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Inventory Periodic Review Model (nq, R, T) With Quadratic Backorder Costs

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ABSTRACT

At each review, the quantity ordered is a multiple of Q, nQ and the reorder level is R. The backorder cost $C_B(t)$ is taken as a question function of t, the length of time of the backorder. $C_B(t)$ is $b_1+b_2+b_3t^3$.

The paper firstly gives the basic mathematics required for the analysis. The demand for, is assumed to follow a normal distribution.

The expected backorder cost is derived.

The inventory costs are derived superlatively for the factors b_1, b_2, and b_3 of the quadratic costs. By setting the b_1, b_2 and b_3 to zero we have the inventory cost that is not time dependent.

Setting b_3 alone to zero, we obtain the inventory costs when the backorder cost is a linear function of the time of backorder.

Key words: Backorder costs.

INTRODUCTION

In this paper the cost depending upon the length of time for which the backorder exists is taken as a quadratic cost. Without inventories to meet orders, customers would have to wait until under orders were filled from a source or were manufactured. The time lag could result in quadratic cost.

Organizations that stored thousands of products could face severe inventory costs when the backorder cost is a quadratic function.

LITERATURE REVIEW

The simple models of economic backorder inventory control model and the (nQ,R,T) model for linear backorder model were extensively dealt with by Hadley and Whitin (1972).

Uthayakumar and Parrathi (2009) investigated a continuous review inventory model to reduce lead time, yield variability and set up costs simultaneously through capital investments. The backorder rate is depending on the lead time through the amount of shortage. Zhang and Dathwo (2003) developed a hybrid inventory system with a time limit in backorders.

BASIC MATHEMATICS

Basic Mathematics is well developed in Hadley and Whitin (2009)

$$Let\ Z_n\ (x_l T) = \int_0^T \frac{t^n}{\sqrt{2x\sigma^2 t}} esp = \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt \tag{1}$$

Integrating by parts we have

$$Z_n(x_l T) = \frac{t^{n+\frac{1}{2}}}{n+\frac{1}{2}} \frac{1}{\sqrt{2t\sigma^2}} esp - \frac{1}{2}\left(\frac{x-DL}{\sqrt{\sigma^2 t}}\right)^2\Bigg]_0^T$$

$$-\int_0^T \frac{2t^{n\frac{1}{2}}}{\sqrt{2x\sigma^2\left(n+t\frac{1}{2}\right)}}\left(\frac{x-DT}{\sqrt{\sigma^2 t}}\right)\left(\frac{D}{\sqrt{\sigma^2 t}} + \frac{(x-DT)}{2\sigma t^{3}/_{2}}\right) esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Simplifying

$$= \frac{2t^{n\frac{1}{2}}}{\sqrt{2\pi\sigma^2(2n+1)}} esp\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\Bigg]_0^T + \frac{D^2}{(2_{n+1})}\sigma^2 \int_0^T \frac{t^{n+1}}{\sqrt{2\pi\sigma^2 t}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

$$-\frac{x^2}{\sigma^2}\int_0^T \frac{t^{n-1}}{\sqrt{2\pi\sigma^2 t(2_{n+1})}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Hence

$$Z_n(x,T)(2n+1)\frac{\sigma^2}{D^2}$$

$$= \frac{2\sigma^2}{D^2}\frac{T^{n+1}}{\sqrt{2\pi\sigma^2 T}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 + Z_{n+1}(x,T) - \frac{X^2}{D^2} Z_{n-1}(x,T) \tag{2}$$

Hence we have

$$Z_{n+1}(x,T) = \frac{2\sigma^2 T^{n+1}}{D^2\sqrt{2\pi\sigma^2 T}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \tag{3}$$

$$+\left(\frac{2_{n+1}}{D^2}\right)\sigma^2 Z_n(x,T) + \frac{X^2}{D^2} Z_{n-1}(x,T)$$

$$n = 0,1,2 \ldots \ldots \ldots$$

Let $F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) = \int_x^\infty \frac{1}{\sqrt{2\pi\sigma^2 t}} esp \frac{-1}{2}\left(\frac{v-Dt}{\sqrt{\sigma^2 t}}\right)^2 dv$

Let $R_n(x,T) = \int_0^T t^n F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) dt \qquad , n = 0,1,2 \ldots$

