

Additional Intermediate Tension Coefficients in Cylindrical Liquid Storage Tanks under Hinged Base Condition

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Hoop tension coefficients for analysis of cylindrical liquid storage tanks are available in codes of practice of various countries and also in Portland Cement Association (PCA). Hoop tension coefficients values are dependent on the dimension parameters, $\frac{H^2}{Dt}$ ranging from 0.4 to 16. Since these coefficients are available in irregular intervals with the assumption that the intermediate $\frac{H^2}{Dt}$ coefficients can be calculated by interpolation. This paper shows graphically and with the table providing in the appendix by solving the fourth order differential equation that the hoop tension coefficient values, calculated from interpolation is not accurate with the increasing $\frac{H^2}{Dt}$ from 0.4 to 16. Availability of these intermediate hoop tension coefficients for $\frac{H^2}{Dt}$ from 0.4 to 16 with interval of 0.1 will not only facilitate analysis and design process for variety of tank dimensions with better accuracy but also provides ease to the practicing engineers.

Keywords: Cylindrical Liquid storage tanks, hoop tension, design codes, tension coefficients.

1. Introduction

Storage reservoirs and overhead tanks are used to store water, liquid petroleum, petroleum products and similar liquids. In general, there are three kinds of storage tanks viz tanks resting on the ground, underground tanks and elevated tanks. These tanks are either rectangular or cylindrical in shape. In this paper, the scope is limited to cylindrical tanks resting on the ground. Some of the examples of tanks resting on the ground are clear water reservoirs, settling tanks and aeration tanks. The walls of these tanks are subjected to hydrostatic pressure and the base is subjected to the weight of liquid and upward soil pressure. The design of liquid storage cylindrical tanks involves calculations of the vertical bending moment, shear force, and hoop tension. These design forces (hoop tension, bending moment and shear force) are calculated from readily available coefficients in Portland Cement Association (PCA) [1-3]. The coefficients are dependent on the dimension parameter, $\frac{H^2}{Dt}$, where H , height of the tank, D , diameter of the tank and, t , thickness of the tank. The values of this dimension parameter, $\frac{H^2}{Dt}$, range from 0.4 to 16. However, these hoop tension coefficients are available in irregular

intervals. This paper addresses hoop tension and contributes by providing additional intermediate hoop tension coefficients values for $\frac{H^2}{Dt}$ from 0.4 to 16 with interval of 0.1. The wall of the tank is considered hinged at base and free at top, as these boundary conditions give maximum hoop tension across the height of the wall.

2. Need for the Study

Hoop tension causes tensile cracks in the walls which are undesirable as they are cause of leakage. Hoop tension coefficients for the cylindrical tank are readily available for $\frac{H^2}{Dt}$ of 0.4, 0.8, 1.2, 1.6, 2, 3, 4, 5, 6, 8, 10, 12, 14 and 16. Prediction can be made by analyzing the behavior of the wall that there will be no prominent deviations. Sometimes the coefficients can be managed within the available $\frac{H^2}{Dt}$ limits by the designer which leads to faulty design of the liquid storage cylindrical tanks. Considerable efforts have been made in the last few decades that aimed at the development of adequate design of liquid storage cylindrical tanks. However, most of the proposed techniques concern only very specific cases and/or are based on numerous simplifying assumptions that give rise to appreciably inaccurate results.

2.1 Calculation of hoop tension

The hoop tension can be calculated by [1-3]

$$T = \text{coefficient} \times \gamma \cdot H \cdot r \quad (1)$$

Let us take an example of tank whose height is 4m, diameter is 12m and thickness of wall is 0.175m, for this tank dimension; $\frac{H^2}{Dt}$ is 7.6. In absence of this $\frac{H^2}{Dt}$ values, the practicing engineers are forced to take either next highest coefficients or interpolate linearly between the available $\frac{H^2}{Dt}$ values i.e between 6 and 8 (as per PCA table).

The problem by taking the next higher coefficients is that the design will be uneconomical. Below Table shows the comparison in the coefficients values by interpolation and analysis. Values shown in the table 1 is for the maximum hoop tension coefficient for the $\frac{H^2}{Dt}$, 6 and 8.

