

## Students' Thinking Process in Constructing the Concepts of Linear Quantities Two Variables

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**Abstract:** This reseach aimed at analyzing the students' thinking process in cunstructing the concepts of linear quantities two variables. The participants were 6 students of grade 8 on junior high school in Jember Regency, East Java Indonesia. It focused on three concepts, i.e definition concept, modelling, and Linear Quantities Two Variables (*LQTV*) solving. Participants are given question that includes it the concepts. The result show that 2 participants succeed in constructing definition concepts, 4 participants succeed in constructing modelling concept, and 1 participants succed in constructing solving concept. Participants' thinking process in constructing *LQTV* concepts consisted of 3 steps, i.e: planning, monitoring, and evaluation. Generally, participants' thinking process were categorized into three types, i.e type A consist of planning (activating relevant facts, selecting goal, and selecting strategies), monitoring, and evaluation; type B consist of planning (selecting goal, activating relevant facts, and selecting strategies), monitoring, and evaluation; type C consist of planning (activating relevant facts, selecting goal, selecting strategies, activating relevant facts, and selecting strategies), monitoring, and evaluation.

**Keywords:** *Thinking process, Concept of LQTV.*

### 1. Introduction

The potrait of mathematics education in Indonesia is still in an unoptimal condition. Based on the results of TIMSS (Trends in International Mathematics and Science Study) survey, the rank of Indonesian students, especially in the field of mathematics study, is still in the top 10 of the world's lowest rank. In 1999 from 38 countries, Indonesia was on the 34th sharply above Chile, Philippines, Morocco and South Africa. In 2003 Indonesia ranked 34 out of 45 countries. In 2007, Indonesia ranked 35 out of 46 countries and in 2011 Ranked 41 of 45 countries (Setiadi et al., 2012). Indonesia's achievements in TIMSS 2011 are still below the international average. Indonesian students are only able to answer correctly as much as 24%

for content numbers, 22% for Algebra, 24% for geometry and 29% for data content and probabily. Indonesia's performance is far below the international average. That is 43% content of numbers, 37% algebra, 39% geometry and 45% for data content and probability (Shodiq et al., 2015).

Based on the above data, algebra becomes the lowest content of achievement. The low achievement of algebra content can not be separated from the role of the learning process in schools where teachers still focuses on how students solve problems. Consequently, the students just memorize the completion procedure but they could not understand the substance of the process that they do. Teacher's concern about how students construct an understanding of algebraic concepts has not been

fully addressed. In fact, thinking about how students think (thinking about thinking) is a reflective step to build high-order thinking skills on students in understanding the concept of mathematics.

Various studies on students' algebraic abilities in schools are widely practiced. Kieran (2004) says that students tend to develop traditional arithmetic programs to study algebra. In the transition from arithmetic to algebra, students must make some adjustments, even for students who are proficient in arithmetic. Lian and Idris (2006) in their article mention that 62% of students are possible to success less than 50%. Most students can only use one data (unistruktural) and they can use some data but they can not find the relationship of data found (multistruktural). In understanding algebra, It's difficult for the students to generalize their arithmetic thinking through the use of algebraic symbols. Widadah (2013) conducted a study aimed at describing the metacognition of reflective and impulsive cognitive students in solving the Problem of linear quantities two variables system (LQTVS). The results of the study showed that students who has reflective cognitive style perform metacognition activities in accordance with the indicators in each metacognition activity, which develops planning, monitoring the implementation, and evaluating the action. Students of impulsive cognitive styling have not performed any activity that suits the indicators in each metacognition activity. Based on the description, a research on the analysis of students' thinking process with the title "Students' Thinking Process in Constructing the Concepts of Linear Quantities Two Variables" was carried out.

Khodijah (2006) states Thinking is processing information mentally or cognitively. More formally,

thinking is the rearrangement or cognitive manipulation of both the information of the environment and the symbols stored in long term memory. Marpaung (1986) states that the process of thinking is a process that begins with the discovery of information (from outside or student self), processing, storage, and recalling information from students' memories. According to Suryabrata (2004), there are three stages of thinking process in general, namely the formulation of understanding, the formulation of opinion and the formulation of decision or drawing conclusions. The formulation of understanding is to remove the general characteristics of things so that it's only specific characteristic of the things left. The formulation of opinion is the process by which the mind combines (decipher) some definitions so that it becomes a sign of trouble. Decision formulation or conclusion is the thought of combining opinions and drawing conclusion from other decisions.

The definition of thinking or thinking process is often associated with the term metacognition. The definition of metacognition stated by Flavel (1979) is as follows.

