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Fuzzy Algebra and Fuzzy Automata

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Abstract
In this paper we represent a fuzzy algebra of standard basis, using fuzzy matrix. Any finitely generated subspace are fuzzy algebra (£) has a unique standard basis and have the same cardinality, and the standardbasis can be uniquely expressed as a linear combination of the standard basis vector, and also we have to compute the standard fuzzy linear combination of the basis vector (0.5, 0.5, 0.5) in terms of the standard basis and also we derived fuzzy automata using relations.

Keywords: Fuzzy matrix, Fuzzy Algebra, Standard Basis.

Standard Basis
Definition: 1.1

A basis C over the fuzzy algebra £ is a standard basis iff whenever

$$c_i = \sum a_{ij} c_j \quad \text{for all } c_i , c_j \in C \text{ and } a_{ij} \in £$$

Then $a_{ij}c_i = c_i$.

Theorem: 1.1.1

The fuzzy algebra £ = [0,1] any two bases for a finitely generated subspace have the same cardinality. Any finitely generated subspace over £ has a unique standard basis.

Proof:

(i).First we show that for any finite basis c, there exists a

standard basis having the same cardinality.

Let s be the set of all fuzzy vectors each of whose entries equals some entry of a vector of c, then s is a finite set.

Suppose c is not a standard basis then

$c_i = \sum a_{ij}c_j$ for some $c_i \in C$ and $a_{ij} \in £ = [0,1]$ with $c_i \neq a_{ij}c_j$.

ie, $c_i \neq \min \{a_{ij},c\}$

$a_{ij}c_i < c_i$

Let c_1 be the set obtained from c by replacing c_i by $a_{ij}c_i$.

Then ,

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$| c | = | c_1 |$ and $< c > = < c_1 >$

It can be verified that c_1 is independent set and all vector of $c_1 \epsilon$ s.

We define,

$F_1 \leq F_2$ for finite subsets F_1 and F_2 of s.

If, weight of $F_1 \leq$ weight of F_2.

Clearly, this is a partial order relation on finite subsets of s.

Since, $a_{ij} c_i < c_i$, $c_1 \leq c$ and $|c_1| = |c|$ is finite.

If c_1 is a standard basis, then c_1 is the required standard basis with the same cardinality C.

If not then repeat the process of replacing c_1 by a basis c_2 and proceed.

Therefore after replacing bases of the form c by bases of the form c_1.

This can happen only if we have obtained a standard basis with the same cardinality c.

(ii).Let c be a standard basis,

Suppose,

$c_k \epsilon$ c,

We have, $c_k \sum_j a_j$

Where $a_j \epsilon < c >$, then a_j can be expressed as a linear combination of basis vector in c.

ie. $a_j = \sum b_{ji} c_i$, $b_{ji} \epsilon$ £ and $c_i \epsilon$ C.

$c_k = \sum_j a_j = \sum_{i,j} b_{ji} c_i = \sum_i (\sum_j b_{ji} c_i)$

Since c is standard basis.

By definition,

We have $(\sum_j b_{jk}) c_k = c_k$. The fuzzy series is maximum,

$b_{jk} c_k = c_k$ for some j,

From, $a_j = \sum b_{ji} c_i$

We get, $a_j \geq c_k$.

From, $c_k = \sum a_j$

We get, $c_k = a_j$ for some j.

Thus we conclude that whenever c_k equals some summand a_j.

Next to prove the uniqueness,

If possible,

Let as assume that c and c' are two standard basis with $| c | = | c' |$.

Since, c′ is a basis, each element of c can be expressed as a linear combination of c′.

By the proceeding argument, each element a_j of c must be multiple of some element of b_j of c′′.

Since fuzzy multiplication is minimum, it follows that $a_i \leq b_j$

In the same manner, by using that each element of c' is a multiple of some element of c and | c | = | c'|.

It follows that $a_i = b_j$

Hence proved the uniqueness of the standard basis.

Definition: 1.2

The dimension of the finitely generated subspace s of a vector space v_n over £, denoted by dim(s). It is defined to be the cardinality of the standard basis of s.

Example: 1

Let s={(1,0,0),(0,1,0),(0,0,1)} be the set of a vector space v_n over the fuzzy algebra £.

It form the standard basis for v_3,

dim(v_3) = 3

Example: 2

Let s={(0.5,0.5,0.5),(0,1,0.5),(0,0.5,1)} be a standard basis of v_3 over the fuzzy algebra £.

The subspace of v_3 generated by s is w = < s >.

W= {(x, y ,z) /0 ≤ x ≤ 0.5 ≤ y, z≤ 1}U {(x, y, z) /0 ≤ x ≤ y = z ≤ 0.5}

dim(w) = 3

< s > = 3

ie. dim(w) = | s | = 3.