Table 1. Comparison of coefficients

$\frac{H^2}{Dt}$	Coefficients value with Interpolation	Coefficients value with Analysis
6	0.643	0.643
7.6	0.686	0.690
8	0.697	0.697

From equation (1), it is clear that hoop tension is dependent on the coefficient value of the $\frac{H^2}{Dt}$. And the above table shows the difference in the coefficient of the analysis value and the interpolation values.

3. Methodology

In this section, the systematic outline of the analysis is provided.

3.1 Modeling

The tank is modeled as an elastic cylindrical tank of diameter D , height H , and thickness t . The Young's modulus and Poisson's ratio of the tank material are E and μ , respectively. The density of liquid contained in the tank is γ . The problem is

axisymmetric and the scope is limited to tanks resting on ground level only.

3.2 Loads

As the tank is considered to be resting on ground level, there is no uplift force on bottom raft due to ground water table. Walls are subjected to hydrostatic pressure from the inner side of the tank. The vertical load of the water body and self-weight of the base slab of the tank are balanced by the reaction from the soil.

3.3. Analytical modeling

Consider a circular cylindrical tank of finite diameter and height and subjected to axisymmetric, torsion-free loading. The hydrostatic pressure causes radial expansion and vertical bending of the tank wall. The governing equation of displacement, y may be written the following way if the thickness of the tank, t , is constant [4-6].

$$\frac{Et^3}{12(1-\mu^2)} \frac{d^4y}{dx^4} = \gamma(H-x) - y \frac{4tE}{D^2} \quad (2)$$

The general solution of the above fourth order ordinary differential equation of constant coefficients may be written as [5].

$$y = e^{kx}(A\cos[kx] + B\sin[kx]) + e^{-kx}(C\cos[kx] + D\sin[kx]) + \gamma(H-x) \frac{D^2}{4Et} \quad (3)$$

Where; A , B , C , and D are the four integration constants to be determined from the boundary conditions.

Note that all the other variables rotation, Θ , bending moment, BM and shearing force, SF can be expressed in terms of displacement, y by the following equations

$$\Theta = \frac{dy}{dx} \quad (4)$$

$$BM = \frac{d^2y}{dx^2} \quad (5)$$

$$\text{SF} = \frac{d^3 y}{dx^3} \quad (6)$$

Several different cylindrical liquid storage tanks are possible based on the dimension parameters. So for the each tank we need to solve the equation (2). The PCA provides tables to calculate design forces based on the $\frac{H^2}{Dt}$ range from 0.4 to 16. We have calculated the hoop tension coefficients values for $\frac{H^2}{Dt}$ range from 0.4 to 16 with the interval of 0.1 by solving equation (3) for several times.

3.4 Calculation of coefficients

Hoop tension coefficients for the cylindrical wall for $\frac{H^2}{Dt}$ range from 0.4 to 16 was taken for verification problem by solving the differential equation (2). Input parameters taken to solve the differential equation (2) are as follows, the density of the liquid, γ is 10 kN/m³; Poisson's ratio, μ is 0.3 and modulus of elasticity of concrete, E is 2000000 kPa. Appendix shows the table of hoop tension coefficients for cylindrical wall hinge base, free top and subjected to hydrostatic load for $\frac{H^2}{Dt}$ range from 0.1 to 16.

4. Result and Discussion

Figure 1 shows the plot of maximum hoop tension coefficient for $\frac{H^2}{Dt}$ range from 0.4 to 16 (plotted with the table from Appendix). It is observed that though the value of maximum hoop tension coefficients increases with the increasing $\frac{H^2}{Dt}$ from 0.4 to 16, but the increment is not linear, which means the value of maximum hoop tension coefficient for intermediate $\frac{H^2}{Dt}$ between any two available $\frac{H^2}{Dt}$ through interpolation may not be accurate.