*“Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, .... metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal”.*

Based on the above definition, it can be seen that there are two aspects of metacognition. The first, metacognition as a person's knowledge about process and its cognitive results, and the second

metacognition as monitoring and self-regulation of its own cognitive activity.

Furthermore, Livingston (1997) describes the sequences of cognitive processes performed.

*“Metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning. Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metakognitive in nature”.*

There are three cognitive activities in thinking process, namely planning the learning task approach given, monitoring understanding, and evaluating the progress of task completion. In the context of learning, students know how to learn, know the ability and learning modalities they have, and know the best learning strategies effectively.

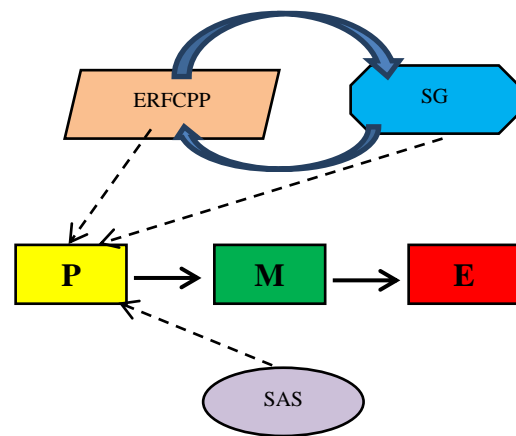
Based on the above explanation, we can create a framework of thinking process to observe students’ thinking process in constructing the concept of linear quantities two variables. The framework of the students’ thinking process can be seen in Table 1.

**Table 1:** A framework for viewing thinking process

Metacognition activity	Indicator	Translation of metacognitive activity
Planning (P)	<ul style="list-style-type: none"> <li>- Setting the goals</li> <li>- Enabling relevant facts, concepts, principles and procedures.</li> <li>- Selecting appropriate strategy</li> </ul>	<ul style="list-style-type: none"> <li>- Participant determines the purpose of the problem</li> <li>- Participant enables facts, concepts, principles and procedures required.</li> <li>- Participant determines the strategy to be used</li> </ul>
Monitoring (M)	<ul style="list-style-type: none"> <li>- Cheking understanding</li> <li>- Checking the objectives to be achieved</li> </ul>	<ul style="list-style-type: none"> <li>- Participant thinks how good the strategy s(he) is applying applying</li> </ul>

	- Selecting an improvement strategy if the selected strategy	- Participant thinks whether is necessary to change the
Metacognition activity	Indicator	Translation of metacognitive activity
	is not working.	strategy being applied - Participant thinks whether is necessary to fitting the goals with the strategy being applied
Evaluation (E)	<ul style="list-style-type: none"> <li>- Determining the level of understanding</li> <li>- Selecting the right strategy</li> </ul>	<ul style="list-style-type: none"> <li>- Participant determines how good the strategy s(he) has applied.</li> <li>- Participant determines what s(he) has done well.</li> <li>- Participant determines what s(he) has not done well.</li> <li>- Participant determines what h(he) should change.</li> </ul>

Abstraction about thinking process done by students in finding the concepts of linear quantities two variables, can be seen in Figure 1.



**Figure 1:** The diagram of students thingking process in constructing the concept of mathematics learning

**Description:**

P : Planning            SG : Setting Goals  
 M : Monitoring        SAS : Selecting Appropriate Strategy  
 E: Evaluation        ERFCPP: Enabling Relevant Facts, Concepts, Principle, and Procedures.

**2. Research Method**

This research is a qualitative research, which aims to describe the thinking process of students in constructing the concept of linear quantities two variables (LQTV). This research focused on three concepts, namely: definition concept of LQTV, the concept of LQTV modeling, and the concept of completion of LQTV. The participants consisted of six students of grade VIII SMP. Participants were selected from two different schools, namely three students from SMPN 1 Jember and three students from SMPN 1 Balung, Jember District East Java Province, Indonesia. Selection of participants was based on the results of sheet instrument test at both schools. From several tested participants, participants who have the correct answers or who have uniqueness answers that can be used to reveal the students' thinking processes in constructing the concept of LQTV were selected. In addition, input from mathematics teachers about students' communication skills is also a consideration in selecting the research participant. Then, interview was held to explore the process of thinking. The results of written answers and interviews of the six participants were then analyzed to obtain a description of the thinking process undertaken.

