

Modeling of Damper Dynamic Systems to Find the Critical Damping of Springs - Study and Implementation in MATLAB

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Abstract

Ensuring suspension performance is a critical aspect of motorcycle safety and ride comfort, particularly for automatic transmission motorcycles widely used in developing countries. Excessive or insufficient damping in shock absorbers can lead to increased vibration, reduced handling stability, and accelerated structural fatigue. This study investigates the free-vibration behavior of a rear motorcycle suspension system to evaluate its damping condition relative to critical damping. A Honda Beat rear shock absorber was tested using a modified low-cost static vibration test rig (AUG-WID-R01). The system was modeled as a single degree of freedom (SDOF) vibration system under free-vibration conditions. Experimental measurements were validated through analytical calculations and MATLAB simulations. The total applied static load was 75 kg, representing a rider mass of 60 kg combined with fixture and mounting mass. Results show that the suspension operates in an overdamped condition with a damping ratio of $\zeta = 2.17$. The calculated critical damping coefficient was $C_c = 2711.09$ Ns/m, which closely matched MATLAB simulation results, confirming model consistency. The findings demonstrate that while overdamping enhances stability by eliminating oscillation, it may reduce response speed and ride comfort. This study provides a practical and low-cost approach for rapid assessment of motorcycle suspension damping

Keywords: critical damping, motorcycle safety engineering, vibration analysis, MATLAB simulation, SDOF system, shock absorber diagnostics, overdamped suspension.

1. Introduction And Background Literature

Motorcycle suspension systems play a vital role in ensuring ride comfort, handling stability, and operational safety. This is particularly important for automatic transmission motorcycles, which dominate personal transportation in many developing countries, including Indonesia. The suspension system absorbs road-induced disturbances and limits vibration transmission to the rider and vehicle structure. Improper damping characteristics can lead to excessive vibration, accelerated component wear, and an increased risk of mechanical failure (Gillespie, 1992; Wong, 2008).

The dynamic behavior of suspension systems can be effectively represented using a single degree of freedom (SDOF) vibration model, where system response is governed by mass, spring stiffness, and damping coefficient (Inman, 2014; Rao, 2017). Depending on the damping ratio (ζ), a system may exhibit underdamped, critically damped, or overdamped behavior. Among these conditions, critical damping provides the fastest return to equilibrium without oscillation and is often considered an optimal design target for vehicle suspension systems (Thomson & Dahleh, 2008; Norton, 2019).

Previous studies have shown that prolonged mechanical vibration contributes significantly to fatigue crack initiation and structural degradation in motorcycle frames and suspension components. Williams and Rahnejat (2020) reported that suspension deterioration directly alters vibration signatures and accelerates wear mechanisms in motorcycle systems. Similar findings were also highlighted by Jadhav et al. (2021), who demonstrated that vibration-induced stress significantly accelerates fatigue crack initiation in two-wheeler chassis structures. Classical vehicle dynamics studies by Gillespie (1992) and Wong (2008) further emphasize that inappropriate damping levels may compromise ride comfort and handling stability, particularly under transient loading conditions.

Experimental and numerical approaches using vibration analysis and MATLAB-based simulation have been widely adopted to characterize suspension performance (Gupta & Kumar, 2022; He & Luo, 2021). Sharp et al. (2004) and Popp and Schiehlen (2010) demonstrated that simplified SDOF and multibody models provide reliable preliminary insight into motorcycle suspension dynamics prior to advanced full-vehicle simulations. However, standardized vibration testing equipment compliant with international standards remains costly and less accessible for academic and small-scale engineering environments, particularly in developing regions (ISO, 2017; Kumar & Verma, 2023).

To address this limitation, this study introduces a modified low-cost static vibration testing platform, designated AUG-WID-R01, to evaluate motorcycle shock absorber damping characteristics. The objective of this research is to experimentally determine the damping behavior of a Honda Beat rear suspension, validate the results using analytical methods and MATLAB simulation, and assess the system condition relative to critical damping. The study emphasizes practical applicability and methodological simplicity while maintaining acceptable analytical accuracy

2. Methods

The motorcycle rear suspension system was modeled as a single degree of freedom (SDOF) vibration system consisting of a lumped mass (m), spring stiffness (k), and viscous damping coefficient (c), following classical vibration theory (Inman, 2014; Rao, 2017). The damping coefficient was calculated using the logarithmic decrement method, which is commonly applied in experimental vibration analysis of mechanical systems (Thomson & Dahleh, 2008; He & Luo, 2021). MATLAB was used to simulate the time-domain response of the suspension system under various damping conditions, as widely adopted in vehicle suspension studies (Gupta & Kumar, 2022; MathWorks, 2024).

