

Expected Time to Recruitment in Man Power Planning Using EE-Distribution

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Abstract: In any organization, successive exit of personnel leads of wastage in the form of man hours. The breakdown of the organization is triggered whenever the cumulative loss of manpower exceeds a particular level called threshold, beyond which the organization cannot sustain due to loss of protectively Gupta and Kundu [2001] developed an Exponentiated Exponential (EE) distribution with two parameters namely scale and shape parameter. The current work is focused on shape parameter of EE distribution. To compute the expected time to reach the recruitment and its variance is found. The analytical results are numerically illustrated by assuming specific distributions.

Keywords: Expected time, Manpower planning, Threshold, Wastage, EE distribution

1. Introduction

Manpower planning depends on the highly unpredictable human behavior and the uncertain social environment in which the system functions. The main factors which determine the behavior of a manpower system are recruitment, promotion, and wastage. The size of various grades, which respond to the expansion, promotions and wastages are maintained at the desired level at any time by a process called recruitment. Vacancies that arise in the lower grade are filled up by recruitments whereas those in the higher grades are filled up by promotions. Wastages is termed as when employees move from one grade to another, they are exposed to different factors influencing them to leave the organization.

Esary et al. [1973] discussed that any component or device when exposed to shocks which cause damage to the device or system is likely to fail when the total accumulated damage exceeds a level called the threshold. The rate of accumulation of damage determines the life time of the component or device. More works on manpower planning one can see in Bartholomew [1971], Bartholomew and Forbes [1979].

Generalized Exponential distribution (location, scale, shape) discussed by Gupta and Kundu [1999] which has an

increasing or decreasing failure rate depending on the shape parameter. Also Gupta and Kundu [2001] discussed about Exponentiated Exponential distribution with two parameters namely scale and shape parameter

The distribution function, $F_E(x, \alpha, \lambda)$,

$$F_E(x, \alpha, \lambda) = (1 - e^{-\lambda x})^\alpha \quad \alpha, \lambda, x > 0$$

The density function is

$$f_E(x, \alpha, \lambda) = \alpha \lambda (1 - e^{-\lambda x})^{\alpha-1} e^{-\lambda x}$$

The corresponding survival function is

$$S_E(x, \alpha, \lambda) = 1 - (1 - e^{-\lambda x})^\alpha$$

When the shape parameter of the exponentiated exponential distribution ($\alpha = 1$), it represents the exponential distribution. Damodaran and Gopal [2009] stated that, for the simplicity and for a single parameter distribution, the Generalised Exponential Distribution with shape parameter ($\alpha = 2$) was considered. Parthasarathy et al., [2010] discussed when threshold follows Gamma distribution. Further the threshold depicts SCBZ carried out by Sahtyamoorthy and Parthasarathy[2003]. In this direction this paper made an attempt that the threshold hold EE distribution when the shape parameter $\alpha = 3$ for which the mean and variances are derived.

2. ASSUMPTION OF THE MODEL

1. Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
2. The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
3. The process of depletion is linear and cumulative.
4. The inter arrival times between successive occasions of wastage are i.i.d. random variables.
5. If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
6. The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

3. NOTATIONS

- X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i=1,2,\dots,k$ and X_i 's are i.i.d and $X_i = X$ for all i .
- Y : a continuous random variable denoting the threshold level having Exponentiated Exponential distribution.
- $g(\cdot)$: The probability density functions of X .
- $g_k(\cdot)$: The k -fold convolution of $g(\cdot)$ i.e., p.d.f. of $\sum_{i=1}^k X_i$
- T : a continuous r.v denoting time to breakdown of the system.
- $g^*(\cdot)$: Laplace transform of $g(\cdot)$.
- $g_k^*(\cdot)$: Laplace transform of $g_k(\cdot)$.
- $h(\cdot)$: The p.d.f. of random threshold level which has Exponentiated Exponential distribution and $H(\cdot)$ is the corresponding c.d.f.
- U : a continuous random variable denoting the inter-arrival times between decision epochs.
- $f(\cdot)$: p.d.f. of random variable U with corresponding c.d.f. $F(\cdot)$.
- $F_k(t)$: The k -fold convolution functions of $F(\cdot)$.
- $S(\cdot)$: The survivor function i.e. $P [T > t]$.
- $L(t)$: $1 - S(t)$
- $F_k(t)$: Probability that there are exactly 'k' policies decisions in $(0, t]$.

4. RESULTS

Here exponentiated exponential distribution with $\alpha = 3$, is considered

Let Y be the random variable which has the cdf defined as

$$H_E(x, \alpha, \lambda) = (1 - e^{-\lambda x})^3 \quad \alpha, \lambda, x > 0$$

Therefore it has the density function

$$h_E(x, \alpha, \lambda) = 3\lambda(1 - e^{-\lambda x})^2$$

The corresponding survival function is

$$S_E(x, \alpha, \lambda) = 1 - (1 - e^{-\lambda x})^3$$

Now,

$P(X_1 + X_2 + \dots + X_k < Y) = P$ [the system does not fail, after k epochs of exits].

$$\begin{aligned} & P\left(\sum X_i < Y\right) \\ &= \int_0^\infty g_k(x) \bar{H}(x) dx \\ &= \int_0^\infty g_k(x) [3e^{-\lambda x} - 3e^{-2\lambda x} + e^{-3\lambda x}] \\ &= 3g_k^*(\lambda) - 3g_k^*(2\lambda) + g_k^*(3\lambda) \\ &= 3[g^*(\lambda)]^k - 3[g^*(2\lambda)]^k + [g^*(3\lambda)]^k \end{aligned}$$

The survival function $S(t)$ which is the probability that an individual survives for a time t .