**Discussion**

Based on the written answers of the six participants studied, two participants (P1 and P4) were able to correctly answer the 1<sup>st</sup> item on the

concept of defining the LQTV, four participants (P1, P2, P3 and P4) were able to answer the 2<sup>nd</sup> item about the concept of LQTV modeling, and only one participant (P1) who were able to answer correctly the 3<sup>rd</sup> about LQTV solution concepts. Furthermore, the researchers conducted an analysis on the participants' thinking processes participants who answered correctly in constructing the concepts of LQTV. The analysis was done by comparing the written answer of the participant with the interview result. The results of the analysis are presented in Table 2.

In constructing the definitions of LQTV, P1 and P4 observed the pattern of addition on the number of pentagon and the vertex presented in the item and generalized the pattern. Then, the participants set the pentagon and the vertex as a variable, pentagon as the variable  $x$  and the vertex as  $y$  variable. They set 4 as the coefficients of the variables  $x$  and 1 as constants. They establish the relationship between variables where the number of vertices ( $y$ ) is influenced by the number of a pentagon ( $x$ ) multiplied by 4 added by 1, to obtain a formula  $y = 4x + 1$ . From the formula obtained, the participant concluded that the formula obtained is a form of quantities in which it contains linear two variables. Based on the explanation, it can be seen that in constructing the concept of LQTV definition, in the planning stage, P1 and P4 did ERFCPP process by determining variables, coefficients, constants, operations to establish relationships between variables, and generalized patterns of relationships between variables into a general form. The SG and SAS processes were performed when the participant composed the general form. The monitoring (M) and evaluation (E) stage done by the two participants

were carried out by substituting the number of pentagons ( $x$  variable values) into the obtained LQTV form.

In constructing the concept of LQTV modelling, P1 and P2 began the planning stage with ERFCPP process, that is observing the problems presented in the item, then setting the variable to be used. Motor as variable  $x$  and car as variable  $y$ . They set the number of motor wheels (2 pieces) as coefficients for variable  $x$  and number of car wheels (4 pieces) as coefficients for variable  $y$ . The participant then determined the number of vehicles and the number of wheels of the two types of vehicles as the constants of 50 and 100. The participant also specified the operations used to establish relationships between the two variables. The next process, P1 and P2 did SG and SAS process. So, there would be two quantities models. Those are  $x + y = 50$  and  $2x + 4y = 120$ . The monitoring and evaluation stage were done by both participants by completing the two quantities, so that the value of each of the variables  $x$  and  $y$  was obtained. They substituted the values of the two variables in the two models of the quantities arranged. Substitution results showed that the left side is equal to the right side. This proved that the quantities were arranged correctly.

In constructing the concept of modeling, the difference of thinking process undertaken by P3 and P4 lied in the planning stage, whereas in the stages of monitoring and evaluation of P3 and P4 did the same process with monitoring and evaluation undertaken by P1 and P2. At the planning stage, P3 did SG process and ERFCPP process, and SAS. The thinking steps undertaken by P3 in the process of ERFCPP was similar to the thinking process

undertaken by P1 and P2. The planning stage undertaken by P4 was longer than the previous three participants. After the ERFCPP, SG, and SAS processes, P4 suffers from disequilibrium, so that he repeated the ERFCPP and SAS processes. The thinking steps in the ERFCPP process were similar to the thinking process undertaken by P1 and P2.

In constructing the concept of modeling, in the first stage of planning P1 observed the given problem to set the goal to be achieved (SG). Furthermore, P1 did the ERFCPP process. Based on the given problem, P1 set the variable to be used. The boat speed was as the variable  $x$  and the current velocity of the stream was as the variable  $y$ . Then P1 determined the relationship between the two variables in accordance with the problems given. Based on the relationship between the two variables, the sum operation was obtained for the same direction of the river flow and the subtraction operation was for the opposite river flow, so that the two equations were  $x + y = 46$  and  $x - y = 18$ . After obtaining both equations, P1 chose a strategy (SAS) using a mixed (substitution-elimination) method to derive the value of each variable. The monitoring and evaluation stage was done by substituting the values of the two variables into both quantities. When the left side was equal to the right side then P1 ensured that the strategy used was correct. In constructing the LQTV completion concept, only P1 succeeded, while the other participant failed. Failure was due to the problem modeling process, the participants could not find the relationship between the two variables, so they could not define the appropriate operation to describe the problems given.