Integrating by parts we have

$$R_n(x,T) = \frac{t^{n+1}}{n+1} F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\Bigg]_0^T$$

$$\frac{-1}{\sqrt{2\pi}}\int_0^1 \frac{t^{n+1}}{(n+1)\sqrt{\sigma^2 T}}\left(\frac{D}{\sqrt{\sigma^2 t}} + \frac{(x-Dt)}{2\sigma t^{3}/_{2}}\right) esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Hence substituting in $Z_{n+1} and T_n$ *(X,T)*

$$R_n(x,T) = \frac{T^{n+1}}{n+1} F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\frac{-D}{2(n+1)} Z_{n+1}(x,T)$$

$\frac{-x}{2(n+1)} Z_n(x,T) \qquad n = 0,1,2 \ldots \ldots \ldots \qquad (4)$

$$Z_0(x,T) = F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) - esp\left(\frac{20x}{\sigma^2}\right) F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$Z_1(x,T) = \int_0^T t\, F(x,Dt)dt$$

$$Z_0(x,T) = \int_0^T \frac{1}{\sqrt{2\pi\sigma^2 t^{1/2}}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

$$= \int_0^T \frac{t^{-1/2}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Integrating by parts and applying

$$\partial\left(esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right) = \left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\left(\frac{\mathrm{D}}{\sqrt{\sigma^2 \mathrm{t}}} + \frac{(\mathrm{x-Dt})}{2\sigma \mathrm{t}^{3/2}}\right) \mathrm{esp} \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$Z_0(x,T) = \left[\frac{2t^{1/2}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T$$

$$-2\int_0^T \frac{t^{1/2}}{\sqrt{2\pi\sigma^2}}\left(\frac{(x-Dt)D}{\sigma^2 t} + \frac{(x-Dt)^2}{2\sigma^2 t^2}\right) esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$$

Simplifying

$$Z_0(x,T) = \left.\frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T + \frac{D^2}{\sigma^2}\int_0^T \frac{t}{\sqrt{2\pi\sigma^2 t}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$$

$$-\frac{x^2}{\sigma^2}\int_0^T \frac{t^{-1/2}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$$

Since

$$Z_1(x,T) = \int_0^T \frac{t}{\sqrt{2\pi\sigma^2 t}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$$

Hence substituting Z_1 (x, T) into Z_0 (x,T)

$$Z_0(x,T) = \left.\frac{2t^{1/2}}{\sqrt{2\pi\sigma^2 t}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T + \frac{D^2}{\sigma^2} Z_1(x,T)$$

$$-\frac{x^2}{\sigma^2}\int_0^T \frac{t^{-1/2}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$$

But $\frac{\partial Z_0(x,T)}{\partial x} = -\frac{1}{\sqrt{2\pi}}\int_0^T \left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 \mathrm{dt}$

Hence substituting into Z_0(x,T) and simplifying

$$Z_0(x,T) = \frac{D^2}{\sigma^2} Z_1(x,T) + \frac{x\partial Z_0}{\partial x}(x,T) - \frac{Dx}{\sigma^2} Z_0(x,T) + \left[\frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T$$

Hence

$$Z_1(x,T) = \frac{\sigma^2}{D^2}\left[\frac{2t_2^1}{2\pi\sigma_2} esp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T + \frac{\sigma^2}{D^2}\left(1+\frac{Dx}{\sigma^2}\right) Z_0(x,T) - \frac{\sigma^2 x}{D^2}\frac{Dz_0(x,T)}{\partial x}$$

This gives Z,(x,T) in known quantities

Since $Z_0(x,T) = \frac{1}{D}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right) - \text{esp}\left(\frac{2Dx}{\sigma^2 T}\right) - \text{esp}\left(\frac{2Dx}{\sigma^2}\right) F\left(\frac{x-DT}{T}\right)\Big)$ (5)

Hence differentiating with respect to x

$$\frac{\partial Z_0(x,T)}{\partial x} = -esp\frac{-1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 - \frac{2D}{\sigma^2} esp\left(\frac{2Dx}{\sigma^2}\right) F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right) + \frac{1}{D\sqrt{2\pi\sigma^2 T}}\text{esp}\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

Hence substituting $Z_0(x,T) and \frac{\partial Z_0}{\partial x}$ into Z_1(x,T) and simplifying we have

$$Z_1(x,T) = \frac{-2\sqrt{\sigma^2 T}}{D^2} esp\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 + \frac{\sigma^2}{D^3}\left(1+\frac{Dx}{\sigma^2}\right) F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right) + \frac{1}{D^2}\left(x - \frac{\sigma^2}{D}\right) esp\left(\frac{2Dx}{\sigma^2 T}\right) F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) \quad (6)$$