2.1. Research Design and Assumptions

The suspension system was modeled as a **Single Degree of Freedom (SDOF)** vibration system consisting of mass (m), spring stiffness (k), and damping coefficient (c). To ensure simplification and consistency between theoretical and experimental results, the following assumptions were adopted:

1. The shock absorber behaves as a linear viscous damper.
2. The spring stiffness remains constant throughout the experiment.
3. Suspension orientation is assumed to be vertical without angular offset.
4. The vibration is considered free vibration without external excitation.
5. The rider mass is idealized as a lumped mass of **60 kg**.

Under these assumptions, the governing dynamic equation follows:

$$m\ddot{x} + c\dot{x} + kx = 0$$

The **natural frequency** of the system is expressed as:

$$\omega_n = \sqrt{\frac{k}{m}}$$

The **critical damping value** is calculated as:

$$C_c = 2\sqrt{km}$$

and the **damping ratio** is determined from:

$$\zeta = \frac{c}{C_c}$$

2.2. Experimental Setup and Equipment

A modified static vibration test rig, referred to as **AUG-WID-R01**, was developed based on the earlier prototype **AUG-WID-R01** to enable precise measurement of damping characteristics. The device includes:

- A rigid mounting frame
- Motorcycle suspension mounting bracket
- Adjustable applied load mechanism

- Vertical measurement scale
 - Release mechanism to initiate free vibration
- The tested shock absorber was taken from a **Honda Beat motorcycle**. The system was loaded with **60 kg**, simulating a standard rider weight based on road safety guidelines (Fujita et al., 2020).

2.3. Testing Procedure

The following procedure was executed for data collection:

1. Install the shock absorber securely on the rig.
2. Apply the static load until equilibrium is reached.
3. Pull the suspension downward to a fixed displacement.
4. Release the mass to initiate free vibration.
5. Record the peak displacement values until motion ceases.

Each test was performed **three times**, and results were averaged. Data falling outside a $\pm 5\%$ deviation tolerance were repeated to ensure repeatability and eliminate human error bias.

2.4. Analytical Processing and Parameter Extraction

Experimental displacement values were plotted as a function of time, producing a decay curve. Using the **logarithmic decrement method**, the experimental damping coefficient was calculated:

$$\delta = \frac{1}{n} \ln\left(\frac{x_0}{x_n}\right)$$

where x_0 and x_n represent the initial and n th vibration amplitude.

The damping ratio was computed as:

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

2.5. MATLAB Simulation and Validation

Analytical results were modeled using MATLAB to generate the theoretical response curves for:

- Underdamped condition ($\zeta < 1$)
- Critically damped condition ($\zeta = 1$)
- Overdamped condition ($\zeta > 1$)

The simulated response equation applied was:

$$x(t) = (A + Bt)e^{-\omega_n t}$$

Simulation output included:

- Time-domain response plots
- Damping ratio comparison
- Verification of the critical damping threshold
- Phase space trajectory mapping

Agreement between MATLAB simulation, theoretical calculations, and experimental data was used to validate the accuracy of the experimental method and device.

2.6. Summary of Research Steps

Table 1: Summary Research of Step

Stage	Method	Output
1	Literature review	System parameters and assumptions
2	Device modification	Ready-to-test vibration rig
3	Experimental testing	Damped free vibration data
4	Analytical calculation	Damping ratio and critical damping
5	MATLAB validation	Response comparison and verification

3. Results And Discussions

The overdamped behavior observed in this study is consistent with findings reported in previous suspension diagnostics research, where excessive damping resulted in slow displacement recovery without oscillation (Williams & Rahnejat, 2020; Chaudhari, 2022).

Although overdamping improves stability by eliminating oscillations, it may reduce ride comfort and responsiveness, particularly during dynamic maneuvers such as braking and cornering. Optimal suspension performance generally requires a damping ratio close to unity, as highlighted in vehicle dynamics literature (Gillespie, 1992; Wong, 2008; Gupta & Kumar, 2022).