Even, sometimes, the location of the maximum hoop tension also changes along the height of the tank wall. Figure 2 shows the plot of location of maximum hoop tension along the height of the tank wall. This location is very crucial with respect to the tension cracks developed due to the hydrostatic loading. Hence, availability of these intermediate hoop tension coefficients for $\frac{H^2}{Dt}$ from 0.4 to 16 with

interval of 0.1 will not only facilitate analysis and design process for variety of tank dimensions with better accuracy but also provides ease to the practicing engineers.

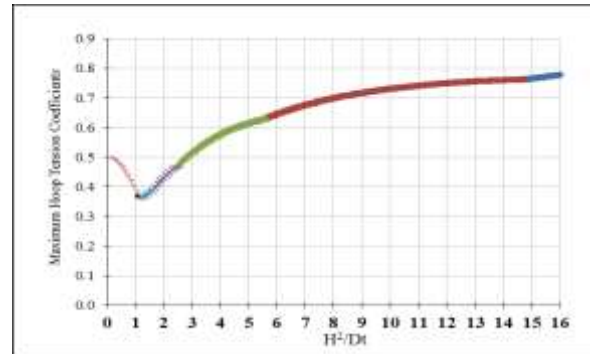


Fig.1 Maximum hoop tension coefficients for $\frac{H^2}{Dt}$ from 0.4 to 16

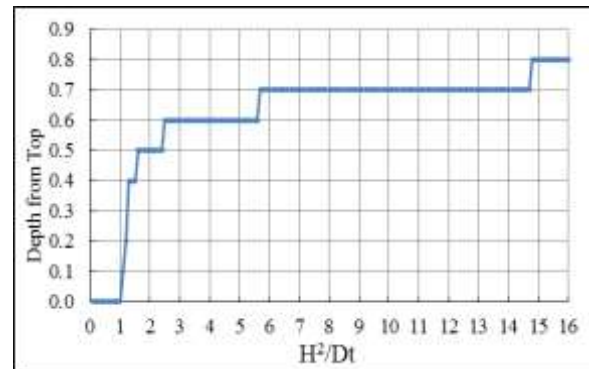


Fig.2 Location of maximum hoop tension for $\frac{H^2}{Dt}$ from 0.4 to 16

Nomenclature

- D = diameter of tank
- H = height of tank
- t = thickness of the wall
- I = moment of inertia per unit length
- μ = Poisson's ratio
- γ = unit weight of liquid
- E = modulus of elasticity of concrete
- y = deflection due to radial expansion
- r = radius of the tank

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Appendix: Tension coefficients for cylindrical wall hinge base, free top and subjected to hydrostatic load