From equation 2 letting n = 1

$$Z_2(x,T) = \frac{2\sigma^2 T^2}{D^2\sqrt{2\pi\sigma^2 T}}\text{esp}\frac{-1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 + \frac{3\sigma^2 Z_1}{D^2} Z_1(x,T) + \frac{x^2}{D^2} Z_0(X,T) \quad (7)$$

Similarly

$$Z_3(x,T) = \frac{2\sigma^2 T^3}{D^2\sqrt{2\pi\sigma^2 T}}\text{esp}\frac{-1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 + \frac{5\sigma^2 Z_2(x,T)}{D^2} + \frac{x^2}{D^2} Z_1(x,T)$$

Substituting for Z_2 (x,T) in Z_3 (x, T) we have

$$Z_3(x,T) = -\frac{2\sigma^3 T^3}{D^2\sqrt{2\pi\sigma^2 T}} esp\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 + \frac{5\sigma^2}{D^2}\left(\frac{-2\sigma^2 T^2}{D^2\sqrt{2\pi\sigma^2 T}}\text{esp}\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right.$$

$$+\frac{3\sigma^2}{D^2} Z_,(x,t) + \frac{x^2}{D^2} Z_,(x,T)$$

Simplifying and remembering that

$$g\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) = \frac{-1}{\sqrt{2\pi}}\text{esp}\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

We have

$$Z_3(x,T) = \frac{2\sqrt{\sigma^2 T}}{D^2}\left(T^2 + \frac{5\sigma^2 T}{D^2}\right) g\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) + Z_1(x,T)\left(\frac{x^2}{D^2}\right) + \frac{15\sigma^4}{D^4} Z_0(x,T) \quad (8)$$

For the periodic review model, the variance for a time interval of length t = $\sigma^2 t$ if the inventory position of the system is R + y immediately after the review at time t, then the expected backorder costs at time t + L

$$= \frac{1}{Q}\int_0^Q D\int_0^1 D\int_0^1 \frac{C_B(t-z)}{\sqrt{\sigma^2 T}} g\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) dzdtdY \quad (9)$$

Similarly the expected backorder costs at time t + L + T

$$= \frac{1}{Q}\int_0^Q D \int_0^L \frac{C_B(t-z)}{\sqrt{\sigma^2 T}}\, g\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) dzdtdY \qquad (10)$$

Nothing that $C_B\,(t) = b_1 + b_2 t + b_3 t^2$

And substituting into 9 and 10

Expected backorder costs at time t + L

$$= \frac{1}{Q}\int_0^Q D \int_0^1 D \int_0^1 \frac{C_B(t-z)}{\sqrt{\sigma^2 T}}\, g\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) dzdtdY \qquad (11)$$

And time t + L

$$= \frac{1}{Q}\int_0^Q D \int_0^{L+T} D \int_0^1 \frac{b_1+b_2(t-z)+b_3(t-2)^2}{\sqrt{\sigma^2 T}}\, \mathrm{g}\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) dzdtdY \qquad (12)$$

dealing with first integral is

$$= \frac{D}{Q}\int_0^Q \int_0^L (b_1 \frac{b_1\sigma^2 t}{D^2} - \frac{b_3\sigma^2 t}{D^2}\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right)^2 - \frac{b_2\sqrt{\sigma^2 t}}{D}\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$F\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) + \frac{\mathrm{D}}{\mathrm{Q}}\int_0^Q \int_0^L \left(\frac{\sqrt{\sigma^2 t b_3}}{D} - \frac{b_3\sigma^2 t}{D^2}\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right)\right)$$

$$\mathrm{g}\left(\frac{R+Y-Dt}{\sqrt{\sigma^2 t}}\right) dtdY \qquad (13)$$

Integrating with respect to Y we have

$$\frac{D\sqrt{\sigma^2 L}}{Q} b_1 \int_0^L \left(\mathrm{g}\left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right) - \left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right) F\left(\frac{R-Dt}{L}\right)\right) dt$$

$$\frac{D\sqrt{\sigma^2 L}}{Q} b_1 \int_0^L \left(\mathrm{g}\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 L}}\right) - \left(\frac{R+Q-Dt}{\sqrt{\sigma^2 L}}\right) F\left(\frac{R+Q-Dt}{L}\right)\right) dt$$

$$+ \frac{Db_2\sigma^2}{2DQ}\int_0^L t\left(\left(1+\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)^2\right) F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) - \left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\mathrm{g}\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt$$