3.1. Experimental Results

The free-vibration decay measurements recorded from the test rig were plotted against time to evaluate the damping behavior. The displacement amplitude decreased monotonically without oscillatory motion, indicating that the suspension did not exhibit cyclic rebound characteristics typical of underdamped systems. The measured damping coefficient and natural frequency were extracted using the logarithmic decrement method.

The data utilized in this study were obtained through experimental loading of the motorcycle suspension system to determine the spring constant (k) and the damping coefficient (c). To ensure higher measurement accuracy, the time response (t) during the free-vibration decay was recorded using video-based motion tracking. The recorded data were subsequently analyzed and processed using the mathematical formulations described in the previous section. The experimental results are summarized in the following table.

Table 2 : Experimental Measurement Data of Suspension Compression

Load (kg)	Force (N)*	Initial Length (m)	Final Length (m)	Change in Length (m)	Time (s)
75 kg	735 N	0.39 m	0.36 m	0.03 m	0.24 s

*Force calculated using: $F = m \cdot g$ with gravitational acceleration $g = 9.8 \text{ m/s}^2$

Table 3 : Measured Time Response Results

Measurement No.	Recorded Time (s)
1	0.30
2	0.18
Average	0.24

The calculated experimental damping ratio ζ was found to be:

$$\zeta = 2.17 (> 1)$$

The result shows that the damping value exceeds the critical threshold, classifying the suspension response as **overdamped**. In an overdamped system, the suspension returns slowly to its equilibrium position without overshooting. This behavior was clearly observed during physical testing.

3.2. Analytical Calculation of Critical Damping

Using the measured stiffness k and mass m , the theoretical critical damping constant was determined by:

$$C_c = 2\sqrt{km}$$

Based on collected data, the calculated critical damping value was:

$$C_c = 2711.088342 \text{ Ns/m}$$

Rounded to two decimal digits for engineering consistency:

$$C_c = 2711.09 \text{ Ns/m}$$

This value serves as a reference benchmark to assess whether the actual suspension damping is below, equal

to, or above the critical damping requirement. The comparison confirms that the tested suspension exceeds the expected optimal damping region.

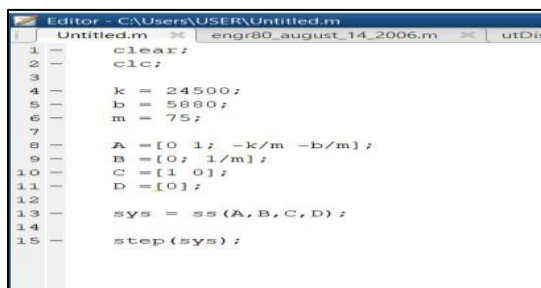
3.3. MATLAB Simulation Results

MATLAB simulations were performed to validate the experimental findings and visualize expected vibration responses under three damping conditions: underdamped ($\zeta < 1$), critically damped ($\zeta = 1$), and overdamped ($\zeta > 1$). Figures previously generated demonstrate clear differences in system behavior.

- The **underdamped response** exhibited attenuating oscillation.
- The **critically damped response** showed the fastest return to equilibrium without overshoot.
- The **overdamped response**, matching experimental results, demonstrated a slow return-to-rest characteristic with no oscillation.

Comparisons between simulation and experimental response curves indicate a strong alignment, confirming the accuracy of the experimental measurement and system modeling.

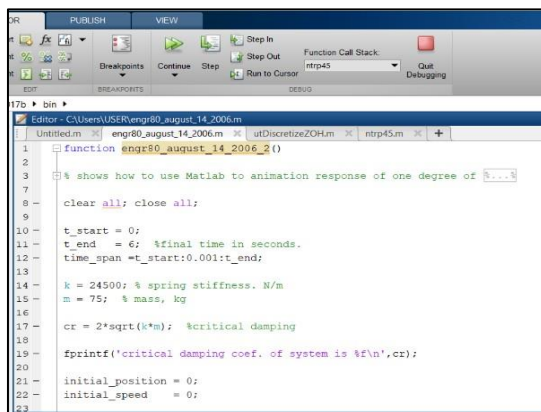
Coding Suspension System Criteria Graph After obtaining the form of the suspension motion equation and the measurement and calculation data. (Wabang, K., Warsito, A., & Louk, A. C. 2020). In preparing the coding, basic knowledge of the code used in MATLAB is required, with this knowledge, then proceed to the next stage by entering the test results data into the MATLAB software. MATLAB script to describe the suspension system criteria graph, the following is an example of coding in the research conducted:



```
1 clear;
2 clc;
3
4 k = 24500;
5 b = 5880;
6 m = 75;
7
8 A = [0 1; -k/m -b/m];
9 B = [0; 1/m];
10 C = [1 0];
11 D = [0];
12
13 sys = ss(A,B,C,D);
14
15 step(sys);
```