$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.1	0.499	0.449	0.400	0.350	0.301	0.251	0.201	0.151	0.101	0.050
0.2	0.495	0.447	0.399	0.351	0.302	0.253	0.204	0.154	0.103	0.052
0.3	0.488	0.442	0.397	0.351	0.305	0.257	0.208	0.158	0.107	0.054
0.4	0.474	0.440	0.395	0.352	0.308	0.264	0.215	0.165	0.111	0.057
0.5	0.467	0.430	0.392	0.353	0.313	0.269	0.223	0.172	0.118	0.060
0.6	0.454	0.421	0.388	0.354	0.318	0.277	0.232	0.182	0.125	0.064
0.7	0.439	0.412	0.385	0.356	0.323	0.286	0.243	0.192	0.134	0.069
0.8	0.423	0.402	0.381	0.358	0.330	0.297	0.249	0.202	0.145	0.076
0.9	0.405	0.391	0.376	0.359	0.336	0.307	0.267	0.216	0.153	0.080
1	0.386	0.379	0.371	0.360	0.343	0.318	0.280	0.229	0.164	0.086
1.1	0.367	0.367	0.366	0.362	0.351	0.329	0.294	0.243	0.175	0.092
1.2	0.350	0.355	0.361	0.362	0.358	0.342	0.309	0.256	0.186	0.098
1.3	0.328	0.342	0.356	0.365	0.365	0.353	0.322	0.271	0.198	0.105
1.4	0.308	0.330	0.351	0.366	0.373	0.364	0.337	0.285	0.210	0.112
1.5	0.289	0.318	0.345	0.368	0.380	0.376	0.351	0.300	0.222	0.119
1.6	0.271	0.303	0.341	0.369	0.385	0.386	0.362	0.314	0.233	0.124
1.7	0.252	0.294	0.335	0.370	0.394	0.399	0.378	0.328	0.245	0.132
1.8	0.234	0.283	0.330	0.371	0.400	0.409	0.392	0.342	0.256	0.139
1.9	0.217	0.272	0.325	0.372	0.406	0.420	0.405	0.355	0.268	0.145
2	0.205	0.260	0.321	0.373	0.411	0.434	0.419	0.369	0.280	0.151
2.1	0.185	0.251	0.316	0.374	0.418	0.439	0.429	0.381	0.289	0.158
2.2	0.170	0.242	0.311	0.374	0.423	0.448	0.441	0.393	0.300	0.164
2.3	0.156	0.232	0.307	0.375	0.428	0.457	0.452	0.405	0.310	0.170
2.4	0.142	0.224	0.303	0.375	0.432	0.465	0.463	0.416	0.320	0.176
2.5	0.130	0.215	0.299	0.375	0.437	0.473	0.473	0.427	0.330	0.182
2.6	0.118	0.208	0.295	0.375	0.440	0.480	0.483	0.438	0.339	0.188
2.7	0.107	0.200	0.291	0.375	0.444	0.487	0.492	0.448	0.348	0.193
2.8	0.097	0.193	0.288	0.375	0.447	0.493	0.501	0.458	0.357	0.199
2.9	0.087	0.187	0.284	0.375	0.450	0.500	0.510	0.468	0.366	0.204
3	0.074	0.179	0.281	0.375	0.449	0.506	0.519	0.479	0.375	0.210
3.1	0.070	0.175	0.278	0.374	0.456	0.511	0.525	0.486	0.382	0.214
3.2	0.062	0.169	0.275	0.374	0.458	0.516	0.533	0.494	0.390	0.219
3.3	0.055	0.164	0.272	0.373	0.460	0.520	0.539	0.503	0.398	0.224
3.4	0.048	0.159	0.269	0.372	0.462	0.524	0.546	0.510	0.405	0.228
3.5	0.042	0.155	0.266	0.372	0.463	0.529	0.552	0.518	0.412	0.233
3.6	0.036	0.150	0.264	0.371	0.465	0.532	0.558	0.525	0.420	0.238
3.7	0.031	0.146	0.261	0.370	0.466	0.536	0.564	0.533	0.426	0.242
3.8	0.026	0.143	0.259	0.369	0.467	0.539	0.569	0.539	0.433	0.246
3.9	0.021	0.139	0.256	0.368	0.468	0.542	0.574	0.546	0.440	0.251
4	0.017	0.137	0.253	0.367	0.469	0.545	0.579	0.553	0.447	0.256
$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4.1	0.013	0.133	0.252	0.366	0.469	0.547	0.584	0.558	0.452	0.259