$$\frac{b_2\sigma^2 D}{2DQ}\int_0^L t\left(\left(1+\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)^2\right) F\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right) - \left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right) * \mathrm{g}\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt$$

$$\frac{+b_3\sigma^2 D}{2D^2 Q} L\int_0^L t^{3/2}\left(\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 + 2\right)\mathrm{g}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) - \left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) *$$

$$\left(1+\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)^2\Big) F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\Big) dt$$

$$\frac{+b_3\sigma^2 D}{3^{D^2}Q}\int_0^L t^{3/2}\left(\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)^2\right) + 2)\mathrm{g}\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$-\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)\left(1+\left(\frac{R+QDt}{\sqrt{\sigma^2 t}}\right)^2\right)\mathrm{F}\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)\Big) dt \qquad (14)$$

Integrating the b_2 factor first

$$\frac{\sigma^2 L b_1}{DQ}\int_0^L \left(\mathrm{g}\left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right) - \left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right)\mathrm{F}\left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right)\right) \mathrm{dt}$$

Let $\mathrm{V} = \frac{R-Dt}{\sqrt{\sigma^2 L}}$

Then we have

$$-\frac{\sqrt{\sigma^2 Lb_1}}{DQ}\sqrt{\sigma^2 LD}\int_{\frac{R}{\sqrt{\sigma^2 t}}}^{\frac{R-Dt}{\sqrt{\sigma^2 L}}}\big(g(V)-VF(V)\big)dV$$

Nothing that

$$\int_k^\infty x^n F(x)dx = \frac{x^{n+1}}{n=1}F(x)\Big]_k^\infty + \int_k^\infty \frac{x^{n+1}}{n+1}g(x)dx$$

When n = 0

$$\int_k^\infty F(x)dx = g(k) - kF(k)$$

When n = 1

$$\int_k^\infty F(x)\,dx = \frac{1}{2}\big((1-k^2)F(k)+kg(k)\big)$$

When n = 2

$$\int_k^\infty x^2(x)\,dx = \frac{1}{2}\big((1-k^2)F(k)+kg(k)\big)$$

Then we have

$$-\frac{\sqrt{\sigma^2 Lb_1}\sqrt{\sigma^2 LD}}{DQ}\int_{Rl\sqrt{\sigma^2 L}}^{\frac{R-Dt}{\sqrt{\sigma^2 L}}}(\,g(V)-VF(V)dV \qquad (15)$$

Integrating and applying the above results we have

$$\frac{-b_1\sigma^1 LD}{DV}\Big[F(V)-\frac{1}{2}(1-V^2)F(V)+Vg\Big]_{\frac{R}{\sqrt{\sigma^2 L}}}^{\frac{R-Dt}{\sqrt{\sigma^2 L}}} \qquad (16)$$

Simplifying then we have

$$\frac{\mathrm{Db_1\sigma^2 L}}{\mathrm{2DQ}}\left(\left(1+\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)^2\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right)$$

Then the b_1 factor gives (14) and (16)

$$\frac{\mathrm{b_1\sigma^2 LD}}{\mathrm{2DQ}}\left(\left(1+\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)^2\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right)$$

$$\frac{\mathrm{-b_1\sigma^2 DL}}{\mathrm{2DQ}}\left(\left(1+\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 L}}\right)^2\right)F\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)-\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right) \qquad (17)$$

Take the b_2 factor and the expression for

$\frac{R-Dt}{\sqrt{\sigma^2 L}}$ only

We have

$$\frac{b_2}{2Q}\int_0^L \sigma^2 t\left(\left(1+\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)^2\right)\right)F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)g\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right)dt \qquad (18)$$

Simplifying we have

$$\frac{b_2 D}{2DQ}\int_0^L Q^2 t\left(\left(1+\frac{R^2}{\sigma^2 t}-\frac{2DRT}{\sigma^2 t}+\frac{D^2 t^2}{\sigma^2 t}\right)F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)-\left(\frac{R}{\sqrt{\sigma^2 t}}-\frac{Dt}{\sqrt{\sigma^2 t}}\right)\right)dt \quad (19)$$

$$Zn(R,L)= \int_0^L \frac{t^n}{\sqrt{\sigma^2 t}}\,\mathrm{g}\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)dt$$

and

$$R_n,(R,L)=\int_0^L t^n F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)dt$$