Remark: 1

For vector spaces over a field

dim (s) = dim (w) iff s=w

Let s ={ (1,0,0), (0,1,0), (0,0,1)} be the set form the standard basis for v_3.

ie. dim (v_3) = 3 ………………… (1)

Let w = {(x,y,z) / 0 ≤ x ≤ 0.5 ≤ y, z ≤ 1} U {(x,y,z) / 0 ≤ x ≤ y = z = 0.5}

And s = {(0.5,0.5,0.5), (0,1,0.5), (0,0.5,1)} be a standard basis for v_3.

ie. dim (w) = 3 …………………. (2)

from (1) and (2),

$\dim(v_3) = \dim(w) = 3$. But $w \neq v_3$.

Theorem: 1.1.2

Let s be a finitely generated subspace of v_n and let { $c_1, c_2, c_3, \ldots\ldots c_n$} be the standard basis for s, then any vector x ϵ s can be expressed uniquely as a linear combination of the standard basis vectors.

Proof:

Let s be a finitely generated subspace of v_n and let { $c_1, c_2, \ldots. c_n$} be the standard basis for s.

x is a linear combination of the standard basis vectors.

$$x = \beta_1 c_1 + \beta_2 c_2 + \ldots\ldots\ldots + \beta_n c_n$$

$$x = \sum_{j=1}^{n} \beta_j c_j$$

where, $\beta_j \epsilon$ £

The coefficient of $\beta_j^{'}$s are not unique. If we write this in the matrix form as

$$x = (\beta_1, \beta_2, \ldots\ldots, \beta_n).c$$

where, c is the matrix whose rows are the basis vectors.

Then, x = p.c has a solution ($\beta_1, \beta_2, \ldots\ldots\ldots.., \beta_n$)

By the theorem,

"For the equation xA = b, Ω (A_1b) ≠ Φ iff $\hat{x}$ = [$\hat{x}$ / j ϵ N_m] defined as

$\hat{x}_j = \min \sigma(a_{jk}, b_k)$, $(a_{jk}.b_k) = \begin{cases} b_k; & \text{if } a_{jk} > b_k \\ 1; & \text{otherwise} \end{cases}$ is the maximum solution of the equation x.A =b".

It follows that this equation has a unique maximal solution ($p_1, p_2, p_3, \ldots\ldots., p_n$)

Then, $x = \sum_{j=1}^{n} p_j c_j$ with $p_j \epsilon$ £ is the unique representation of the vector x.

Hence Proved.

Theorem: 1.1.3

Let s be a vector space over £ and be the linear span of the vectors

$x_1, x_2, x_3, \ldots. x_m$. If some x_i is a linear combination of $x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots., x_m$ then the vectors $x_1, x_2, \ldots.., x_{i-1}, x_{i+1}, \ldots.., x_m$ also spans s.

Proof:

Let w = {$x_1, x_2, \ldots\ldots, x_m$}

such that, s = < w >

Since, x_i is a linear combination of $x_1, x_2, \ldots.., x_{i-1}, x_{i+1}, \ldots.. x_m$ there exists $\beta_j^{'}$s for i=1,2,3……m and j ≠ i ϵ £.

such that,

$$x_j = \sum_{\substack{j=1 \\ j \neq i}}^{m} \beta_j x_j.$$

Since s = < w >, any vector y ϵ s can be expressed as

$$y = \propto_1 x_1 + \propto_2 x_2 + \ldots\ldots + \propto_{i-1} x_{i-1} + \propto_{i+1} x_{i+1} + \ldots\ldots + \propto_n x_m$$

$$= \sum_{\substack{j=1 \\ j\neq i}}^{m} \propto_j x_j + \propto_i x_i$$

$$= \sum_{\substack{j=1 \\ j\neq i}}^{m} \propto_j x_j + \propto_i \left(\sum_{\substack{j=1 \\ j\neq i}}^{m} \beta_j x_j \right)$$

$$= \sum_{\substack{j=1 \\ j\neq i}}^{m} \propto_j x_j + \sum_{\substack{j=1 \\ j\neq i}}^{m} \propto_i \beta_j x_j$$

$$= \sum_{\substack{j=1 \\ j\neq i}}^{m} (\propto_j + \beta_j \propto_i) x_j$$

$$y = \sum_{\substack{j=1 \\ j\neq i}}^{m} \gamma_j x_j$$

where,

$\gamma_j = \propto_j + \beta_j \propto_j$ for j=1,2,…..m and $j\neq i$ are elements in £.

Since y is arbitrary vector in s.

We have, s = < w / $\{x_i\}$ >

Thus the vectors $x_1, x_2, \ldots\ldots, x_{i-1}, x_{i+1}, \ldots\ldots x_m$ spans s.

Hence the theorem.