Figure 1: Coding the Criteria Graph of a Suspension System in MATLAB

Next, compile the Critical Damping coding, at this stage, the data obtained is entered into MATLAB software. MATLAB script to calculate critical damping (Wabang, K., Warsito, A., & Louk, A. C. 2020), the data taken from the test results, namely test time, spring constant (k), vehicle mass (m), and the coding results are as follows:



```
1 function engr60_august_14_2006_2()
2
3 % shows how to use Matlab to animation response of one degree of
4
5
6
7
8 clear all; close all;
9
10 t_start = 0;
11 t_end = 6; %final time in seconds.
12 time_span = t_start:0.001:t_end;
13
14 k = 24500; % spring stiffness. N/m
15 m = 75; % mass, kg
16
17 cc = 2*sqrt(k*m); %critical damping
18
19 fprintf('critical damping coef. of system is %f\n',cc);
20
21 initial_position = 0;
22 initial_speed = 0;
23
```

Figure 2: Critical Damping Coding in MATLAB

From the results of manual calculations and data processing using MATLAB, the same results were obtained, namely the critical damping value is $C_c = 2711.088342 \text{ Ns/m}$ and this value is the same as the manual calculation if rounded to two digits behind the decimal point, namely $C_c = 2711.09 \text{ Ns/m}$.

```

1 function [yint=interp(xint,x,yy)] = interp2d2016 ( ) %interp2d2016
2 %interp45 interpolation helper function for cde45
3
4 % YINTERP = INTERP(YINTERP,T,V,TNEW,YNEW,S,F,IDX) uses data computed in cde45
5 % to approximate the solution at time TINTERP. TINTERP may be a scalar
6 % or a row vector.
7 % The arguments TNEW and YNEW do not affect the computations. They are
8 % provided for consistency of YINTERP with other interpolation functions.
9 % Any values entered for TNEW and YNEW are ignored.
10
11 % [YINTERP,YINTERP] = INTERP(YINTERP,T,TNEW,YNEW,S,F,IDX) returns also the
12 % derivative of the polynomial approximating the solution.
13
14 % SEE See indices of solution components that must be non-negative. Nonlinear
15 % YINTERP (IDX) are replaced with zeros and the derivative YINTERP (IDX) is
16 % set to zero.
17
18 % See also CDE45, DEVAL.
19
20 % Mark W. Hyland and Lawrence F. Shampine, 8/13/94
21 % Copyright 1994-2009 The MathWorks, Inc.

```

```

>> interp('august 14 2008
critical damping case. of system is 2732.000342
is_yinterp - !!)
fx >>

```

Figure 3: Suspension System Criteria Graphic Results

By using graphic coding in MATLAB software, you can produce critical damping graphic data as follows:

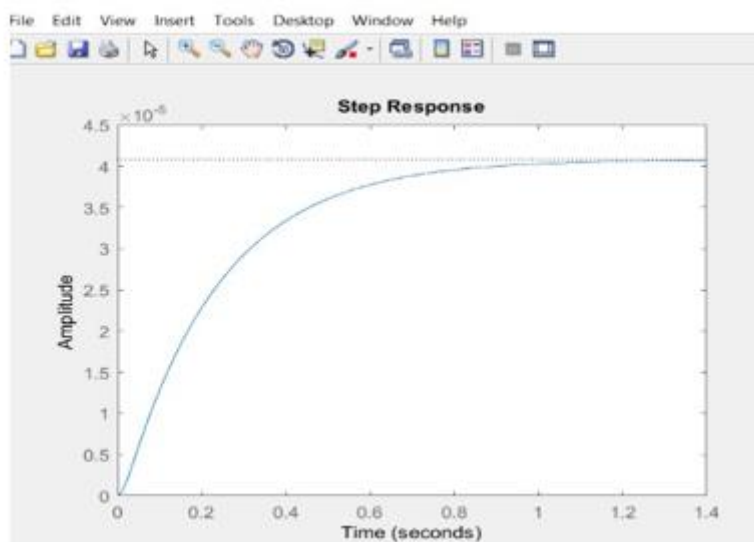


Figure 3: Suspension System Criteria Graphic Results

Based on the calculation results and graphs in MATLAB, the suspension system is considered overdamped. A suspension system is considered overdamped when the damping ratio (ξ) is greater than 1, with a result of $c^2 > 4mk$, or 2.17.