Then integrating (18) we have

$$\frac{Db_2}{2QD}\left(R_2(R,L)+(\sigma^2-2DR)R_2(R,L)+D^2R_2(R,L)-\sigma^2 RZ_1(R,L)+D\sigma^2 Z_2(R,L)\right)$$

substitute for R_0 (R,L), R_1 (R, L) and Z_2 (R,L)

from equation 6, 7, respectively

$$\frac{Db_2}{2QD}[R^2\left(LF\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\frac{\mathrm{D}}{2}\mathrm{Z}_1(\mathrm{R,L})-\frac{\mathrm{R}}{2}\mathrm{Z}_0(\mathrm{R,L})\right)+(\sigma^2+2DR)(\frac{L^2}{2}F\left(\frac{R-DL}{\sqrt{\sigma^2 t}}\right)+\frac{2\sqrt{\sigma^2 L}}{4D^2}.L.\mathrm{g}\left(\frac{R-DL}{\sqrt{\sigma^2 t}}\right)-$$
$$\frac{3\sigma^2}{4\mathrm{D}}Z_1(R,L)-\frac{R^2 Z_0}{4D}(R,L)-\frac{R}{4}Z_1(R,L)$$

$$+D^2\left(\frac{L^2}{3}F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)+\frac{2\sqrt{\sigma^2 L}}{D^2}\mathrm{g}\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{DL^2}{6}+\frac{s\sigma^2 L}{D}+\frac{RL}{6}\right)\right)$$

$$-D\left(\left(\frac{R^2}{6D}+\frac{13\sigma^2}{6D^3}+\frac{3\sigma^2 R}{2D^2}\right)Z_1(R,L)+\left(\frac{3\sigma^2 R^2}{6D^3}+\frac{R^2}{6D^2}\right)Z_0(R,L)\right)$$

$$-\sigma^2 RZ,(R,L)+D\sigma^2(\frac{-2\sqrt{\sigma^2 LL}}{D^2}\mathrm{g}\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)+\frac{3\sigma^2}{D^2}Z,(R,L)+\frac{R^2}{D^2}Z_0(R,L))) \quad (20)$$

simplifying we have

$$\frac{b_2}{2Q}\left(R^2L+(\sigma^2-2DR)\frac{L^2}{2}+\frac{D^2L^3}{3}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

$$-\frac{2b_2}{2QD^2\sqrt{\sigma^2 L}}\left(-\left(RD\frac{DL}{4}-D^2\left(\frac{DL^2}{6}\frac{3\sigma^2 L}{D}+\frac{RL}{6}\right)+D\sigma^2 L\right)*\mathrm{g}\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right.$$

$$\frac{+b_2}{2Q}\left(\frac{-DR}{2}-\frac{3\sigma^2}{4D}(\sigma^2-2DR)-D^2\left(\frac{R^2}{6D}+\frac{15\sigma^4}{6D^3}+\frac{3\sigma^2 R}{6D^2}\right)-\frac{R}{4}(\sigma^2-2DR)-\sigma^2 R+\frac{3\sigma^4}{D}Z,(R,L)\right.$$

$$\frac{b_2}{2Q}\left(R^2L+(\sigma^2-2DR)\frac{L^2}{2}+\frac{D^2L^3}{3}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) \quad (21)$$

substitute for Z, (R,L) and Z_0 (R,L) from 5,6,7 equation respectively then we have

$$\frac{b_2}{2Q}\left(R^2L+(\sigma^2-2DR)\frac{L^2}{2}+\frac{D^2L^3}{3}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

$$-\frac{b_2 2\sqrt{\sigma^2 L}}{2QD^2}\left(\frac{-\sigma^2 DL}{4}+\frac{D^2 RL}{2}-\frac{D^3L^2}{6}-5D\sigma^2 L\frac{-D^2RL}{6}+D\sigma^2 L\right)\mathrm{g}\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

$$+\frac{b_2}{2Q}\left(\frac{-DR^2}{2}-\frac{3\sigma^4}{4D}+\frac{6\sigma^2 R}{4}-\frac{DR^2}{6}-\frac{15\sigma^4}{6D}-\frac{\sigma^2 R}{2}-\sigma^2 R-\frac{3\sigma^4}{D}-\frac{\sigma^2 R}{4}+\frac{2DR^2}{4}\right.$$

$$\left(\frac{\sigma^2}{\sigma^2}\left(\frac{-R^3}{2}-\frac{R^2\sigma^2}{4D}+\frac{R^3}{2}-\frac{3\sigma^2R^2}{6D}-\frac{R^3}{6}+\frac{\sigma^2R^2}{D}\right)*\left(F\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)-\text{esp}\,\frac{2\text{DR}}{\sigma^2\text{L}}\right)\right) \quad (22)$$