Theorem: 1.1.4

Let s be a vector space over £ of dimension n and let $x_1, x_2, \ldots\ldots, x_m$ be linearly independent vectors in s. Then there exist a basis for s containing

$x_1, x_2, \ldots\ldots, x_m$.

Proof:

Let s be a vector space over £ of dimension n and

Let $x_1, x_2, \ldots\ldots, x_m$ be linearly independent vectors in s.

Let $y_1, y_2, \ldots\ldots, y_n$ be unique standard basis for s.

Then the set w = $\{x_1, x_2, \ldots\ldots, x_m, y_1, y_2, \ldots\ldots y_n\}$ is linearly dependent subset of s.

Therefore y_i for some < s > = s, and

Thus s is a linear span of w.

By the theorem,

s = < w / { y_i } > is also spans s.

If the set is linearly independent set and basis for s.

Otherwise we continue the process until we get a basis containing $x_1, x_2, \ldots, x_m$.

Hence proved.

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Remark: 2

The image of the zero linear transformation under the mapping T→ [T] is the zero matrix, all whose entries are zero.

Remark: 3

In particular for the identity transformation on w. since I(ci) = c_i for each basis vector c_i in the standard basis β = { c_1, c_2,….., c_n} of w.

Example: 3

Let w be the subspace. Compute the standard fuzzy linear combination of the basis vector (0.5, 0.5, 0.5) in terms of the standard basis

B = { (0.5, 0.5, 0.5), (0, 1, 0.5), (0, 0.5, 1) of the subspace w of v_3 generated by B the identity mapping I : w → w

Since I(x) = x for x ϵ w, the standard fuzzy linear combination of the standard basis vector (0.5, 0.5, 0.5) is

(0.5, 0.5, 0.5) = 1(0.5, 0.5, 0.5) + 0.5(0, 1, 0.5) +0.5(0, 0.5, 1)

Let the standard fuzzy linear combination of the basis vector

$$\mathbf{c_i = \hat{x}_1 c_1 + \hat{x}_2 c_2 + \ldots\ldots + \hat{x}_n c_n}$$

Let (0.5, 0.5, 0.5) = $\hat{x}_1$ (0.5, 0.5, 0.5) + $\hat{x}_2$ (0, 1, 0.5) + $\hat{x}_3$ (0, 0.5, 1)

Where,

$$A = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix};$$

$x = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$, b = (0.5, 0.5, 0.5)

The fuzzy relational equation x.A = b

$$= (\hat{x}_1, \hat{x}_2, \hat{x}_3) \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}$$

Here,

$b_1 = b_2 = b_3 = 0.5$

$a_{11} = a_{12} = a_{13} = a_{23} = a_{32} = 0.5$

$a_{21} = a_{31} = 0$ and

$a_{22} = a_{33} = 1$

Find the minimum solution $x = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$

By the theorem,

$$\hat{x}_j = \min_{k \in K} \sigma(a_{jk}, b_k)$$

Where,

$$\sigma(a_{jk}, b_k) = \begin{cases} b_k ; & \text{if } a_{jk} > b_k \\ 1 ; & \text{otherwise} \end{cases}$$

$$\hat{x}_j = \min_{k \in K} \sigma(a_{jk}, b_k) \quad \text{for } i = 1, \; j = 1, 2, 3 \quad k \in K$$

For $j = 1$

$$\hat{x}_1 = \min_{k \in K} \sigma(a_{jk}, b_k)$$

$= \min \{\sigma(a_{1k}, b_k)\}$

$= \min \{\sigma(a_{11}, b_1), \sigma(a_{12}, b_2), \sigma(a_{13}, b_3)\}$

$= \min \{\sigma(0.5,0.5), \sigma(0.5,0.5), \sigma(0.5,0.5)\}$

$= \min \{1, 1, 1\}$

$= 1$

$\hat{x}_1 = 1$

For $j = 2$

$$\hat{x}_2 = \min_{k \in K} \sigma(a_{jk}, b_k)$$

$= \min \{\sigma(a_{2k}, b_k)\}$

$= \min \{\sigma(a_{21}, b_1), \sigma(a_{22}, b_2), \sigma(a_{23}, b_3)\}$

$= \min \{\sigma(0,0.5), \sigma(1,0.5), \sigma(0.5, 0.5)\}$

$= \min \{1,0.5,1\}$

$= 0.5$

$\hat{x}_2 = 0.5$

For $j = 3$

$$\hat{x}_3 = \min_{k \in K} \sigma(a_{jk}, b_k)$$

$= \min \{\sigma(a_{3k}, b_k)\}$

$= \min \{\sigma(a_{31}, b_1), \sigma(a_{32}, b_2), \sigma(a_{33}, b_3)\}$

$= \min \{\sigma(0,0.5), \sigma(0.5,0.5), \sigma(1,0.5)\}$

$= \min \{1,1,0.5\}$

$= 0.5$

$\hat{x}_3 = 0.5$

Thus $x = (\hat{x}_1, \hat{x}_2, \hat{x}_3) = (1, 0.5, 0.5)$

Hence,

$$(0.5, 0.5, 0.5) = \hat{x}_1(0.5, 0.5, 0.5) + \hat{x}_2(0, 1, 0.5) + \hat{x}_3(0, 0.5, 1)$$

$$= 1(0.5, 0.5, 0.5) + 0.5(0,1, 0.5) + 0.5(0, 0.5,1)$$

is the standard fuzzy linear combination of basis vectors.