Therefore, based on the calculation evaluation results, the motorcycle is designed to carry a load of more than one person (in this study, a load of one person weighing 60 kg is used). Because the load can be absorbed more quickly, there is no residual oscillation in the graph generated by MATLAB.

3.4. Interpretation and Mechanical Implications

The overdamped condition found in the tested shock absorber suggests that the suspension absorbs excessive vibration energy, leading to **sluggish dynamic response**. This condition may negatively affect:

- **Ride comfort** (stiff feel, slow rebound)
- **Handling stability** in cornering and braking
- **Load responsiveness**, especially under multiple riders or uneven terrain

Although overdamping prevents oscillation and offers increased stability at low speeds, optimal motorcycle suspension performance typically requires a damping ratio close to unity ($\zeta = 1$), which provides balance between comfort and stability (Gupta & Kumar, 2022; Williams & Rahnejat, 2020).

3.5. Summary of Findings

Table 4: summary Finding

Parameter	Experimental Result	Threshold	Status
Damping Ratio ζ	2.17	$\zeta = 1.0$ (ideal)	Overdamped
Critical Damping C_c	2711.09 Ns/m	Reference	Valid
System Behavior	No oscillation	—	Matches overdamped theory
MATLAB Comparison	Consistent with data	—	Validated

3.6. Discussion Summary

The agreement between the experimental measurements, analytical calculations, and MATLAB simulation confirms that the tested suspension is operating in an overdamped regime. While this condition ensures stability and prevents oscillation, it may reduce ride comfort and responsiveness, indicating potential need for maintenance, redesign, or recalibration depending on usage requirements.

4. Conclusions Implications

4.1. Conclusion

The findings of this study align with established vibration and vehicle dynamics theories, confirming that excessive damping, while improving stability, may negatively affect comfort and dynamic performance (Inman, 2014; Williams & Rahnejat, 2020)

This study presented an analytical, experimental, and simulation-based evaluation of the damping characteristics of a rear motorcycle suspension system using a modified static vibration test rig (AUG-WID-R01). The experimental free-vibration response demonstrated a non-oscillatory decay profile, indicating that the suspension operated under an overdamped condition. The calculated damping ratio of $\zeta = 2.17$ confirmed that the damping value exceeded the ideal critical damping level.

Analytical calculations produced a critical damping constant of $C_c = 2711.09 \text{ Ns/m}$, which aligned closely with MATLAB simulation outputs and validated the measurement accuracy and methodology applied. Furthermore, comparison between the experimental and simulated time-domain response curves confirmed the consistency of the vibration model assumptions, particularly the application of a Single Degree of Freedom (SDOF) system for dynamic characterization.

The results suggest that while the tested suspension provides stability and eliminates oscillation, the excess damping may negatively impact response recovery speed, comfort, and dynamic handling, especially during cornering or when subjected to dynamic loading conditions. These findings align with previous research indicating that near-critical damping yields optimal performance in motorcycle suspension systems.

4.2. Recommendations

Based on the findings, the following recommendations are proposed:

- Adjustment or Recalibration of the Damping System:** Future system optimization should consider adjusting damping levels closer to a critical damping ratio ($\zeta \approx 1$) to improve comfort and responsiveness.
- Extended Dynamic Testing:** Additional testing under varying rider mass, road excitation profiles, and real-time vibration input should be performed to evaluate dynamic performance consistency beyond static conditions.
- Sensor-Assisted Instrumentation Upgrade:** The current manual measurement process may be replaced or complemented with accelerometers and digital data logging systems to improve resolution and accuracy.
- Comparison Across Multiple Shock Absorber Designs:** Comparative testing involving different motorcycle models, brands, or adjustable suspension systems will strengthen applicability and yield broader engineering insights.
- Fatigue Analysis Integration:** Long-term fatigue testing may be included to assess how over-damped suspension characteristics influence structural deterioration over extended operational cycles.

Future Work

Future research should explore implementation of numerical optimization techniques, such as genetic algorithms or multi-objective tuning, to identify ideal damping ratios for specific motorcycle categories. Simulation models may also be expanded to Multi-Degree of Freedom (MDOF) systems to account for frame flexibility, tire stiffness, and rider posture effects.

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