Simplifying we have

$$+\frac{b_2}{Q}\left(\frac{D^2L^3}{6}-\frac{\sigma^4R}{6D^3}+\frac{DL^2R}{2}-\frac{\sigma^2R^2}{4D^2}+\frac{\sigma^2LR^2}{2}+\frac{R^3}{6D}-\frac{\sigma^6}{8D^4}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)$$

$$+\frac{b_2}{Q}\left(\sqrt{\sigma^2L}\left(\frac{DL^2}{6}-\frac{LR}{3}+\frac{R^2}{6D}+\frac{\sigma^2L}{12D}+\frac{\sigma^2R}{4D^2}+\frac{\sigma^4}{4D^3}\right)\text{g}\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)\right)+\frac{\sigma^6}{8D^4Q}esp\left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)$$

which we define as

$$\frac{b_2}{Q}G_3(R,L) \quad (23)$$

from equation (6) considering the b_3 factor and expression for $\frac{R-Dt}{\sqrt{\sigma^2L}}$ only we have

$$\frac{b_3}{3DQ}\int_0^L Q^2t\,{}^3\!/_2\left(\left(\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)^2+2\right)\text{g}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)-\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)(3+\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)^2)*\text{F}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)\right)dt \quad (24)$$

Expanding

$$=\frac{b_1}{3DQ}\int_0^L\left(\sqrt{\sigma^2t}\,(R^2-2R\,Dt+D^2t^2\right)+2\sigma^2t\right)\text{g}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)$$

$$-\left(\frac{\sigma^2tR}{D^2}-\frac{D\sigma^2t^2}{D^2}+\frac{R^3}{3D^2}-\frac{3R^2Dt}{3D^2}+\frac{3RD^2t^2}{3D^2}-\frac{D^3t^3}{3D^2}\right)F\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)dt \quad (25)$$

Re-arranging in powers of t we have

$$\frac{b_1}{3DQ}\int_0^L\frac{1}{\sqrt{\sigma^2t}}(R^2Q^2t-t^2(2D\sigma^2R-2\sigma^4)+D^2\sigma^2t^3)\text{g}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)dt$$

$$-\frac{b_3}{2D^2Q}\int_0^L((\sigma^2R-DR^2)t+t^2(RD)^2+\frac{R^3}{3}-\frac{t^3D^2}{3})\text{g}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)dt \quad (26)$$

Integrating we have and nothing that

$$\int_0^L t^n\,\text{g}\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)dt=Z_n(R,t)and\ \int_0^L t^n\,F\left(\frac{R-Dt}{\sqrt{\sigma^2t}}\right)dt=R_n(x,L) \quad (27)$$

We have

$$\frac{b_3}{3DQ}\left(R^2\sigma^2Z_1(R,L)-(2D\sigma^2R-2\sigma^4)Z_2(R,L)+D^2\sigma^2Z_3(R,L)\right)$$

$$-\frac{b_3}{DQ}(\sigma^2R-DR^2)R_1(R,L)+(RD^2-D\sigma^2)R_2(R,L)+\frac{R^3}{3}R_0(R,L)-\frac{D^2}{3}R_3(R,L)\Big)$$

Substituting for Z_2(R,L)

Z_3(R,L), R_1(R,L),R_2 (R,L), R_3(R,L) and R_0(R,L) from 8,6,6.6 respectively

Then we have, nothing that

$$\text{g}\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)=\frac{1}{\sqrt{2\pi}}exp\frac{-1}{2}\left(\frac{R-DL}{\sqrt{\sigma^2L}}\right)$$

Equation 27 gives

$$\frac{b_3}{3DQ}\left[\frac{R^2\sigma^4}{D^3}\left(1-\frac{DR}{\sigma^2}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\frac{2R^2\sigma^2\sqrt{\sigma^2 L}}{D^2}g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right]$$

$$+\frac{R^2\sigma^2}{D^2}\left(\frac{R-\sigma^2}{D}\right)esp\frac{2DR}{\sigma^2}F\left(\frac{R-Dt}{\sqrt{\sigma^2 L}}\right)\Bigg]+(2D\sigma^2R-2\sigma^4)\frac{2\sqrt{\sigma^2L}}{D^2}Lg\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\frac{G\sigma^2}{D^2}$$