Next,

$(0, 1, 0.5) = b$

$b_1 = 0, b_2 = 1, b_3 =0.5$

$a_{11} = a_{12} = a_{13} = a_{23} = a_{32} = 0.5$

$a_{21} = a_{31} = 0$ and $a_{22} = a_{33} =1$

For $j = 1$

$$\hat{x}_1 = \min_{k\epsilon K} \sigma(a_{jk}, b_k)$$

$$= \min \{\sigma(a_{1k}, b_k)\}$$

$$= \min \{\sigma(a_{11}, b_1), \sigma(a_{12}, b_2), \sigma(a_{13}, b_3)\}$$

$$= \min \{\sigma(0.5, 0), \sigma(1,1), \sigma(1,0.5)\}$$

$$= \min \{0,1,0.5\}$$

$$= 0$$

$\hat{x}_1 = 0$

For $j = 2$

$$\hat{x}_2 = \min_{k\epsilon K} \sigma(a_{jk}, b_k)$$

$$= \min \{\sigma(a_{2k}, b_k)\}$$

$$= \min \{\sigma(a_{21}, b_1), \sigma(a_{22}, b_2), \sigma(a_{23}, b_3)\}$$

$$= \min \{\sigma(0,0), \sigma(1,1), \sigma(0.5,0.5)\}$$

$$= \min \{1,1,0.5\}$$

$$= 0.5$$

$\hat{x}_2 = 0.5$

For j = 3

$$\hat{x}_3 = \min_{k \in K} \sigma(a_{jk}, b_k)$$

$$= \min \{\sigma(a_{3k}, b_k)\}$$

$$= \min \{\sigma(a_{31},b_1), \sigma(a_{32},b_2), \sigma(a_{33},b_3)\}$$

$$= \min \{\sigma(0,0), \sigma(0.5,1), \sigma(1,0.5)\}$$

$$= \min \{0,0.5,0.5\}$$

$$= 0.5$$

$$\hat{x}_3 = 0.5$$

Thus $x = (\hat{x}_1, \hat{x}_2, \hat{x}_3) = (0,1,0.5)$

Hence,

(0,1,0.5) = 0(0.5,0.5,0.5) + 1(0,1,0.5) + 0.5(0,0.5,1) is the standard fuzzy linear combination of basis vectors.

Next,

$$b = (0, 0.5, 1)$$

$$b_1 = 0, \quad b_2 = 0.5, \quad b_3 = 1$$

For j = 1

$$\hat{x}_1 = \min_{k \in K} \sigma(a_{jk}, b_k)$$

$$= \min \{\sigma(a_{1k}, b_k)\}$$

$$= \min \{\sigma(a_{11}, b_1), \sigma(a_{12}, b_2), \sigma(a_{13}, b_3)\}$$

$$= \min \{\sigma(0.5,0), \sigma(0.5,0.5), \sigma(0.5,1)\}$$

$$= \min \{0,0.5,0.5\}$$

$$= 0$$

$$\hat{x}_1 = 0$$

For j = 2

$\hat{x}_2 = \min_{k \epsilon K} \sigma(a_{jk}, b_k)$

$= \min \{\sigma(a_{2k}, b_k)\}$

$= \min \{\sigma(a_{21}, b_1), \sigma(a_{22}, b_2), \sigma(a_{23}, b_3)\}$

$= \min \{\sigma(0,0), \sigma(1,0.5), \sigma(0.5,1)\}$

$= \min \{1,0.5,1\}$

$= 0.5$

$\hat{x} = 0.5$

For j = 3

$\hat{x}_3 = \min_{k \epsilon K} \sigma(a_{jk}, b_k)$

$= \min \{\sigma(a_{3k}, b_k)\}$

$= \min \{\sigma(a_{31}, b_1), \sigma(a_{32}, b_2), \sigma(a_{33}, b_3)\}$

$= \min \{\sigma(0,0), \sigma(0.5,0.5), \sigma(1,1)\}$

$= \min \{1,1,1\}$

$= 1$

$\hat{x}_3 = 1$

Thus,

$x = (\hat{x}_1, \hat{x}_2, \hat{x}_3) = (0,0.5,1)$

$(0, 0.5,1) = \hat{x}_1(0.5,0.5,0.5) + \hat{x}_2(0,1,0.5) + \hat{x}_3(0,0.5,1)$

$= 0(0.5,0.5,0.5) + 0.5(0,1,0.5) + 1(0,0.5,1)$

is the standard fuzzy linear combination of basis vectors.