4.2	0.010	0.130	0.250	0.365	0.469	0.550	0.588	0.564	0.458	0.263
4.3	0.007	0.128	0.248	0.364	0.470	0.552	0.593	0.570	0.464	0.267
4.4	0.004	0.125	0.246	0.363	0.470	0.554	0.597	0.576	0.470	0.271
4.5	0.001	0.123	0.244	0.362	0.470	0.556	0.600	0.581	0.476	0.275
4.6	-0.001	0.121	0.242	0.361	0.470	0.557	0.604	0.586	0.481	0.279
4.7	-0.004	0.119	0.240	0.360	0.470	0.559	0.607	0.591	0.487	0.282
4.8	-0.005	0.117	0.239	0.358	0.470	0.560	0.611	0.596	0.492	0.286
4.9	-0.007	0.115	0.237	0.357	0.469	0.561	0.614	0.601	0.498	0.290
5	-0.008	0.114	0.235	0.356	0.469	0.562	0.617	0.606	0.503	0.294
5.1	-0.010	0.112	0.234	0.355	0.469	0.564	0.620	0.610	0.508	0.297
5.2	-0.011	0.111	0.233	0.354	0.468	0.564	0.622	0.615	0.513	0.300
5.3	-0.013	0.109	0.231	0.352	0.468	0.565	0.625	0.619	0.517	0.304
5.4	-0.014	0.108	0.230	0.351	0.467	0.566	0.627	0.623	0.522	0.307
5.5	-0.014	0.107	0.229	0.350	0.467	0.567	0.630	0.627	0.527	0.311
5.6	-0.015	0.106	0.227	0.349	0.466	0.567	0.632	0.631	0.532	0.314
5.7	-0.016	0.105	0.226	0.348	0.465	0.568	0.634	0.635	0.536	0.317
5.8	-0.016	0.104	0.225	0.346	0.464	0.568	0.636	0.638	0.540	0.321
5.9	-0.017	0.103	0.224	0.345	0.464	0.568	0.638	0.642	0.545	0.324
6	-0.017	0.103	0.223	0.343	0.463	0.568	0.640	0.645	0.549	0.327
6.1	-0.017	0.102	0.222	0.343	0.462	0.568	0.641	0.649	0.553	0.330
6.2	-0.018	0.101	0.221	0.342	0.461	0.569	0.643	0.652	0.558	0.333
6.3	-0.018	0.100	0.220	0.341	0.461	0.569	0.645	0.655	0.562	0.337
6.4	-0.018	0.100	0.219	0.340	0.460	0.569	0.646	0.658	0.566	0.340
6.5	-0.018	0.099	0.218	0.338	0.459	0.569	0.648	0.661	0.570	0.343
6.6	-0.018	0.099	0.217	0.337	0.458	0.569	0.649	0.664	0.573	0.346
6.7	-0.018	0.098	0.216	0.336	0.457	0.569	0.650	0.667	0.577	0.349
6.8	-0.018	0.098	0.215	0.335	0.456	0.569	0.651	0.670	0.581	0.352
6.9	-0.018	0.098	0.214	0.334	0.455	0.568	0.653	0.673	0.585	0.354
7	-0.018	0.097	0.214	0.333	0.454	0.568	0.654	0.675	0.588	0.357
7.1	-0.017	0.097	0.213	0.332	0.453	0.568	0.655	0.678	0.592	0.360
7.2	-0.017	0.097	0.212	0.331	0.452	0.568	0.656	0.681	0.596	0.363
7.3	-0.017	0.096	0.211	0.330	0.452	0.567	0.657	0.683	0.599	0.366
7.4	-0.017	0.096	0.211	0.329	0.451	0.567	0.657	0.685	0.603	0.369
7.5	-0.016	0.096	0.210	0.328	0.450	0.567	0.658	0.688	0.606	0.371
7.6	-0.016	0.096	0.210	0.327	0.449	0.566	0.659	0.690	0.609	0.374
7.7	-0.016	0.096	0.209	0.326	0.448	0.566	0.660	0.692	0.613	0.377
7.8	-0.015	0.096	0.208	0.325	0.447	0.565	0.660	0.694	0.616	0.380
7.9	-0.015	0.095	0.208	0.325	0.446	0.565	0.661	0.697	0.619	0.382
8	-0.015	0.095	0.208	0.324	0.445	0.564	0.661	0.697	0.621	0.386
8.1	-0.014	0.095	0.207	0.323	0.444	0.564	0.662	0.701	0.625	0.388
$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
8.2	-0.014	0.095	0.206	0.322	0.443	0.563	0.663	0.703	0.628	0.390

