from the origin i to the destination j. For any given feasible solution. $X = \{ x_{ij} : i = 1,2,\ldots, m$ and $j = 1,2,\ldots,n\}$ of the problem the time transportation is the maximum of t_{ij}'s among the cells in which there are positive allocations.

This time of the transportation remains independent of the amount of commodity sent so long as $x_{ij} > 0$.

2.1 Blocking Method

We, now introduce a new method namely, the blocking method for finding an optimal solution to bottleneck transportation problems.

The blocking method proceeds as follows.

Algorithm:

Step 1: Find the maximum of the minimum of each row and column of the given transportation table. Say, T

Step 2: Construct a reduced transportation table from the given table by blocking all cells having time more than T.

Step 3: Check if each column demand is less

								Supply
	12	13	34	7	8	29	19	15
	7	18	36	40	38	6	10	7
	11	20	30	21	21	29	31	45
	27	12	39	31	5	36	12	30
	15	17	32	36	22	16	14	12
	17	38	16	33	23	30	29	16
Demand	20	13	11	27	9	5	40	

than to the sum of the supplies in the reduced transportation problem obtained from the Step 2.. Also, check if each row supply is less than to sum of the column demands in the reduced transportation problem obtained from the Step 2.. If so, go to Step 6. (Such reduced transportation table is called the active transportation table).

If not, go to Step 4.

Step 4: Find a time which is immediately next to the time T. Say U.

Step 5: Construct a reduced transportation table from the given transportation table by blocking all cells having time more than U and then, go to the Step 3..

Step 6: Do allocation according to the following rules:

(a) Allot the maximum possible to a cell which is only one cell in the row / column. Then, modify the active transportation table and then, repeat the process till it is possible or all allocations are completed.

(b) If (a) is not possible, select a row / a column having minimum number of unblocked cell and allot maximum possible to a cell which helps to reduce the large supply and / or large demand of the cell.

Step 7: This allotment yields a solution to the given bottleneck transportation problem.

Now, we prove the solution to a BTP obtained by the blocking method is an optimal solution to the BTP.

2.2 Theorem :

A solution obtained by the blocking method to the BTP, is optimal.

Proof :

Let T_0 be the time transportation of a feasible solution of the given problem.

Let W be the time transportation of the feasible solution to the given BTP by the blocking method. It means that all transportation can be made in the time of W. In the active table, maximum time is W. As per the Step 3 to 5, W is the minimum time to transport all items from the origins to destinations.

If $T_0 \geq W$, the solution obtained by the blocking method is optimal.

If $T_0 \geq W$, it means that all transporting work can be made in the time of T_0 and $T_0 \geq W$. Therefore, T_0 is a time for transportation which is less than W. This is not possible.

Thus, the time for transportation obtained using the blocking method to the bottleneck transportation problem is optimal.

Hence the theorem.

2.3 Numerical Example

The blocking method for solving a BTP is illustrated by the following example.

Consider the following bottleneck transportation problem :

Now, the maximum of minimum of each row and the minimum of each column = 16.

Now, using the Step 1, to the Step 5., we have the following complete allocation table:

								Supply
	12	13		7	8		19	15
	7	18				6	10	7
	11	20		21	21			45
		12			5		12	30
	15	17				16	14	12
	17		16					16
Demand	20	13	11	27	9	5	40	

Now, using the Step 6., the optimal solution to the bottleneck problem is given below :

								Supply
					8 (9)		19 (6)	15
						6 (5)	10 (2)	7
	11 (15)	20 (3)		21 (27)				45
							12	30

							(30)	
		17 (10)					14 (2)	12
	17 (5)		16 (11)					16
Demand	20	13	11	27	9	5	40	

and the minimum time transportation is 21.

3. Mathematical Formulation of Fuzzy Bottleneck Transportation Problems:

Minimize $z_1 = \sum_{i}^{m} \sum_{j}^{n} c_{ij} x_{ij}$

Minimize z_2 = [Maximize t_{ij} / $x_{ij} > 0$]

Subject to

$$\sum_{j}^{n} x_{ij} = a_i \text{; i = 1,2, …., m}$$

$$\sum_{i}^{m} x_{ij} = b_j \text{; j = 1,2,….., n}$$

$$x_{ij} \geq 0, \text{ i = 1, 2, …, m; j = 1, 2, …, n;}$$

Where a_i is the supply available at i^{th} sources; b_j is the demand required at j^{th} destination. x_{ij} is the number of white shipped from i^{th} source to j^{th} destination. c_{ij} is the cost of transportation a unit from i^{th} source to j^{th} destination t_{ij} is the time of transporting goods from i^{th} source to j^{th} destination. m is the number of sources and n is the number of destinations.

3.1 Definition : A point (X,T) where $X = [x_{ij}; i = 1,2,….,m$ and $j = 1,2,3,….,n]$ and T is a time, is said to be a feasible solution of (MP) if X satisfies the conditions (1) to (3).

3.2 Definition : A feasible point (X0,T0) is said to be efficient for (MP) if there exists no other feasible point (X,T) in (MP) such that $z_1(X) \leq z_1(X_0)$ and $z_2(T) < z_2(T_0)$ or $z_1(X) \leq z_1(X_0)$ and $z_2(T) < z_2(T_0)$.

3.3 Definition : A cost transportation problem of a BCTP is said to be active for any time M if the minimum time transportation corresponding to the cost transportation problem is M.

3.4 Blocking Zero point method:

The blocking zero point method proceeds as follows:

Algorithm:

Step 1: Construct the time transportation problem from the given BCTP.

Step 2: Solve the time transportation problem by the blocking method. Let the optimal solution be T_0.