Similarly,

The standard fuzzy linear combination of the other basis vectors are

$(0,1,0.5) = 0(0.5,0.5,0.5) + 1(0,1,0.5) + 0.5(0,0.5,1)$

$(0,0.5,1) = 0(0.5,0.5,0.5) + 0.5(0,1,0.5) + 1(0,0.5,1)$

Hence,

The matrix [I] corresponding to the identity linear transformation with respect to the standard basis β = {(0.5,0.5,0.5), (0,1,0.5), (0,0.5,1)} is

$$[I]_\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

Hence solved.

Theorem: 1.1.5

If β' is the standard basis of w obtained from $\beta = \{ c_1, c_2, \ldots, c_n\}$ by the basis vectors in β, then for any linear transformation T on w, $[T]_{\beta'}$ are similar matrixes in $£_n$.

Proof:

Let $\beta = \{ c_1, c_2, \ldots c_n\}$ be the standard basis vector and

$\beta' = \{ c_{\sigma(1)}, c_{\sigma(2)}, \ldots c_{\sigma(n)}\}$ be the standard basis of w for some permutation σ on s = {1,2,…..,n}.

Let,

$[T]_\beta = [\propto_j]$ and

$[T]_{\beta'} = [\gamma_{\sigma(i),\sigma(j)}]$

$T(c_j) = \sum_i \propto_{ij}.c_j$ ………… (1)

and $T(c_{\sigma(j)}) = \sum_i \gamma_{\sigma(i),\sigma(j)}\, c_{\sigma(i)}$ .... (2)

For any j ∈ s,

$T(c_k) = \sum_i \propto_{ik}\, c_i$ ………… (3)

$T(c_{\sigma(j)}) = T(c_k) = \sum_i \gamma_{\sigma(i)k}.\, c_{\sigma(i)}$ ....(4)

From, $T(c_k) = \sum_i \gamma_{\sigma(i)k}\, .\, c_k$ is the standard fuzzy linear combination of $T(c_k)$.

Since, the coefficients $\propto_{ik}$'s are uniquely determined by $T(c_k)$.

The coefficients $\gamma_{rk} = \propto_{ik}$ for σ(i)=r.

Hence, the coefficients $\gamma_{\sigma(i)k}$'s i =1,2,…..n are the rearrangement according to σ of the coefficients $a_{1k}, a_{2k}, \ldots, a_{nk}$.

Thus,

The entries of the j^{th} column of $[T]_\beta{}' [T]_\beta$ permuted according to σ.

Since, j϶s is arbitrary each column of $[T]_\beta{}'$ is obtained by permuting suitably the entries of some other column of $[T]_\beta{}'$.

Hence,

$$[T]_\beta{}' = P^T [T]_\beta P.$$

Where,

P is the permutation matrix corresponding to the permutation σ on s.

Since, the permutation matrices are invertible in $£_n$.

∴ $[T]_\beta{}'$ and $[T]_\beta$ are similar matrix.

Hence proved.

Fuzzy Automata

Introduction:

A finite state machine (or) sequential machine is a dynamic system operating in discrete time that transforms sequence of input states (stimuli) received at the input of the system to sequence of output states (responses) produced at the output of the system.

The sequences may be finite (or) countably infinite. The transformation is accomplished by the concept of a dynamically changing internal state.

At each time, the response of the system is determined on the basis of the received stimulus and the internal state of the system. At the same time, a new internal state is determined, which replaces its predecessor.

The new internal state is stored in the system to used subsequently. A finite state machine (or) finite automation is called **fuzzy automata**, when its states are characterized by fuzzy sets, the production of response and next states is facilitated by suitable fuzzy relations.

Definition: 1.3

A finite fuzzy automata A is a fuzzy relational system defined by the quintuple

A = < X, Y, Z, R, S >

Where,

X is a non-empty finite set of input states (stimuli)

Y is a non-empty finite set of output states (responses)

Z is a non-empty finite set of internal states

R is a fuzzy relation on X×Y (response relation)

S is a fuzzy relation on X×Y×Z (state transformation relation)

Problem: 1

Consider a fuzzy automaton with $X = \{x_1, x_2\}$,

$Y = \{y_1, y_2, y_3\}$, $Z = \{z_1, z_2, z_3, z_4\}$ whose output relation.