8.3	-0.014	0.095	0.206	0.321	0.442	0.563	0.663	0.705	0.631	0.393
8.4	-0.013	0.095	0.205	0.321	0.442	0.562	0.664	0.706	0.634	0.395
8.5	-0.013	0.095	0.205	0.320	0.441	0.562	0.664	0.708	0.637	0.398
8.6	-0.013	0.095	0.205	0.319	0.440	0.561	0.664	0.710	0.640	0.400
8.7	-0.012	0.095	0.204	0.318	0.439	0.561	0.665	0.712	0.643	0.403
8.8	-0.012	0.095	0.204	0.318	0.438	0.560	0.665	0.713	0.646	0.405
8.9	-0.012	0.095	0.203	0.317	0.437	0.560	0.665	0.715	0.649	0.408
9	-0.011	0.095	0.203	0.316	0.437	0.559	0.666	0.717	0.651	0.410
9.1	-0.011	0.095	0.203	0.316	0.436	0.559	0.666	0.718	0.654	0.413
9.2	-0.011	0.095	0.202	0.315	0.435	0.558	0.666	0.720	0.657	0.415
9.3	-0.010	0.095	0.202	0.314	0.434	0.557	0.666	0.721	0.659	0.417
9.4	-0.010	0.095	0.202	0.314	0.433	0.557	0.667	0.723	0.662	0.420
9.5	-0.010	0.095	0.201	0.313	0.433	0.556	0.667	0.724	0.665	0.422
9.6	-0.009	0.095	0.201	0.313	0.432	0.556	0.667	0.726	0.667	0.425
9.7	-0.009	0.095	0.201	0.312	0.431	0.555	0.667	0.727	0.670	0.427
9.8	-0.009	0.095	0.201	0.311	0.430	0.554	0.667	0.728	0.672	0.429
9.9	-0.008	0.095	0.200	0.311	0.429	0.553	0.666	0.729	0.675	0.431
10	-0.008	0.095	0.200	0.311	0.428	0.552	0.666	0.730	0.678	0.433
10.1	-0.008	0.095	0.200	0.310	0.428	0.552	0.666	0.732	0.679	0.436
10.2	-0.007	0.095	0.200	0.309	0.427	0.552	0.666	0.733	0.682	0.438
10.3	-0.007	0.095	0.200	0.309	0.427	0.551	0.667	0.734	0.684	0.440
10.4	-0.007	0.096	0.199	0.309	0.426	0.551	0.667	0.735	0.686	0.443
10.5	-0.006	0.096	0.199	0.308	0.425	0.550	0.667	0.736	0.688	0.445
10.6	-0.006	0.096	0.199	0.308	0.425	0.550	0.667	0.738	0.691	0.447
10.7	-0.006	0.096	0.199	0.307	0.424	0.549	0.667	0.739	0.693	0.449
10.8	-0.006	0.096	0.199	0.307	0.424	0.548	0.667	0.740	0.695	0.451
10.9	-0.005	0.096	0.199	0.306	0.423	0.548	0.667	0.741	0.697	0.453
11	-0.005	0.096	0.198	0.306	0.422	0.547	0.667	0.741	0.699	0.455
11.1	-0.005	0.096	0.198	0.306	0.422	0.546	0.666	0.742	0.701	0.458
11.2	-0.005	0.096	0.198	0.305	0.421	0.546	0.666	0.743	0.703	0.460
11.3	-0.004	0.096	0.198	0.305	0.421	0.545	0.666	0.744	0.705	0.462
11.4	-0.004	0.096	0.198	0.305	0.420	0.545	0.666	0.745	0.707	0.464
11.5	-0.004	0.096	0.198	0.304	0.419	0.544	0.666	0.746	0.709	0.466
11.6	-0.004	0.096	0.198	0.304	0.419	0.543	0.666	0.747	0.711	0.468
11.7	-0.004	0.096	0.198	0.304	0.418	0.543	0.665	0.747	0.713	0.470
11.8	-0.003	0.096	0.198	0.303	0.418	0.542	0.665	0.748	0.715	0.472
11.9	-0.003	0.097	0.198	0.303	0.417	0.542	0.665	0.749	0.717	0.474
12	-0.003	0.097	0.197	0.302	0.417	0.541	0.664	0.750	0.720	0.477
12.1	-0.003	0.097	0.197	0.302	0.416	0.541	0.664	0.750	0.721	0.478
12.2	-0.003	0.097	0.197	0.302	0.416	0.540	0.664	0.751	0.723	0.480
$\frac{H^2}{Dt}$	Coefficients at point									
(1)	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	
12.3	-0.003	0.097	0.197	0.302	0.415	0.539	0.664	0.752	0.724	0.482
12.4	-0.002	0.097	0.197	0.302	0.415	0.539	0.664	0.752	0.726	0.483
12.5	-0.002	0.097	0.197	0.302	0.414	0.538	0.664	0.753	0.728	0.485