Step 3: Construct the cost transportation problem from the given BCTP.

Step 4: Solve the cost transportation problem by the zero point method and also, find the corresponding time transportation. Let it be T_m.

Step 5: For each time M in [T_0, T_m], compute $\alpha = \frac{T_m - M}{T_m - T_0}$ which is the level of time satisfaction for the time M.

Step 6: Construct the active cost transportation problem for each time M in [T_0, T_m] and solve it by zero point method.

Step 7: For each time M, an optimal solution to the cost transportation problem, X is obtained from the Step 6 with the level of time satisfaction α, Then, the vector (X,M) is an efficient solution to BCP.

3.5 Numerical Example

Now, the blocking zero point method is illustrated by the following example.

Consider the following 3 x 4 bottleneck-cost transportation problem. The upper left corner in each cell gives the time of transportation on the corresponding route and the lower right corner in each cell gives the unit transportation cost per unit on that route.

					Supply
	10 5	68 6	73 10	52 11	8
	66 6	95 7	30 12	21 14	19
	97 14	63 11	19 9	23 7	17
Demand	11	3	14	16	

Now, the time transportation problem of BCTP is given below:

					Supply
	10	68	73	52	8
	66	95	30	21	19
	97	63	19	23	17
Demand	11	3	14	16	

Using the blocking method, we have that the optimal solution of the time transportation problem of BTP is 66.

Now, the cost transportation table of BCTP is given below.

					Supply
	5	6	10	11	8
	6	7	12	14	19
	14	11	9	7	17
Demand	11	3	14	16	

By zero point method, the optimal solution is $x_{13} = 8$; $x_{21} = 11$; $x_{22} = 3$; $x_{23} = 5$; $x_{33} = 1$ and $x_{34} = 16$ with the minimum transportation cost is 348 and the minimum time transportation is 95.

Now, we have $T_0 = 66$; $T_m = 95$ and the time M = 66,68,73,95}

Now, the active cost transportation problem of BCTP for M = 66 is given below:

					Supply
	5	-	-	11	8
	6	-	12	14	19
	-	11	9	7	17

Demand	11	3	14	16	

Using zero point method, the optimal solution is x_{11} = 6; x_{14} = 2; x_{21} = 5; x_{23} = 14; x_{32} = 3 and x_{34} = 14 with total minimum transportation cost = 381.

Now, the active cost transportation problem of BCTP for M = 68 is given below

					Supply
	5	6	-	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using zero point method, the optimal solution is x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1 and x_{34} = 16 with the minimum transportation cost 356.

Now, the active cost transportation table of BCTP for M = 73 is given below.

					Supply
	5	6	10	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using zero point method, the optimal solution is x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1 and x_{34} = 16 with the minimum total transportation cost = 351.

Now, the efficient solutions to the BCTP is given below:

S.No.	Efficient solution of BCTP	Objective value of BCTP	Satisfaction Level α
1	x_{11} = 6; x_{14} = 2; x_{21} = 5; x_{23} = 14; x_{32} = 3 and x_{34} = 14 with time 66	(381, 66)	1
2	x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1 and x_{34} = 16 with time 68	(356, 68)	$\frac{27}{29} \approx 0.93$
3	x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1 and x_{34} = 16 with time 73	(351, 73)	$\frac{22}{29} \approx 0.76$
4	x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1 and x_{34} = 16 with time 95	(348, 95)	

4. Conclusion

In many transportation problems the important factor is travel time. The blocking method is quite simple to understand and apply. And blocking zero point method, we obtain a sequence of optimal solutions to a fuzzy bottleneck transportation problem for a sequence of various time in a time interval. The blocking zero point method enables the decision maker to evaluate and correct the managerial decisions.

References

[1]. Alexandra Tkachenko and Artiom Alhazov,"The Multiobjective Bottleneck Transportation Problem " Computer Science Journal of Moldova, vol.9, no.3(27), 2001

[2]. Y.P.Aneja and K.P.K.Nair, Bi-criteria transportation problem, Management Sci., 25(1979), 73-79.

[3]. H.L.Bhatia, Kanti Swaroop and M.C. Puri, A procedure for time minimization transportation problem, Presented in 7th Annual Conference of ORSI at Kanpur, 1974.

[4]. R.S.Garfinkel and M.R.Rao, The bottleneck transportation problem, Naval Research Logistics Quarterly, 18(1971), 465-472.

[5]. T.Glickman and p.D. Berger, Cost – Completion – Date Tradeoffs in the Transportation Problem, O. Res. 25 (1977), 163-168.

[6]. R.Gupta, Time-Cost Transportation Problem, Econ-math overview, 4(1977), 431-443.

[7]. P.L.Hammer, Time minimizing transportation problems, Naval Research Logistics Quarterly, 16 (1969), 345-357.

[8]. Ilija Nikolic, Total time minimizing transportation problem, Yugoslav Journal of Operations Research, 17 (2007), 125-133.

[9]. H. Issermann, Linear bottleneck transportation problem, Asia Pacific Journal of Operational Research, 1 (1984), 38 – 52.

[10]. Pandian.P and Natarajan.G, "A New Method for solving Bottleneck-cost transportation problems", International Mathematical Forum, 6 (2011); 451-460.

[11]. Rita Malhotra and M.C.Puri, pricing of bottlenecks at optimal time in a transportation problem, Combinatorial Optimization : Some aspects, Rita Malhotra et al.(Eds), Narosa Publishing House, New Delhi, India, 2007.

[12].W. Szware, Some remarks on the transportation problem, Naval Research Logistics Quarterly, 18(1971), 473-485.