R =

	y_1	y_2	y_3
z_1	1	0	0
z_2	0	1	0
z_3	0	0	1
z_4	0.5	1	0.3

State transition relation δ are defined by the following matrices respectively for the input states x_1 and x_2.

$\delta A_1(z_i,z_j)$ =

	z_1	z_2	z_3	z_4
z_1	0	0.4	0.2	1
z_2	0.3	1	0	0.2
z_3	0.5	0	0	1
z_4	0	0	0	1

$$\delta A_2(z_i,z_j) = \begin{array}{c} \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{array} \begin{array}{cccc} Z_1 & Z_2 & Z_3 & Z_4 \\ \hline \end{array}$$

$$\delta A_2(z_i,z_j) = \begin{array}{c|cccc|} & Z_1 & Z_2 & Z_3 & Z_4 \\ Z_1 & 0 & 0 & 1 & 0 \\ Z_2 & 0.2 & 0 & 0 & 1 \\ Z_3 & 0 & 0 & 0 & 1 \\ Z_4 & 1 & 0.3 & 0 & 0.6 \end{array}$$

Generate sequences of two fuzzy internal states and output states under the following conditions,

(i). The initial fuzzy state is $c_1 = [1\ \ 0.8\ \ 0.6\ \ 0.4]$

The input fuzzy states are,

$A_1 = [1\ \ 0.4]$, $A_2 = [0\ \ 1]$

Solution:

Let us assume that the initial state of the automaton is $c_1 = [1\ 0.8\ \ 0.6\ \ 0.4]$ and its fuzzy input states is $A_1 = [1\ \ 0.4]$ are given ,

We know that the equation,

$$\delta A_t\ (z_i,z_j) = \max_{k\epsilon\{1,2\}} [\min(A_t(x_k),\delta_k(z_i,z_j)]$$

Let us compute the 4×us compute the 4×4 matrix Δa_1

$\delta A_1(z_1,z_1) = \max\ \{\min[A_1(x_1),\delta x_1(z_1,z_1)],\ \min[A_2(x_2),\delta x_2(z_1,z_1)$

$= \max\ \{\min(1,0),\min(0.4,)$

$= \max\ (0,0)$

$= 0$

$\delta A_1(z_1,z_2) = \max\ \{\min(1,0.4),\min(0.4,0)\}$

$= \max\ (0.4,0)$

$= 0.4$

$\delta A_1(z_1,z_3) = \max\ \{\min[A_1(x_1),\delta x_1(z_1,z_3)],\ \min[A_1(x_2),\delta x_2(z_1,z_3)]\}$

$\delta A_1(z_1,z_4) = \max\ \{\min[A_1(x_1),\delta x_1(z_1,z_4)],\ \min[A_1(x_2),\delta x_2(z_1,z_4)]\}$

$= \max\ \{\min(1,1),\ \min(0.4,0)\}$

$= \max\ (1,0)$

$= 1$

Thus the first row of δA_1 is [0 0.4 0.4 1]

$\delta A_1(z_2,z_1) = \max \{\min[A_1(x_1),\delta x_1(z_2,z_1)], \min[A_1(x_2),\delta x_2(z_2,z_1)]\}$

$= \max \{\min(1,0.3), \min(0.4,0.2)\}$

$= \max (0.3,0.2)$

$= 0.3$

$\delta A_1(z_2,z_2) = \max \{\min[A_1(x_1),\delta A_1(z_2,z_2)], \min[A_1(x_1),\delta A_2(z_2,z_2)]\}$

$= \max \{\min(1,1), \min(0.4,0)\}$

$= \max (1,0)$

$= 1$

$\delta A_1(z_2,z_3) = \max \{\min[A_1(x_1),\delta A_1(z_2,z_3)], \min[A_1(x_2),\delta A_2(z_2,z_3)]\}$

$= \max \{\min(1,0), \min(0.4,0)\}$

$= \max (0,0)$

$= 0$

$\delta A_1(z_2,z_4) = \max \{\min[A_1(x_1),\delta A_1(z_2,z_4)], \min[A_1(x_2),\delta A_2(z_2,z_4)]\}$

$= \max \{\min(1,0.2), \min(0.4,1)\}$

$= \max (0.2, 0.4)$

$= 0.4$

Thus the second row δA_1 is [0.3 1 0 0.4]

$\delta A_1(z_3,z_1) = \max \{\min[A_1(x_1),\delta A_1(z_3,z_1)], \min[A_1(x_2),\delta A_2(z_3,z_1)]\}$

$= \max \{\min(1, 0.5), \min(0.4, 0)\}$

$= \max (0.5, 0)$

$= 0.5$

$\delta A_1(z_3,z_2) = \max \{\min[A_1(x_1),\delta A_1(z_3,z_2)], \min[A_1(x_2),\delta A_2(z_3,z_2)]\}$