**Table 2:** Students' thinking process in constructing LQTV concept

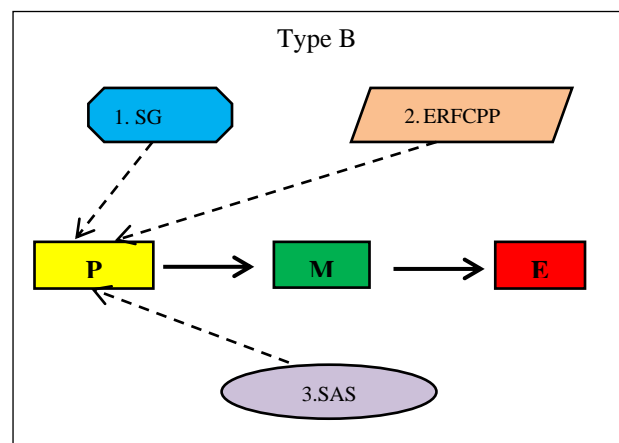
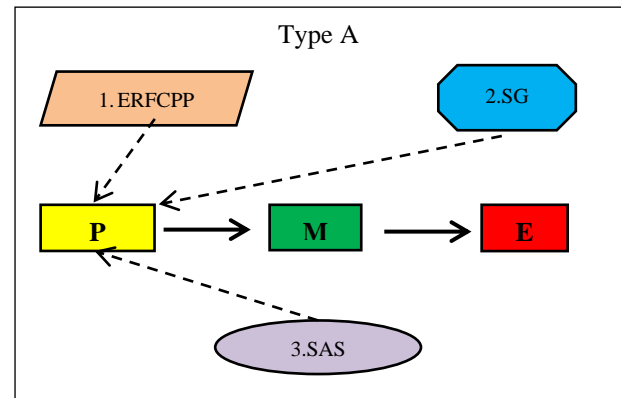
No	Participant	Concept Constructed	Thinking Process
1.	P1	LQTV definition	P(ERFCPP→SG→SAS)→M→E
2.	P4	LQTV definition	P(ERFCPP→SG→SAS)→M→E
3.	P1	LQTV modelling	P(ERFCPP→SG→SAS)→M→E
4.	P2	LQTV modelling	P(ERFCPP→SG→SAS)→M→E
5.	P3	LQTV modelling	P(SG→ERFCPP→SAS)→M→E
6.	P4	LQTV modelling	P(ERFCPP→SG→SAS→ERFCPP→SAS)→M→E
7.	P1	LQTV completion	P(SG→ERFCPP→SAS)→M→E

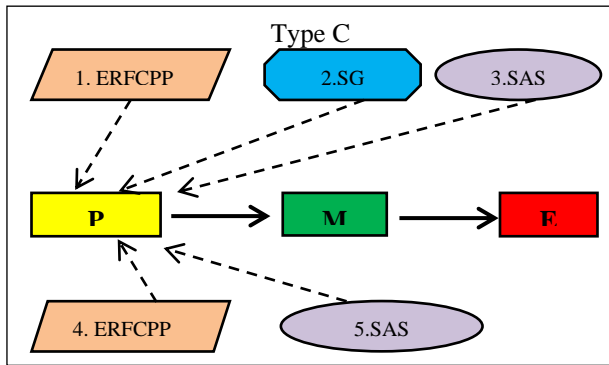
Based on the data on Table 2, it is known that from the three concepts studied, the concept of completion of LQTV was the most difficult concept because only one participant which is P1 who managed to construct the concept. The concept of modeling was on the second in which there were two successful participants, then the concept of modeling as many as four participants. Based on Table 2, it is known that only P1 succeeded in constructing the three LQTV concepts. P4 succeeded in constructing two concepts (the definition of LQTV and LQTV modeling), while P3 and P4 only succeeded in constructing the concept of LQTV modeling. Based on Table 2 it is also known that in constructing LQTV concepts, the thinking stage undertaken by the research participant consisted of three stages, namely the planning stage, the monitoring stage, and the evaluation stage. The difference in constructing LQTV concepts lies in the planning stage. In general, students' thinking processes in constructing LQTV concepts are

divided into three types of thought processes as follows.

- Type A. It consists of the following processes: Planning (ERFCPP, SG, and SAS), Monitoring, and Evaluation.
- Type B. It consists of the following processes: Planning (SG, ERFCPP, and SAS), Monitoring, and Evaluation.
- Type C. Consists of the following process: Planning (ERFCPP, SG, SAS, ERFCPP, and SAS), Monitoring, and Evaluation.

The abstraction of three types thinking process are presented in Figure 2.