$S(t) = P(T > t) =$ Probability that the system survives beyond t .

$$\begin{aligned} &= \sum_{k=0}^\infty P \{ \text{there are exactly } k \text{ instants of exists in } (0, t] \\ &\quad * P \text{ (the system does not fail in } (0, t]) \} \end{aligned}$$

It is also known from renewal theory that

$$\begin{aligned} & P(\text{exactly } k \text{ policy decision in } (0, t]) \\ &= F_k(t) - F_{k+1}(t) \quad \text{with } F_0(t) \\ &= 1 \\ &= \sum_{k=0}^\infty F_k(t) P(X_i < Y) \end{aligned}$$

$$\begin{aligned} &= \left\{ 3 \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(\lambda)]^k \right. \\ &\quad - 3 \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(2\lambda)]^k \\ &\quad \left. + \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(3\lambda)]^k \right\} \end{aligned}$$

Now

$$\begin{aligned} & P(T < t) = L(T) = 1 - S(t) \\ &= 1 - \left\{ 3 \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(\lambda)]^k \right. \\ &\quad - 3 \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(2\lambda)]^k \\ &\quad \left. + \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] [g^*(3\lambda)]^k \right\} \end{aligned}$$

$$L(t) = 1 - \{ 3[1 - [1 - g^*(\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(\lambda)]^{k-1}] -$$

$$3[1 - g^*(2\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(2\lambda)]^{k-1} -$$

$$+ [1 - g^*(3\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(3\lambda)]^{k-1} \}$$

$$= 1 - 3 + 3[1 - g^*(\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(\lambda)]^{k-1} + 3 -$$

$$3[1 - g^*(2\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(2\lambda)]^{k-1} - 3 + [1 -$$

$$g^*(3\lambda)] \sum_{k=1}^\infty F_k(t) [g^*(3\lambda)]^{k-1}$$

Taking laplace transform of $L(t)$, we get

$$\begin{aligned} L^*(s) &= \frac{3[1 - g^*(\lambda)] f^*(s)}{[1 - g^*(\lambda)] f^*(s)} - \frac{3[1 - g^*(2\lambda)] f^*(s)}{[1 - g^*(2\lambda)] f^*(s)} \\ &\quad + \frac{[1 - g^*(3\lambda)] f^*(s)}{[1 - g^*(3\lambda)] f^*(s)} \end{aligned}$$

Where $[f^*(s)]^k$ is laplace transform of $F_k(t)$ since the inter arrival times are i.i.d. The above equation can be rewritten as,

$$= 3c[1 - g^*(\lambda)][c + s - g^*(\lambda)c]^{-1} - 3c[1 - g^*(2\lambda)][c + s - g^*(2\lambda)c]^{-1} + c[1 - g^*(3\lambda)]^{-1}$$

$$E(T) = -\frac{d}{ds} L^*(S) \text{ given } s=0$$

$$E(T^2) = \frac{d^2 L^*(S)}{ds^2}$$

From which $V(T)$ can be obtained.

Let the random variable U denoting inter arrival time which follows exponential with parameter c . Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation we get,

$$E(T) = \frac{3}{c[1 - g^*(\lambda)]} - \frac{3}{c[1 - g^*(2\lambda)]} + \frac{1}{c[1 - g^*(3\lambda)]}$$

$$E(T^2) = \frac{d}{ds} \left[-\frac{3[1 - g^*(\lambda)]c}{[c + s - g^*(\lambda)c]^2} + \frac{3[1 - g^*(2\lambda)]c}{[c + s - g^*(2\lambda)c]^2} - \frac{3[1 - g^*(3\lambda)]c}{[c + s - g^*(3\lambda)c]^2} \right]$$

$$= \frac{6}{c^2[1 - g^*(\lambda)]^2} - \frac{6}{c^2[1 - g^*(2\lambda)]^2} + \frac{3}{c^2[1 - g^*(3\lambda)]^2}$$

$$V(T) = \frac{6}{c^2[1 - g^*(\lambda)]^2} - \frac{6}{c^2[1 - g^*(2\lambda)]^2} + \frac{3}{c^2[1 - g^*(3\lambda)]^2} - \left[\frac{3}{c[1 - g^*(\lambda)]} - \frac{3}{c[1 - g^*(2\lambda)]} + \frac{1}{c[1 - g^*(3\lambda)]} \right]^2$$