$= \max \{\min(1,0), \min(0.4, 0)]$

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$= \max(0,0)$

$= 0$

$\delta A_1(z_3,z_3) = \max\{\min[A_1(x_1),\delta A_1(z_3z_3)], \min[A_1(x_2),\delta A_2(z_3,z_3)]\}$

$= \max\{\min(1,0), \min(0.4,0)\}$

$= \max(0,0)$

$= 0$

$\delta A_1(z_3,z_4) = \max\{\min[A_1(x_1),\delta A_1(z_3,z_4)], \min[A_2(x_2),\delta A_2(z_3,z_4)]\}$

$= \max\{\min(1,1), \min(0.4,1)\}$

$= 1$

Thus the third row of δA_1 is [0.5 0 0 1]

$\delta A_1(z_4,z_1) = \max\{\min[A_1(x_1),\delta A_1(z_4,z_1)], \min[A_1(x_2),\delta A_2(z_4,z_1)]\}$

$= \max\{\min(1,0), \min(0.4,1)\}$

$= \max(0, 0.4)$

$= 0.4$

$\delta A_1(z_4,z_2) = \max\{\min[A_1(x_1),\delta A_1(z_4,z_2)], \min[A_1(x_1),\delta A_2(z_4,z_2)]\}$

$= \max\{\min(1, 0), \min(0.4, 0.3)\}$

$= \max(0, 0.3)$

$= 0.3$

$\delta A_1(z_4,z_3) = \max\{\min[A_1(x_1),\delta A_1(z_4,z_3)], \min[A_1(x_1),\delta A_2(z_4,z_3)]\}$

$= \max\{\min(1,0), \min(0.4, 0)\}$

$= \max(0,0)$

$= 0$

$\delta A_1(z_4,z_4) = \max\{\min[A_1(x_1),\delta A_1(z_4,z_4)], \min[A_1(x_1),\delta A_2(z_4,z_4)]\}$

$= \max\{\min(1,1), \min(0.4,0.6)\}$

$= \max(1, 0.4)$

= 1

Thus the last row of δA_1 is [0.4 0.3 0 1]

The matrix δA_1 is

$$\delta A_1 = \begin{bmatrix} 0 & 0.4 & 0.4 & 1 \\ 0.3 & 1 & 0 & 0.4 \\ 0.5 & 0 & 0 & 1 \\ 0.4 & 0.3 & 0 & 1 \end{bmatrix}$$

To calculate the fuzzy next state E, and the fuzzy output state B, of the fuzzy automaton.

$\mathbf{E_1 = C_1.\delta A_1}$

$$= [1\ \ 0.8\ \ 0.6\ \ 0.4] \begin{bmatrix} 0 & 0.4 & 0.4 & 1 \\ 0.3 & 1 & 0 & 0.4 \\ 0.5 & 0 & 0 & 1 \\ 0.4 & 0.3 & 0 & 1 \end{bmatrix}$$

E_1 = [max(0, 0.3, 0.5, 0.4) max(0.4, 0.8, 0, 0.3) max(0.4, 0, 0, 0)

max(1, 0.4, 0.6, 0.4)]

E_1 = [0.5, 0.8, 0.4 1]

$\mathbf{B_1 = C_1.R}$

$$= [1\ \ 0.8\ \ 0.6\ \ 0.4] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 0.3 \end{bmatrix}$$

= [max(1, 0 0 0.4) max(0, 0, 0.6, 0.3)

max(0, 0, 0.6, 0.3)]

B_1 = [1 0.8 0.6]

Assume that the next fuzzy input state is given A_2 = [0, 1]

Then compute the matrix δA_2,

Using the equation,

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$$\delta A_t(z_i,z_j) = \max_{k\in\{1,2\}} \{ \min [A_t(x_k), \delta_x(z_i,z_j)]\}$$

$\delta A_2(z_1,z_1) = \max\{\min[A_2(x_1),\delta A_1(z_1,z_1)], \min[A_2(x_2),\delta A_2(z_1,z_1)]\}$

$= \max \{\min(0, 0.), \min(1,0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_1,z_2) = \max \{\min[A_2(x_1),\delta A_1(z_1,z_2)], \min[A_2(x_2),\delta A_2(z_1,z_2)]\}$

$= \max \{\min(0, 0.4), \min(1, 0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_1,z_3) = \max \{\min[A_2(x_1),\delta A_1(z_1,z_3)], \min[A_2(x_2),\delta A_2(z_1,z_3)]\}$

$= \max \{\min(0, 0.2), \min(1, 1)$

$= \max (0, 1)$

$= 1$

$\delta A_2(z_1,z_4) = \max \{\min[A_2(x_1),\delta A_1(z_1,z_4)], \min[A_2(x_2),\delta A_2(z_1,z_4)]\}$