12.6	-0.002	0.097	0.197	0.301	0.414	0.538	0.663	0.754	0.730	0.487
12.7	-0.002	0.097	0.197	0.301	0.414	0.537	0.663	0.754	0.731	0.489
12.8	-0.002	0.097	0.197	0.301	0.413	0.537	0.663	0.755	0.733	0.491
12.9	-0.002	0.097	0.197	0.301	0.413	0.536	0.662	0.755	0.735	0.493
13	-0.002	0.097	0.197	0.301	0.412	0.536	0.662	0.756	0.736	0.495
13.1	-0.001	0.097	0.197	0.300	0.412	0.535	0.662	0.756	0.738	0.497
13.2	-0.001	0.097	0.197	0.300	0.412	0.534	0.661	0.757	0.740	0.498
13.3	-0.001	0.098	0.197	0.300	0.411	0.534	0.661	0.757	0.741	0.500
13.4	-0.001	0.098	0.197	0.300	0.411	0.533	0.661	0.758	0.743	0.502
13.5	-0.001	0.098	0.197	0.300	0.410	0.533	0.661	0.758	0.744	0.504
13.6	-0.001	0.098	0.197	0.300	0.410	0.532	0.660	0.759	0.746	0.506
13.7	-0.001	0.098	0.197	0.299	0.410	0.532	0.660	0.759	0.747	0.507
13.8	-0.001	0.098	0.197	0.299	0.409	0.531	0.660	0.760	0.749	0.509
13.9	-0.001	0.098	0.197	0.299	0.409	0.531	0.659	0.760	0.750	0.511
14	-0.001	0.098	0.197	0.299	0.408	0.531	0.659	0.761	0.752	0.513
14.1	0.000	0.098	0.197	0.299	0.408	0.530	0.658	0.761	0.753	0.514
14.2	0.000	0.098	0.197	0.299	0.408	0.529	0.658	0.761	0.755	0.515
14.3	0.000	0.098	0.197	0.299	0.408	0.529	0.658	0.761	0.756	0.516
14.4	0.000	0.098	0.197	0.299	0.407	0.528	0.657	0.762	0.758	0.518
14.5	0.000	0.098	0.197	0.298	0.407	0.528	0.657	0.762	0.759	0.520
14.6	0.000	0.098	0.197	0.298	0.407	0.528	0.657	0.762	0.760	0.521
14.7	0.000	0.098	0.197	0.298	0.407	0.527	0.656	0.763	0.762	0.523
14.8	0.000	0.098	0.197	0.298	0.406	0.527	0.656	0.763	0.763	0.525
14.9	0.000	0.098	0.197	0.298	0.406	0.526	0.656	0.763	0.764	0.526
15	0.000	0.098	0.197	0.298	0.406	0.526	0.655	0.763	0.766	0.530
15.1	0.000	0.098	0.197	0.298	0.405	0.525	0.655	0.764	0.767	0.530
15.2	0.000	0.099	0.197	0.298	0.405	0.525	0.655	0.764	0.768	0.531
15.3	0.000	0.099	0.197	0.298	0.405	0.524	0.654	0.764	0.769	0.531
15.4	0.000	0.099	0.197	0.298	0.405	0.524	0.654	0.764	0.771	0.532
15.5	0.000	0.099	0.197	0.298	0.404	0.524	0.653	0.765	0.772	0.533
15.6	0.000	0.099	0.197	0.298	0.404	0.523	0.653	0.765	0.773	0.533
15.7	0.001	0.099	0.197	0.298	0.404	0.523	0.653	0.765	0.774	0.534
15.8	0.001	0.099	0.197	0.298	0.404	0.522	0.652	0.765	0.775	0.534
15.9	0.001	0.099	0.197	0.297	0.404	0.522	0.652	0.765	0.776	0.535
16	0.002	0.100	0.198	0.299	0.403	0.521	0.650	0.764	0.776	0.536