**Figure 2:** The abstraction of students' thinking process in constructing the concepts of linear quantities two variables

#### 4. Conclusion and Suggestion

Based on the results of data analysis and the discussion, it can be concluded students' thinking process in constructing the concepts of linear quantities two variables consisted of three stages of thinking, namely: planning, monitoring, and evaluation. The planning stage included three stages of thinking, i.e setting the goals (SG), enabling relevant facts, concepts, principles and procedures (ERFCPP), and selecting the right strategy (SAS). In general, the students' thinking process in constructing the concept of LQTV is divided into three types, as follows.

a. Thinking process type A was done by P1 and P4 in constructing the concept of LQTV definition, and constructing the concept of LQTV modeling was done P1 and P2. The thinking process started from the planning stage. At this stage, students did the planning by understanding the problem and activating the mathematical objects (facts, concepts, principles, and procedures) that were relevant. Furthermore, students determined what problems exist in the item to determine the goals to be achieved in the process of thinking. After understanding the problem, students determined the appropriate completion strategy. In the process of applying the appropriate strategy, the

students monitored their thinking process. Supervision was carried out to see the correctness of thinking steps undertaken, then the students performed an evaluation to ensure the accuracy of the selected strategy.

b. Thinking process type B was done by P3 in constructing the concept of LQTV modelling and constructing LQTV completion concepts was done by P1. The thinking process started from the planning stage. At the planning stage, the student first identified the problem in question, it was done to determine the purpose of thinking that he did. After understanding the problem, students observed whatever information existed and related to the problem. It was done to enable the relevant mathematical objects (facts, concepts, principles, and procedures). The final step in the planning stage was to choose the appropriate strategy to get the right solution. In the process of applying the appropriate strategy, the students monitored their thinking process. Supervision was carried out to see the correctness of thinking steps undertaken, then the students performed an evaluation to ensure the accuracy of the selected strategy.

c. Thinking process type C was done by P4 in constructing the concept of LQTV modeling. The thinking process started from the planning stage. At this stage, student did the planning by understanding the problem and enabling the mathematical objects (facts, concepts, principles, and procedures) that were relevant. Furthermore, student determined what problems exist in the item to set the goals to be achieved in the process of thinking. After understanding the problem, student determined the appropriate completion

strategy. In the process of applying the selected strategy, the participant had difficulty thinking so that he returned to the process of understanding problems and enabling mathematical objects (facts, concepts, principles, and procedures) that were relevant. After the difficulty of thinking was resolved, the student resumed the strategy he selected to get the right results. In the process of applying the appropriate strategy, the student monitored their thinking process. Supervision was carried out to see the correctness of thinking steps undertaken, then the student performed an evaluation to ensure the accuracy of the selected strategy.

## 5. References

1. Flavell, J. H. 1979. Metacognition and Cognitive Monitoring. A New Area Cognitive-Developmental Inquiry. *AMERICAN PSYCHOLOGIST*, 34(10): 906-911.
2. Khodijah, N. 2006. *Psikologi Belajar*. Palembang: IAIN Raden Fatah Press Suriasumantri (ed). (online on <http://www.andragogi.com>, retrieved on May 17, 2016).
3. Kieran, C. (2004,). Algebraic Thinking in the Early Grades. *The Mathematics Educator*, 139 - 151.
4. Lian, L. H., Indris, N. (2006). Assesing Algebraic Solving Ability of Form Four Students. *International Electronic Journal of Mathematics Education*, 56-76.
5. Livingston, J.A. 1997. Metaconition: An Overview. [online]: <http://www.gse.buffallo.edu/fas/shuell/cep564/metacog.htm>. [February 17, 2017].

6. Marpaung, Y. 1986. *Proses Berpikir Siswa dalam Pembentukan Konsep Algoritma Matematis*. Makalah Pidato Dies Natalis XXXI IKIP Sanata Dharma Salatiga, 25 Oktober 1986.
7. Setiadi, H. et al., (2012). *Kemampuan Matematika Siswa SMP Indonesia Menurut Benchmark Internasional TIMSS 2011*. Jakarta: Pusat Penilaian Pendidikan Badan Penelitian dan Pengembangan Kementerian Pendidikan dan Kebudayaan.
8. Shodiq, L. J. et al. (2015). Analisis Soal Matematika TIMSS 2011 dengan Indeks Kesukaran Tinggi. *Seminar Nasional Pendidikan* (hal. -). Jember: University of Jember.
9. Suryabrata, S. 2004. *Psikologi Pendidikan*. Jakarta: PT. Raja Grafindo Persada.
10. Widadah, S. et al. (2013, April Senin). Profil Metakognisi Siswa dalam Menyelesaikan Soal Sistem Persamaan Linear Dua Variabel Berdasarkan Gaya Kognitif. *Vol.1, No.1*, 13-24.

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