$g^*(.) \sim \exp(\mu), g^*(\lambda) = \frac{\mu}{\mu + \lambda}, g^*(2\lambda) = \frac{\mu}{\mu + 2\lambda}, g^*(3\lambda) = \frac{\mu}{\mu + 3\lambda}$

$$E(T) = \frac{3}{c \left[1 - \frac{\mu}{\mu + \lambda} \right]} - \frac{3}{c \left[1 - \frac{\mu}{\mu + 2\lambda} \right]} + \frac{1}{c \left[1 - \frac{\mu}{\mu + 3\lambda} \right]}$$

$$= \frac{1}{c^2} \left[\frac{6\mu^2 + 6\lambda^2 + 12\lambda\mu}{\lambda^2} - \frac{6\mu^2 + 12\lambda^2 + 24\lambda\mu}{2\lambda^2} + \frac{2\mu^2 + 6\lambda^2 + 12\lambda\mu}{3\lambda^2} \right] - \frac{1}{c^2} \left[\frac{3\mu + 3\lambda}{\lambda} - \frac{3\mu + 6\lambda}{2\lambda} + \frac{\mu + 3\lambda}{3\lambda} \right]^2$$

on simplification

5. NUMERICAL ILLUSTRATION

From table 1 and the corresponding figure 1 we could observe the difference in the values of $E(T)$ and $V(T)$ when the threshold distribution has Exponentiated exponential distribution with $\mu = 0.4$ and $\lambda = 0.2$ as C increases $E(T)$ as well as $V(T)$ decreases.

From table 2 and the corresponding figure 2 we could observe the difference in the values of $E(T)$ and $V(T)$ when the threshold distribution has Exponentiated exponential distribution. If parameter value is increased by $\mu = 0.7$ and $\lambda = 0.4$ as C increases $E(T)$ and $V(T)$ decreases. In both the cases the behavior is found to be same.

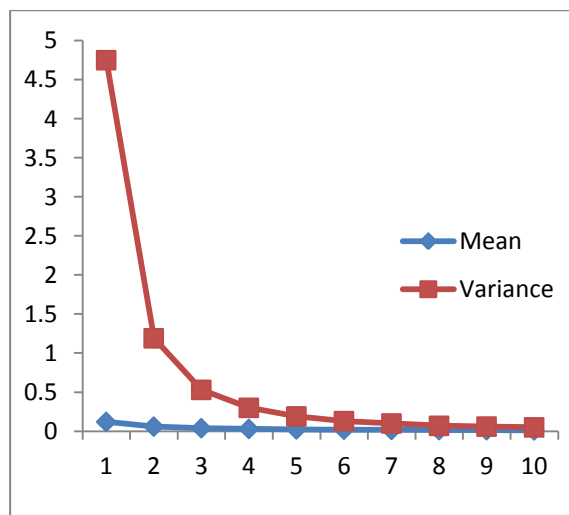


Figure 1.

$$= \frac{1}{c\lambda} \left[\frac{6\lambda\mu + \lambda(6\lambda + 15) - 5\mu}{2\lambda} \right] \text{ on simplification}$$

$$V(T) = \frac{1}{c^2} \left[\frac{6}{[1 - g^*(\lambda)]^2} - \frac{6}{[1 - g^*(2\lambda)]^2} + \frac{3}{[1 - g^*(3\lambda)]^2} - \frac{1}{c^2} \left[\frac{3}{[1 - g^*(\lambda)]} - \frac{3}{[1 - g^*(2\lambda)]} + \frac{1}{[1 - g^*(3\lambda)]} \right]^2 \right]$$

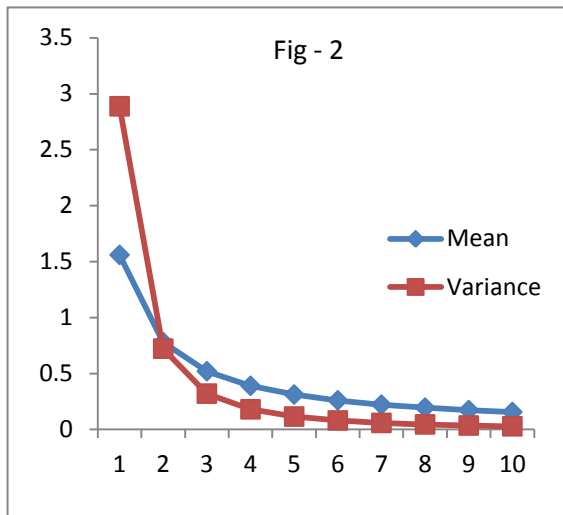


Figure 2.

Table 1 $\mu=0.4, \lambda = 0.2, C=1,2,..10$

C	1	2	3	4	5	6	7	8	9	10
E(T)	0.12	0.06	0.04	0.03	0.024	0.02	.017	.015	.013	.012
V(T)	4.75	1.19	0.53	0.30	0.19	0.13	0.10	0.07	0.06	0.05

Table 2 $\mu=0.7, \lambda = 0.4, c=1,2,..10$

C	1	2	3	4	5	6	7	8	9	10
E(T)	1.56	.78	.52	.39	.312	.26	.222	.195	.173	.156
V(T)	2.88	.72	.32	.18	.115	.08	.058	.045	.035	.028

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