$$(D\sigma^2R-\sigma^4)Z,(R,L)\frac{-2R^2}{D^2}(D\sigma^2R-\sigma^4)Z_0(R,L)$$

$$-2D^2\frac{\sigma^2\sqrt{\sigma^2L}}{D^2}\left(L^2+\frac{5\sigma^2L}{D^2}\right)g\left(\frac{R-D\sigma}{\sqrt{\sigma^2 L}}\right)+D^2\sigma^2\left(\frac{R^2}{D^2}+\frac{15\sigma^4}{D^4}\right)z_1(R,L)+\frac{3D^2\sigma^4}{D_4}R^2Z_0(R,L)$$

$$\frac{Db_3Z_0(R,L)}{Q}\left(\frac{-7\sigma^2R^3}{12D^3}+\frac{2\sigma^4R^2}{3D^4}-\frac{R^3}{12D^2}+\frac{5\sigma^4R^2}{6D^2}+\frac{5\sigma^2R^3}{8D^3}+\frac{R^4}{6D^2}-\frac{R^4}{24D^2}-\frac{35\sigma^4R^2}{24D^4}\right)$$

$$\frac{Db_3Z_1(R,L)}{Q}\left(\frac{7\sigma^2R^3}{12D^3}-\frac{R^2}{12D}-\frac{21\sigma^2R}{12D^2}+\frac{2\sigma^6}{D^4}-\frac{\sigma^2R^2}{4D^2}+\frac{15\sigma^6}{6D^4}+\frac{R^2}{8D}-\frac{15\sigma^4R}{8D^3}-\frac{7\sigma^2R^2}{24D^2}-\frac{105\sigma^6}{24D^4}-\frac{3\sigma^3R^2}{24D^2}\right)$$

$$\frac{2\sqrt{\sigma^2L}}{D^2}g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(-\frac{7\sigma^2RL}{12D^2}+\frac{2\sigma^4L}{3D^2}-\frac{R^2L}{12}+\frac{\sigma^2L^2}{6}+\frac{3\sigma^4L}{6D^2}+\frac{L^2RD}{8}+\frac{3\sigma^4RL}{8D}-\frac{D^2L^3}{24}-\frac{7\sigma^2L^2}{24}\right.$$

$$\left.-\frac{35\sigma^4L}{24D^2}-\frac{R^2L}{24}\right)+\left(\frac{L^2\sigma^2R}{2D^2}+\frac{L^2R^2}{2D}-\frac{L^3R}{3}+\frac{L^3\sigma^2}{3D}+\frac{R^3L}{3D^2}+\frac{L^3D}{12}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) \qquad (28)$$

Simplifying we have

$$\frac{Db_3}{Q}Z_0(R,L)\left(\frac{R^4}{24D^2}+\frac{\sigma^2R^2}{24D^3}+\frac{\sigma^4R^2}{24D^4}\right)+\frac{Db_2}{Q}Z,(R,L)\left(\frac{R^3}{24D}+\frac{\sigma^2R^2}{12D}+\frac{\sigma^4R}{8D^3}+\frac{\sigma^6}{8D^4}\right.$$

$$\frac{-2D\sqrt{\sigma^2L}}{D^2}g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{\sigma^2LD}{24D}+\frac{\sigma^4L}{24D^2}-\frac{R^2L}{8}-\frac{\sigma^2L^2}{8}+\frac{L^2RD}{8}-\frac{D^2L^3}{24}\right)\frac{b_3}{Q}$$

$$-\frac{Db_2}{Q}F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{-L^2\sigma^2R}{2D^2}+\frac{L^2D^2}{2D}-\frac{L^3R}{3}+\frac{L^3\sigma^2}{3D}-\frac{R^3L}{3D^2}+\frac{L^4D}{12}\right) \qquad (29)$$