$= \max \{\min(0, 1), \min(1,0)\}$

$= \max (0, 0)$

$= 0$

Thus the first row of δA_2 is [0, 0, 1, 0]

$\delta A_2(z_2,z_1) = \max \{\min[A_2(x_1),\delta A_1(z_2,z_1)], \min[A_2(x_2),\delta A_2(z_2,z_1)]\}$

$= \max \{\min(0, 0.3), \min(1, 0.2)\}$

$= \max (0, 0.2)$

$= 0.2$

$\delta A_2(z_2,z_2) = \max \{\min[A_2(x_1),\delta A_1(z_2,z_2)], \min[A_2(x_2),\delta A_2(z_2,z_2)]\}$

$= \max \{\min(0, 1), \min(1, 0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_2,z_3) = \max \{\min[A_2(x_1),\delta A_1(z_2,z_3)], \min[A_2(x_2),\delta A_2(z_2,z_3)]\}$

$= \max \{\min(0, 0), \min(1, 0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_2,z_4) = \max \{\min[A_2(x_1),\delta A_1(z_2,z_4)], \min[A_2(x_2),\delta A_2(z_2,z_4)]]$

$= \max \{\min(0, 0.2), \min(1, 1)\}$

$= \max (0, 1)$

$= 1$

Thus the second row of δA_2 is [0.2 0 0 1]

$\delta A_2(z_3,z_1) = \max \{\min[A_2(x_1),\delta A_1(z_3,z_1)], \min[A_2(x_2),\delta A_2(z_3,z_1)]\}$

$= \max \{\min(0, 0.5), \min(1, 0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_3,z_2) = \max \{\min[A_2(x_1),\delta A_1(z_3,z_2)], \min[A_2(x_2),\delta A_2(z_3,z_2)]\}$

$= \max \{\min(0, 0), \min(1, 0)\}$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_3,z_3) = \max \{\min[A_2(x_1),\delta A_1(z_3,z_3)], \min[A_2(x_2),\delta A_2(z_3,z_3)]\}$

$= \max \{\min(0, 0), \min(1, 0)]$

$= \max (0, 0)$

$= 0$

$\delta A_2(z_3,z_4) = \max \{\min[A_2(x_1),\delta A_1(z_3,z_4)], \min[A_2(x_2),\delta A_2(z_3,z_4)]\}$

$= \max \{\min(0, 1), \min(1, 1)\}$

$= \max (0, 1)$

$= 1$

Thus the third row of δA_2 is [0 0 0 1].

$\delta A_2(z_4,z_1) = \max\{\min[A_2(x_1),\delta A_1(z_4,z_1)], \min[A_2(x_2),\delta A_2(z_4,z_1)]\}$

$= \max\{\min(0, 0), \min(1, 1)\}$

$= \max(0, 1)$

$= 1$

$\delta A_2(z_4,z_2) = \max\{\min[A_2(x_1),\delta A_1(z_4,z_2), \min[A_2(x_2),\delta A_2(z_4,z_2)]\}$

$= \max\{\min(0, 0), \min(1, 0.3)\}$

$= \max(0, 0.3)$

$= 0.3$

$\delta A_2(z_4,z_3) = \max\{\min[A_2(x_1),\delta A_1(z_4,z_3)], \min[A_2(x_2),\delta A_2(z_4,z_3)]\}$

$= \max\{\min(0, 0), \min(1, 0)\}$

$= \max(0, 0)$

$= 0$

$\delta A_2(z_4,z_4) = \max\{\min[A_2(x_1),\delta A_1(z_4,z_4)], \min[A_2(x_2),\delta A_2(z_4,z_4)]\}$

$= \max\{\min(0, 1), \min(1, 0.6)\}$

$= \max(0, 0.6)$

$= 0.6$

Thus the last row of δA_2 is [1 0.3 0 0.6].

The matrix δA_2 is,

$$\delta A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix}$$

Then $E_2 = C_2.\delta A_2 = E_1.\delta A_2$

$$= [0.5\ \ 0.8\ \ 0.4\ 1]\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix}$$

= { masx(0, 0.2, 0, 1) max(0, 0, 0, 03) max(0.5, 0, 0, 0)

max(0, 0.8, 0.4, 0.6)}

E_2 = [1 0.3 0.5 0.8]

$B_2 = E_1.R$

$$= [0.5\ \ 0.8\ \ 0.4\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 0.3 \end{bmatrix}$$

= { max(0.5, 0, 0, 0.5) max(0, 0.8, 0, 1) max(0, 0,0.4, 0.3)}

B_2 = [0.5 1 0.4]

Similarly, we can produce large sequences of fuzzy internal state and output states for any given sequences of fuzzy input states of a fuzzy automaton.

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