Substituting 5 for $Z_0\,(R, L)$, 6 for $Z,(R,L)$ we have

$$\frac{b_3}{Q}\left(F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-esp\frac{2DR}{\sigma^2}F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\right)\left(\frac{R_4}{24D^2}+\frac{\sigma^2R^3}{24D^3}+\frac{\sigma^4R^2}{24D^4}\right)$$

$$\frac{b_3D}{Q}\left(\frac{\sigma^2}{D^3}\left(1+\frac{DR}{\sigma^2}\right)F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)-\frac{2\sqrt{\sigma^2L}}{D^2}g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)+\frac{1}{D^2}\left(\frac{R-\sigma^2}{D}\right)esp\frac{2DR}{\sigma^2}F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right)\right.$$

$$\left(\frac{R^3}{24D}+\frac{\sigma^2R^2}{12D^2}+\frac{\sigma^4R}{8D^3}+\frac{\sigma^3}{8D^4}\right)-\frac{2D\sqrt{\sigma^2L}}{D^2}g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{\sigma^2LR}{24D}+\frac{\sigma^4L}{24D^2}+\frac{R^2L}{8}-\frac{\sigma^2L^2}{8}+\frac{L^2RD}{7}-\frac{D^2L^2}{24}\right.$$

$$+DF\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(-\frac{\sigma^2L^2R}{2D^2}+\frac{L^2R^2}{2D}-\frac{L^2R}{3}+\frac{L^2\sigma^2}{3D}-\frac{R^2L}{3D^2}+\frac{L^4D}{12}\right) \qquad (30)$$

Simplifying we have

$$\frac{b_3D}{Q}\left(\frac{R^4}{12D^2}+\frac{\sigma^2R^2}{6D^4}+\frac{\sigma^2R^2}{4D^5}+\frac{\sigma^6R}{4D^8}+\frac{\sigma^8}{8D^7}-\frac{L^2\sigma^2R}{2D^2}+\frac{L^2R^2}{2D}-\frac{RL^3}{3}+\frac{L^3\sigma^2}{3D}-\frac{R^3L}{3D^2}+\frac{L^4D}{12D^4}+\frac{\sigma^4R}{8D^6}+\frac{\sigma^6}{8D^6}\right)$$

$$\frac{-b_3}{Q}2\sqrt{\sigma^2L}.Dg\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left(\frac{\sigma^2RL}{24D^3}+\frac{\sigma^4R}{24D^4}+\frac{R^2L}{8D^2}-\frac{\sigma^2L^2}{8D^2}+L^2R-\frac{L^3}{24}+\frac{R^3}{24D^3}+\frac{\sigma^2\sigma^2}{24D^4}+\frac{\sigma^4R}{8D^6}+\frac{\sigma^6}{8D^6}-\right.$$
$$\frac{1}{8}\frac{\sigma^3}{D^6}\frac{b_3}{Q}esp\frac{2DR}{\sigma^2}F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) \qquad (31)$$

If we define G_{11}(R,L) in such way that

$$\frac{b_3}{Q}G_{11}(R,L)equals\ equation\ 25$$

Hence from 25 expected backorder cost for the b_3 factor equals

$$\frac{b_3}{Q_{11}}G_{11}(R,L)-\frac{b_3}{Q}G_{11}(R+Q,L) \qquad (32)$$

Similarly from (4) expected backorder cost at time t + L + T for the b_3 factor

$$\frac{b_3}{Q}\left(G_{11}\left(R,T+L\right)-G_{11}(R+Q,T+L)\right) \qquad (33)$$

No of cycles = $1/_T$

Hence expected backorder costs per year excluding the cost based on the number of stockout is

$$=\frac{b_1}{QT}\left(G_1(R,T+L)-G_1(R,L)-G_1(R+Q,T+L)+G_1(R+Q,L)\right)$$

$$+\frac{b_2}{QT}\left(G_3(R,T+L)-G_3(R,L)-G_3(R+Q,T+L)+G_3(R+Q,L)\right)$$

$$\frac{b_3}{QT}\left(G_{11}(R,T+)-G_{11}(R,L)-G_{11}(R+Q,T+L)+G_{11}(R+Q,L)\right)$$

The inventory cost for the quadratic backorder costs is equal to the inventory costs for the linear backorder costs plus the factor

$$\frac{RC}{T}+\frac{SPORT}{T}+hc\left(Q+R-DL-\frac{DT}{2}\right)+\frac{b_1}{QT}\left(G_1(L,R+L)-G,(R,L)-G,(R+Q,T+L)+G,(R,L)\right)$$

$$+\frac{(hc+b_2)}{QT}\left(G_3(R,T+L)-G_3(R+Q,T+L)+G_3(R+Q,L)\right)$$

$$+\frac{b_3}{QT}\left(G_{11}(R,T+L)-G_{11}(R,L)-G_{11}(R+Q,T+L)+G_{11}(R+Q,L)\right)$$

$$+\frac{s}{QT}(R,T+L)-G_4(R,L)-G_4(R+Q,T+L)+G_4(R+Q,L)) \qquad (35)$$

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